Orthonormal vectors. $q_i^T \cdot q_j = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$

$Q = [q_1 \cdots q_n]$ $\quad Q^T \cdot Q = I$

If Q is square then $Q^T \cdot Q = I. \Rightarrow Q^T = Q^{-1}$

If Q has orthonormal columns, project onto its column space: $P = \frac{Q \cdot Q^T}{Q^T \cdot Q} = QQ^T = \begin{cases} I, & \text{if } Q \text{ is square} \\ \end{cases}$

$A^T A \hat{x} = A^T b$. now A is Q

so $\hat{x} = Q^T \cdot b$ $\quad (\hat{x}_i = q_i^T \cdot b)$

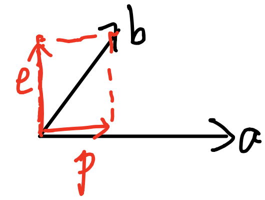
Gram-Schmidt 正交化

independent vectors a, b $\longrightarrow$ orthogonal A, B

$\downarrow$

orthonormal

$q_1 = \frac{A}{\|A\|}$, $q_2 = \frac{B}{\|B\|}$



So. $A = a$, $B = b - \frac{A \cdot A^T}{A^T \cdot A} \cdot b$. $A \perp B$ $A^T B = 0$

$$A^T B = A^T\left(b - \frac{A A^T}{A^T A} b\right) = 0$$

If it is 3-D. to get A, B, C

已知 A, $B \perp A$, C需要 $C \perp A$, $C \perp B$

$$C = c - \frac{AA^T}{A^TA} \cdot c - \frac{BB^T}{B^TB} \cdot c$$

Example: $a=\begin{bmatrix}1\\1\\1\end{bmatrix}$ $b=\begin{bmatrix}1\\0\\2\end{bmatrix}$

$$A=\begin{bmatrix}1\\1\\1\end{bmatrix} \quad B=b-\frac{A^Tb}{A^TA}A$$

$$=\begin{bmatrix}1\\0\\2\end{bmatrix}-\frac{3}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}-\begin{bmatrix}0\\-1\\1\end{bmatrix}$$

$$q_1=\frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix} \quad q_2=\frac{1}{\sqrt{2}}\begin{bmatrix}0\\-1\\1\end{bmatrix}$$

$$A=\begin{bmatrix}1&1\\1&0\\1&2\end{bmatrix}\to Q\begin{bmatrix}1/\sqrt{3}&0\\1/\sqrt{3}&-1/\sqrt{2}\\1/\sqrt{3}&1/\sqrt{2}\end{bmatrix}$$

A.Q属于同一个空间, $A=QR\longrightarrow R$是一个上三角矩阵

$$\begin{bmatrix}a_1&a_2\end{bmatrix}=\begin{bmatrix}q_1&q_2\end{bmatrix}\begin{bmatrix}\cdots&\vdots\\0&\vdots\end{bmatrix}$$