

Detection of 3D circles / Ellipses in a photo

Supervisor: Prof. YAN Hong

WU Zihan

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Electrical Engineering



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Project Target

To detect ellipses in the images/videos.



Figure: Input



Figure: Output

Ellipse I

To describe an ellipse we need 5 parameters:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \text{ where } B^2 - 4AC < 0.$$

Or in another way, we need the coordinates of ellipse's center (x_0, y_0) , semi-major/semi-minor axes (a, b) , and a rotation angle (φ) .

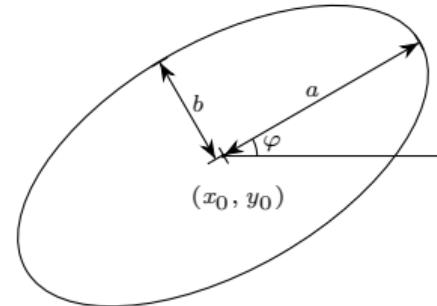


Figure: Parameters for the ellipse

Two practical ways

Hough Transform

- ▶ Slow
- ▶ Sacrifice accuracy for efficiency

Edge Following

- ▶ Derived from Arc-support LS
- ▶ use greyscale image (gradient)
- ▶ Greedy for efficiency

Methods

- ▶ To detect the arc segments;
- ▶ (To form arcs;)
- ▶ To predict the 5 parameters for ellipses;
- ▶ Co-clustering;
- ▶ Validation.

LSD: A Fast Line Segment Detector with a False Detection Control

IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE

- ▶ Finding line-support region (region growing algorithm)
- ▶ Rectangular Approximation of Regions
- ▶ Validation

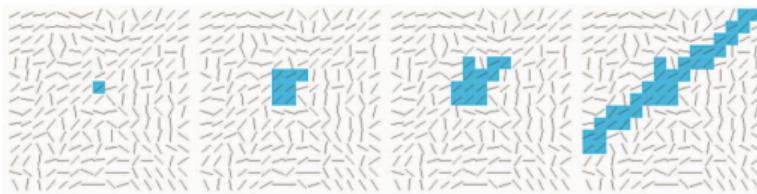


Figure: Region generation

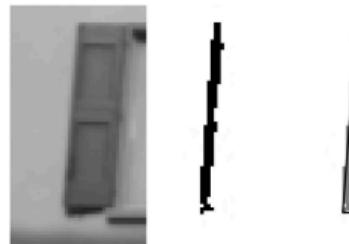


Figure: Rectangular Approximation

Arc segments' result

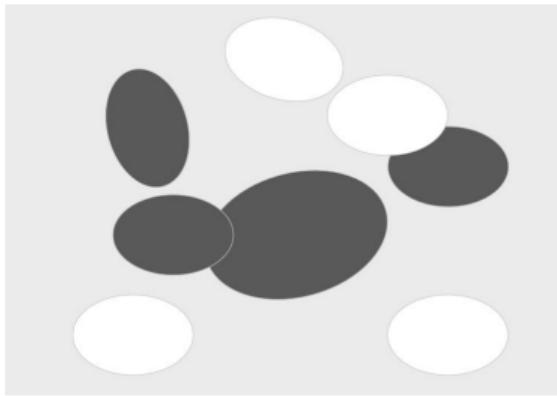


Figure: Source Images

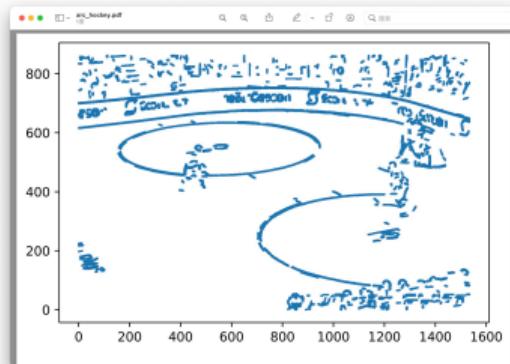
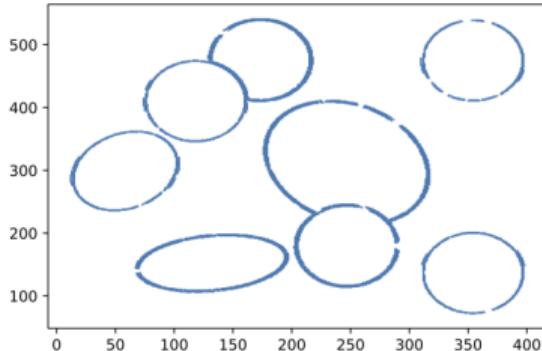
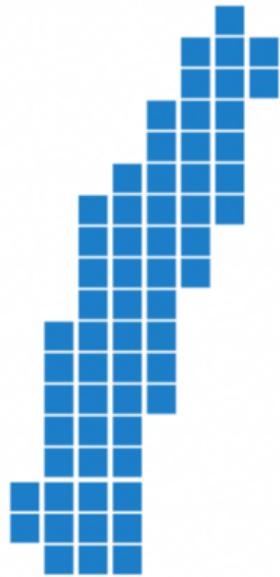


Figure: Arc Detection Results

Caluculating the Parameters



$$x^2 + bxy + cy^2 + dx + ey + f = 0;$$

$$\begin{pmatrix} x & y & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{b}{2} & \frac{d}{2} \\ \frac{b}{2} & c & \frac{e}{2} \\ \frac{d}{2} & \frac{e}{2} & f \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = O_{3 \times 3};$$

$$\begin{pmatrix} x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{b}{2} & \frac{d}{2} \\ \frac{b}{2} & c & \frac{e}{2} \\ \frac{d}{2} & \frac{e}{2} & f \end{pmatrix} \begin{pmatrix} x_1 & \dots & x_n \\ y_1 & \dots & y_n \\ 1 & \dots & 1 \end{pmatrix} = O_{3 \times 3}.$$

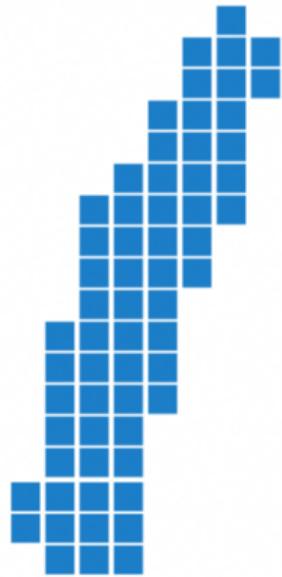
Figure: Arc Segment

Example

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Caluculating the Parameters



We can also alter it into:

$$\mathbf{D}\alpha = \mathbf{0},$$

where

$$\mathbf{D} = \begin{pmatrix} 2 & x_1 y_1 & y_1^2 & x_1 & y_1 & 1 \\ 1 & x_2^2 & x_2 y_2 & y_2^2 & x_2 & y_2 & 1 \\ x_2^2 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^2 & x_n y_n & y_n^2 & x_n & y_n & 1 \end{pmatrix};$$

$$\alpha^T = (1 \ b \ c \ d \ e \ f)$$

Figure: Arc Segment

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Cluster/Co-cocluster I

After all the α_i (i means the i th arc) are predicted, We correlate each two of them, and then do co-cluster.

Advantage of coclustering:

1. Fast for high-demensional cluster jobs;
2. Define codistances in parameter space to represent the relativity.

Cluster/Co-cocluster II

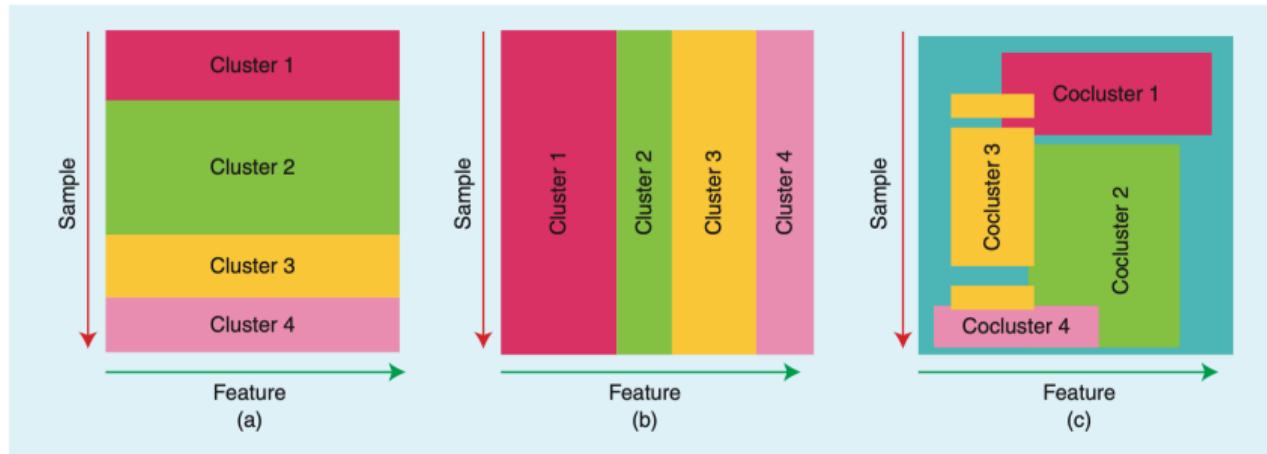


Figure: cocluster method

Cluster/Co-cocluster III

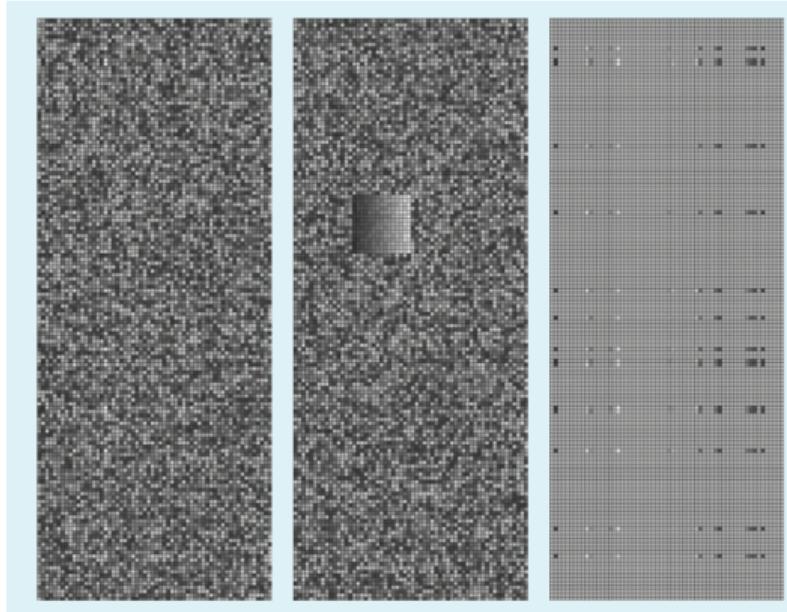


Figure: cocluster result

Direct Least Square Fitting of Ellipses I

PATTERN ANALYSIS AND MACHINE INTELLIGENCE

Lagrange multipliers

$$\mathbf{D} = \begin{bmatrix} x_1^2 & x_1 y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2 y_2 & y_2^2 & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^2 & x_n y_n & y_n^2 & x_n & y_n & 1 \end{bmatrix}_{n \times 6}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & -1 & \cdots & 0 \\ 0 & 2 & 0 & & \\ -1 & 0 & 0 & & \vdots \\ \vdots & & & \ddots & \\ 0 & \cdots & & & 0 \end{bmatrix}_{6 \times 6}$$

Direct Least Square Fitting of Ellipses II

PATTERN ANALYSIS AND MACHINE INTELLIGENCE

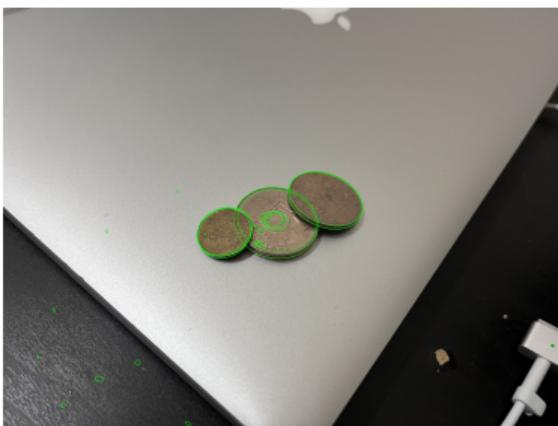
To minimize $\|Da\|^2$

It turns to an optimization problem.

$$2D^T Da - 2\lambda Ca = 0$$

$$a^T Ca = 1$$

Pilot Result



Summary

1. Co-clustering is effective to grouping data with good accuracy taking ellipses for examples.
2. Co-clustering can serve as an unsupervised learning model in grouping problems.

Q&A

Thank you!