

Elimination (消元):

$$\begin{cases} x + 2y + z = 2 \\ 3x + 8y + z = 12 \\ 4y + z = 2 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix}$$

A

augmented matrix 增广矩阵

$$\begin{array}{ccc|ccc} \boxed{1} & 2 & 1 & 2 & & & \\ 3 & 8 & 1 & 12 & \rightarrow & 0 & 2 & -2 & 6 & \rightarrow & 0 & 2 & -2 & 6 \\ 0 & 4 & 1 & 2 & & 0 & 4 & 1 & 2 & & 0 & 0 & 5 & -10 \end{array}$$

pivot 不可以为0

upper triangular

U

A      b

消元 may 失效: pivot 为0, 且无法 exchange row

利用矩阵消元可解得方程组

Matrix:

permutation: 置换矩阵

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix} \quad (\text{换行})$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix} \quad (\text{换列})$$

Inverse matrix: 可逆矩阵

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{E^{-1} \cdot E = I}$$