

Independence:

Definition: vectors x_1, x_2, \dots, x_n are independent if no combination gives zero vector (except all 0 combination)

If there is zero in vectors, it must be dependence.

$$\begin{cases} R(A) = n & \text{independent.} \\ R(A) < n & \text{dependence} \end{cases}$$

Basis $\frac{H}{F}$.

Basis for a vector is a sequenced vectors $v_1, \dots, v_2, \dots, v_k$ has two properties:

①. they are independent

② they can span the space.

Example:

space is \mathbb{R}^3 .

one basis is: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

another:

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 8 \end{bmatrix}$$

↓
a plane, use in that plane

\mathbb{R}^n : 需要 n 个 basis. 用 n 个 basis 构造的 $n \times n$

vector must be inverse.

↓
矩阵要可逆则没有非0解使得 $AX=0$

Every basis for the space has the same number vectors.
the number is called the dimension of the space.

$\text{Rank}(A) = \text{pivot numbers} = \text{dimension of } \underline{\text{column space}}$

$n - R(A) = \text{free variables number} = \text{dimension of } \underline{\text{null space}}$