Detection of Ellipses

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Project Target

To detect ellipses in the images/videos.

Figure: Input

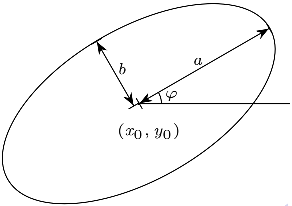
Figure: Output

Ellipse

To describe an ellispe we need 5 parameters:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \text{where } B^2 - 4AC < 0.$$

Or in another way, we need the coordinates of ellipse's center (x_0, y_0), semi-major/semi-minor axes (a, b), and a rotation angle (φ).



Two major ways

Hough Transform

- Slow
- Sacrifice accuracy for efficiency

Edge Following

- Derived from Arc-support LS
- use greyscale image (gradient)
- Greedy for efficiency

Methods

- To detect the arc segements;
- (To form arcs;)
- To predict the 5 parameters for ellipses;
- Co-clustering;
- Validation.

LSD: A Fast Line Segment Detector with a False Detection Control

IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE

- Finding line-support region (region growing algorithm)
- Rectangular Approximation of Regions
- Validation

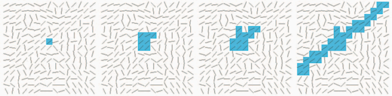


Figure: Region generation

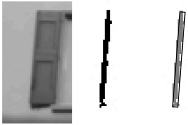


Figure: Rectangular Approximation

Arc segments' result

Figure: Source Images

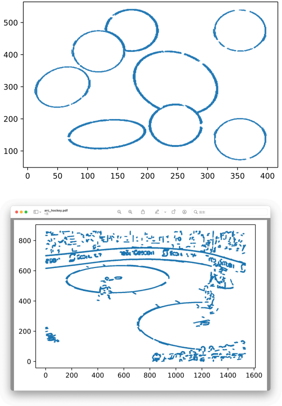


Figure: Arc Detection Results

Arc Segments' Result

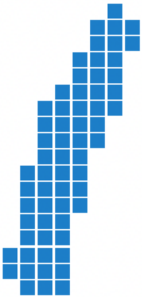


Figure: Arc Segment Example

$$x^2 + bxy + cy^2 + dx + ey + f = 0;$$

$$\begin{pmatrix} x & y & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{b}{2} & \frac{d}{2} \\ \frac{b}{2} & c & \frac{e}{2} \\ \frac{d}{2} & \frac{e}{2} & f \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = O_{3\times3};$$

$$\begin{pmatrix} x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{pmatrix} \begin{pmatrix} 1 & \dfrac{b}{2} & \dfrac{d}{2} \\ \dfrac{b}{2} & c & \dfrac{e}{2} \\ \dfrac{d}{2} & \dfrac{e}{2} & f \end{pmatrix} \begin{pmatrix} x_1 & \dots & x_n \\ y_1 & \dots & y_n \\ 1 & \dots & 1 \end{pmatrix} = O_{3\times3}.$$

Arc Segments' Result

Figure: Arc Segment Example

We can also alter it into:

$$\mathbf{D}\alpha = \mathbf{0},$$

where

$$\mathbf{D} = \begin{pmatrix} {}^2_1 & x_1 y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2 y_2 & y_2^2 & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^2 & x_n y_n & {y_n}^2 & x_n & y_n & 1 \end{pmatrix};$$

$$\alpha^{\mathbf{T}} = \begin{pmatrix} 1 & b & c & d & e & f \end{pmatrix}$$