Exploring Discrete Approximations of Manifolds in Computational Geometry

Biber

1 Introduction

In this paper, we delve into the relationship between continuous manifolds and their discrete approximations, particularly focusing on the mesh functor g_θ and its role in computational geometry.

2 Mesh Functor g_θ and its Properties

The mesh functor g_θ, given a parameter θ, discretizes a continuous manifold M into a finite triangular mesh P. This process is crucial in computational applications where exact manifold representations are infeasible.

2.1 Definition and Continuity

The functor g_θ can be formally defined as follows:

$$g_\theta : \mathbf{Man} \to \mathbf{SimpCplx}$$

where **Man** denotes the category of 3-dimensional manifolds and **SimpCplx** the category of simplicial complexes. This functor preserves essential topological and geometric features of M, albeit in a discretized form.

2.2 Error Analysis

An important aspect of g_θ is the error introduced during the discretization. The Gromov-Hausdorff distance provides a measure of this error, offering insights into the fidelity of P as an approximation of M.

3 Comparative Analysis

3.1 Manifold M versus Mesh P

The fidelity of P in representing M is crucial for algorithms in computational geometry. Despite the inherent approximation, P maintains most of the critical geometric properties of M, allowing for effective algorithmic processing.

3.2 Algorithmic Considerations

The discrete nature of P introduces unique computational challenges and opportunities. Algorithms must be adapted to handle the discrete structure efficiently, balancing accuracy and computational complexity.[1]

4 Conclusion

Our exploration underscores the significance of the mesh functor g_θ in bridging the gap between theoretical manifolds and their practical computational counterparts. This understanding is pivotal for advancing methods in computational geometry and related fields. [2]

References

[1] D. Coeurjolly, S. Miguet, and L. Tougne, "Discrete Curvature Based on Osculating Circle Estimation," in *Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, C. Arcelli, L. P. Cordella, and G. S. Di Baja, Eds., red. by G. Goos, J. Hartmanis, and J. Van Leeuwen, vol. 2059, 2001, pp. 303–312, ISBN: 3-540-42120-3. DOI: 10.1007/3-540-45129-3_27.

[2] Y. Wu, E. Dobriban, and S. B. Davidson, "DeltaGrad: Rapid retraining of machine learning models," in *37th International Conference on Machine Learning (ICML)*, vol. PartF16814, Jun. 30, 2020, pp. 10 286–10 297. DOI: 10.48550/arxiv.2006.14755. arXiv: 2006.14755 [cs, stat].