求解 $Ax=0$：变量 特解

eg: $A=\begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 8 & 10 \end{bmatrix}$

$Ax=0$ ↓①

$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 2 & 4 \end{bmatrix}$

↓②

$\begin{bmatrix} \textcircled{1} & 2 & 2 & 2 \\ 0 & 0 & \textcircled{2} & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ← U (upper)

pivot columns

free columns

秩为2. the rank of A

equals the number of pivots

特解方向 $x_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$

$$x = c\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + d\begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

if $R=2$, free variables is $\underbrace{n-R}_{\text{number of } x}$.

steps: ① elimination to U

② find number of pivots, called R

③ get the n-R special resolution and plus make n-R free variables be 0 or 1

Reduced row echelon form

$$V = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 & -2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

J

pivot cols $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ free cols $\begin{bmatrix} 2 & -2 \\ 0 & 2 \end{bmatrix}$ matlab: rref(A)

I F

$$R = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix} \quad RN = 0 \quad RN = 0$$

$$N = \begin{bmatrix} -F \\ I \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 2 & 6 & 8 \\ 2 & 8 & 10 \end{bmatrix}$$

$$R(A^T) = 2 = R(A)$$

$$I \leftarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow F$$

$$X = C\begin{bmatrix} -F \\ I \end{bmatrix} = C\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$