

Appendix

1 Relationship between the manifolds and discrete approximations

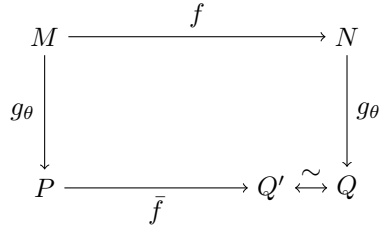


Figure 1:

M : cell surface

N : convex hull of M

P : discrete approximation of M

Q : discrete approximation of N

Q' : convex hull of P

2 Preliminaries

2.1 Hausdorff distance

Let M be a 3-dimensional manifold, and N be the convex hull of M . Since M and N are both compact and Riemann in \mathbb{R}^3 , the Hausdorff distance between M and N can be defined as stated in Chap 7 in [1]:

Definition 1. *Let M and N be two compact subsets of \mathbb{R}^3 . The Hausdorff distance between M and N is defined as*

$$d_H(M, N) = \max \left\{ \sup_{x \in M} \inf_{y \in N} |x - y|, \sup_{y \in N} \inf_{x \in M} |x - y| \right\}$$

We are to prove that the Hausdorff distance between N and Q' is small enough. From the metric geometry, the geometric properties between two man-

ifolds have tiny deviations controlled by the Hausdorff distance between them, in which context the location of minima of the distance function is included.

2.2 Functors between manifold category \mathbf{Man} and simplicial complex category $\mathbf{SimpCplx}$

Denote the category of 3-dimensional manifolds as \mathbf{Man} , and the convex-hull morphism f is defined as

$$\begin{aligned} f : \mathbf{Man} &\rightarrow \mathbf{Man} \\ M &\mapsto N \end{aligned}$$

where N is the convex hull of M . Note that f is a continuous morphism [2].

Given θ , a mesh map g_θ :

$$\begin{aligned} g_\theta : \mathbf{Man} &\rightarrow \mathbf{SimpCplx} \\ M &\mapsto P \end{aligned}$$

where P is the discrete approximation of M with mesh size θ . If Q' were the same as Q , then a good approximation functor would be defined, and any property of M, N can be derived from $P, Q(= Q')$, respectively. Though Q' is not necessarily the same as Q , we can still prove that $d_H(N, Q')$ is small enough.

3 Proof of which Q' is a good approximation of N

Proposition 1. $\lim_{\theta \rightarrow 0} d_H(N, Q') = 0$

Proof. The proof is from the continuity of f . Since g_θ is a θ -approximation, $d_H(M, P) < 2\theta$. Then $\forall \epsilon > 0$, there exists $\theta > 0$ such that $d_H(M, P) < \epsilon$. \square

References

- [1] Burago, D., Burago, Y., Ivanov, S., 2001. *A course in metric geometry*, Graduate Studies in Mathematics. American Mathematical Society, Providence, Rhode Island. <https://doi.org/10.1090/gsm/033>
- [2] Bobenko, A.I. (Ed.), 2016. *Advances in discrete differential geometry*. Springer Berlin Heidelberg, Berlin, Heidelberg. <https://doi.org/10.1007/978-3-662-50447-5>