Detection of 3D circles / Ellipses in a photo

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Catalogue

Project Target

To detect ellipses in the images/videos.

Figure: Input

Figure: Output

Ellipse I

To describe an ellipse we need 5 parameters:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \text{where } B^2 - 4AC < 0.$$

Or in another way, we need the coordinates of the ellipse's center (x_0, y_0), semi-major/semi-minor axes (a, b), and a rotation angle (φ).

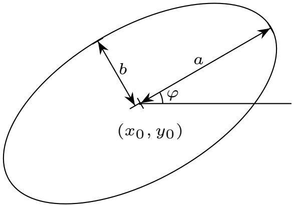

Figure: Parameters for the ellipse

Two practical ways

Hough Transform

- Slow
- Sacrifice accuracy for efficiency

Edge Following

- Derived from Arc-support LS
- use greyscale image (gradient)
- Greedy for efficiency

Methods

- To detect the arc segments;
- (To form arcs;)
- To predict the 5 parameters for ellipses;
- Co-clustering;
- Validation.

LSD: A Fast Line Segment Detector with a False Detection Control

IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE

- Finding line-support region (region growing algorithm)
- Rectangular Approximation of Regions
- Validation

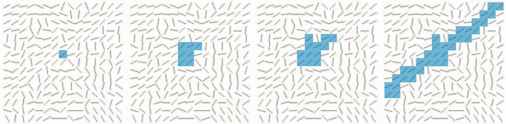

Figure: Region generation

Figure: Rectangular Approximation

Arc segments' result

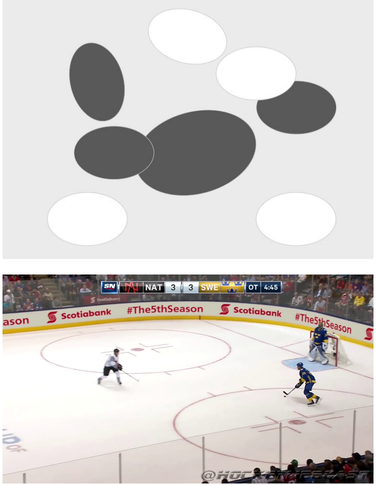

Figure: Source Images

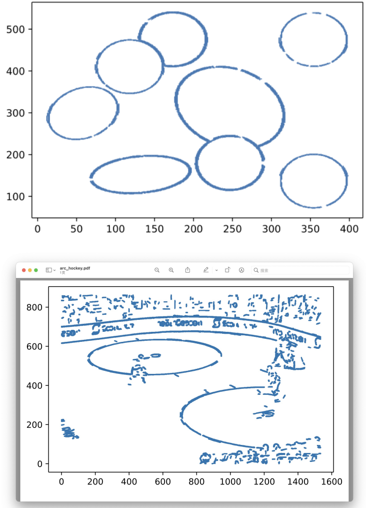

Figure: Arc Detection Results

Caluculating the Parameters

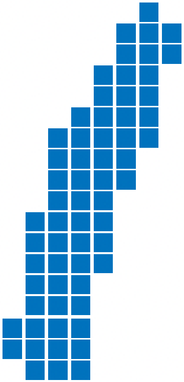

Figure: Arc Segment Example

$$x^2 + bxy + cy^2 + dx + ey + f = 0;$$

$$\begin{pmatrix} x & y & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{b}{2} & \frac{d}{2} \\ \frac{b}{2} & c & \frac{e}{2} \\ \frac{d}{2} & \frac{e}{2} & f \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = O_{3\times 3};$$

$$\begin{pmatrix} x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{pmatrix} \begin{pmatrix} 1 & \dfrac{b}{2} & \dfrac{d}{2} \\ \dfrac{b}{2} & c & \dfrac{e}{2} \\ \dfrac{d}{2} & \dfrac{e}{2} & f \end{pmatrix} \begin{pmatrix} x_1 & \dots & x_n \\ y_1 & \dots & y_n \\ 1 & \dots & 1 \end{pmatrix} = O_{3\times 3}.$$

Caluculating the Parameters

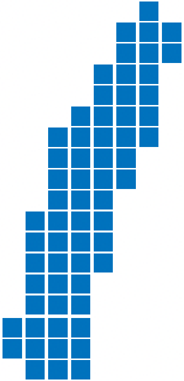

Figure: Arc Segment Example

We can also alter it into:

$$\mathbf{D}\alpha = \mathbf{0},$$

where

$$\mathbf{D} = \begin{pmatrix} {}_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2y_2 & y_2^2 & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^2 & x_ny_n & {y_n}^2 & x_n & y_n & 1 \end{pmatrix};$$

$$\alpha^{\mathbf{T}} = \begin{pmatrix} 1 & b & c & d & e & f \end{pmatrix}$$

Cluster/Co-cocluster I

After all the α_i (i means the ith arc) are predicted, We correlate each two of them and then do co-clustering.

Advantages of coclustering:

1. Fast for high-dimensional cluster jobs;
2. Define co-distance in parameter space to represent the relativity.

Cluster/Co-cocluster II

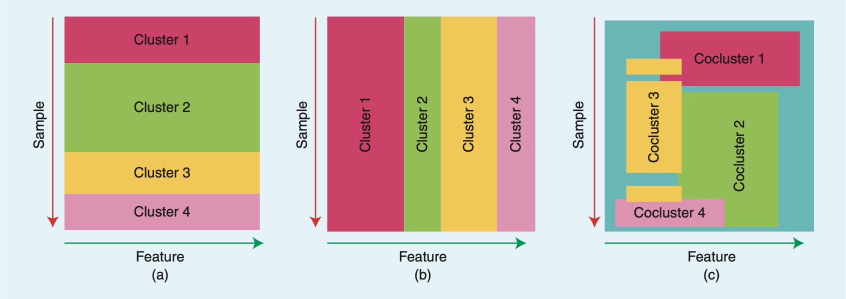

Figure: cocluster method

Cluster/Co-cocluster III

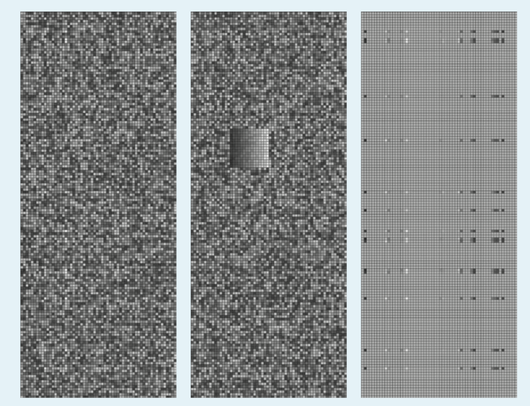

Figure: cocluster result

Lagrange multipliers

$$\mathbf{D} = \begin{bmatrix} x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2y_2 & y_2^2 & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^2 & x_ny_n & {y_n}^2 & x_n & y_n & 1 \end{bmatrix}_{n\times 6}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & -1 & \cdots & 0 \\ 0 & 2 & 0 & & \\ -1 & 0 & 0 & & \vdots \\ \vdots & & & \ddots & \\ 0 & & \cdots & & 0 \end{bmatrix}_{6\times 6}$$

Direct Least Square Fitting of Ellipses II

To minimize $||\mathrm{Da}||^2$

It turns into an optimization problem.

$$\begin{aligned} 2\mathrm{D}^{\mathrm{T}}\mathrm{Da} - 2\lambda\mathrm{Ca} &= 0 \\ \mathrm{a}^{T}\mathrm{Ca} &= 1 \end{aligned}$$

Pilot Result

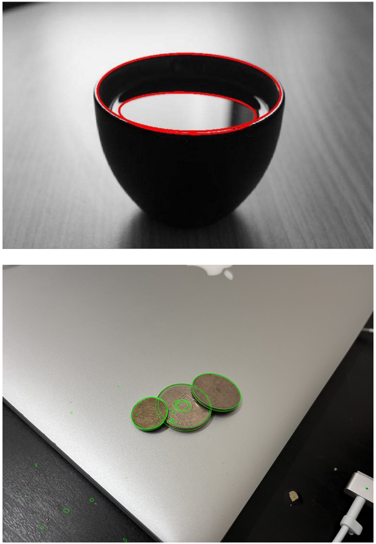

Figure: Good Examples

Figure: Bad Examples

Summary

1. Co-clustering is effective in grouping data with good accuracy taking ellipses for example.
2. Co-clustering can serve as an unsupervised learning model in grouping problems.

Q&A

Thank you!