$Ax = \lambda x$.

S: 由A的特征向量组成，且n个特征向量无关

$$A \cdot S = A \cdot \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix} = \begin{bmatrix} \lambda_1 x_1 & \cdots & \lambda_n x_n \end{bmatrix}$$

$$= \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

$$= S \cdot \Lambda$$

$$S^{-1} \cdot A \cdot S = \Lambda \qquad A = S \Lambda S^{-1}$$

If $Ax = \lambda x$ then $A^2 x = \lambda A x = \lambda^2 x$

$$A^2 = S\Lambda S^{-1} \cdot S \Lambda S^{-1} = S \Lambda^2 S^{-1}$$

A^2 and A, eigenvector is the same. and eigenvalues become the square of it.

So $A^k = S \Lambda^k S^{-1}$

$A^k \to 0$ as $k \to \infty$ if all $|\lambda_i| < 1$

A is sure to have n independent eigenvectors

if all the λ's are different.

Repeated eigenvalues ⇒ may or not have n indep vectors

Equation $u_{k+1} = A \cdot u_k$. to solve it

$u_1 = A u_0 \quad u_2 = A^2 u_0 \quad \cdots \quad u_k = A^k u_0$

solve steps:

write $u_0 = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$

$A u_0 = c_1 \lambda_1 x_1 + \cdots + c_n \lambda_n x_n$

$A^{100} u_0 = c_1 \lambda_1^{100} x_1 + \cdots + c_n \lambda_n^{100} x_n$

$A^{100} u_0 = S \Lambda^{100} c \qquad u_0 = Sc$

because $A u_0 = S \Lambda S^{-1} \cdot (Sc) = S \Lambda c$

Fibonacci : 0, 1, 1, 2, 3, 5 , what is F_{100}

$F_{k+2} = F_{k+1} + F_k$

$F_{k+1} = F_{k+1}$.

$$u_k = \begin{bmatrix} F_{k+1} \\ F_k \end{bmatrix} \qquad u_{k+1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} F_{k+1} \\ F_k \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}}_{A} u_k$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \qquad \lambda^2 - \lambda - 1 = 0 \qquad x_1 = \begin{bmatrix} \lambda_1 \\ 1 \end{bmatrix}$$

$$\lambda = \frac{1 \pm \sqrt{5}}{2} \qquad x_2 = \begin{bmatrix} \lambda_2 \\ 1 \end{bmatrix}$$

$$F_{100} \approx c_1 \left(\frac{1+\sqrt{5}}{2}\right)^{100}$$