Exploring Discrete Approximations of Manifolds in Computational Geometry

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1 Introduction

In this paper, we delve into the relationship between continuous manifolds and their discrete approximations, particularly focusing on the mesh functor g_θ and its role in computational geometry.

2 Mesh Functor g_θ and its Properties

The mesh functor g_θ, given a parameter θ, discretizes a continuous manifold M into a finite triangular mesh P. This process is crucial in computational applications where exact manifold representations are infeasible.

2.1 Definition and Continuity

The functor g_θ can be formally defined as follows:

$$g_\theta : \mathbf{Man} \to \mathbf{SimpCplx}$$

where **Man** denotes the category of 3-dimensional manifolds and **SimpCplx** the category of simplicial complexes. This functor preserves essential topological and geometric features of M, albeit in a discretized form.

2.2 Error Analysis

An important aspect of g_θ is the error introduced during the discretization. The Gromov-Hausdorff distance provides a measure of this error, offering insights into the fidelity of P as an approximation of M.

3 Comparative Analysis

3.1 Manifold M versus Mesh P

The fidelity of P in representing M is crucial for algorithms in computational geometry. Despite the inherent approximation, P maintains most of the critical geometric properties of M, allowing for effective algorithmic processing.

3.2 Algorithmic Considerations

The discrete nature of P introduces unique computational challenges and opportunities. Algorithms must be adapted to handle the discrete structure efficiently, balancing accuracy and computational complexity.

4 Conclusion

Our exploration underscores the significance of the mesh functor g_θ in bridging the gap between theoretical manifolds and their practical computational counterparts. This understanding is pivotal for advancing methods in computational geometry and related fields.