For square:

eigenvalue and eigenvectors：特征值与特征向量.

$Ax = \lambda x$ Eigenvectors parallel to x

投影矩阵: $P = \frac{A \cdot A^T}{A^T \cdot A}$

Any vector in plane $Px = x$. $\lambda = 1$

Any vector $\perp$ plane $Px = 0x$ $\lambda = 0$

$n \times n$ vector has n eigenvectors.

特征值的和等于矩阵对角线之和

特征值的积等于矩阵行列式的值

solve $Ax = \lambda x$

解矩阵 $(A - \lambda I)x = 0$

$|A - \lambda I| = 0$ 行列式为0 （λ可能重复）

step: ① find n λ

② find x

eg: $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ $|A - \lambda I| = \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 - 1 = 0$

$\lambda = 2, 4$

$A - \lambda_1 I = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A - \lambda_2 I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Example: $\underbrace{Q=\begin{bmatrix}0&-1\\1&0\end{bmatrix}}$ 旋转矩阵每个向量会转90°

$$\begin{cases}\lambda_1+\lambda_2=0\\ \lambda_1\lambda_2=1\end{cases}$$

$\underbrace{\lambda_1=i \quad \lambda_2=-i}_{\text{复数}}$

Example: $\underbrace{A=\begin{bmatrix}3&1\\0&3\end{bmatrix}}_{\text{三角矩阵}}$ $(3-\lambda)^2=0$ $\lambda=3$

$A-\lambda I=\begin{bmatrix}0&1\\0&0\end{bmatrix}$ $(A-\lambda I)x=0$

$\underbrace{x_1=\begin{bmatrix}1\\0\end{bmatrix}}$ 只有1个特征向量