

$$A = LU$$

$$\downarrow$$

$$\textcircled{P}A = L\tilde{U} \Rightarrow \text{for any invertible matrix } A$$

permutation matrix

所有的 P 存在 P^{-1} , 且 $P^{-1} = P^T$

$$P^T \cdot P = I$$

Transpose Matrix:

$$A_{ij} = (A^T)_{ji}$$

Symmetric Matrix

$$A_{ij} = A_{ji}, A = A^T$$

$A \cdot A^T$ 是一个对称矩阵. why?

$$\text{eg: } A: m \times n \quad A^T: n \times m$$

$$B = A \cdot A^T: m \times m$$

$$\therefore B_{ij} = \underbrace{\text{row } i \text{ of } A} \times \underbrace{\text{col } j \text{ of } A^T}$$

$$\therefore B_{ji} = \underbrace{\text{row } j \text{ of } A} \times \underbrace{\text{col } i \text{ of } A^T}$$

$$\therefore B_{ij} = B_{ji}$$

so B is a symmetric matrix.

$$\underline{(A \cdot A^T)^T = A \cdot A^T}, \text{ so } A \cdot A^T \text{ is symmetric}$$

Chapter 3 Vector space

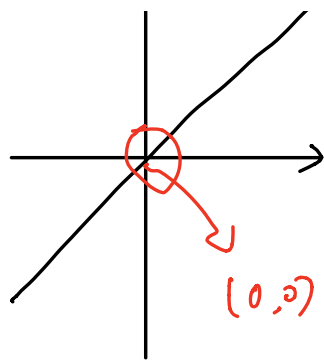
\mathbb{R}^2 : 2-dim vectors

\mathbb{R}^3 : 3-dim vectors.

x-y plane eg: $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

↓
两两相乘, 数乘, 线性组合
仍在 \mathbb{R}^2 内

↑ ↘



a subspace of \mathbb{R}^2

must add and multiply

$(0,0)$ must be in the space

subspace of \mathbb{R}^2 {

- ① \mathbb{R}^2 itself
- ② any line through $(0,0)$
- ③ zero vector alone