Application of determinant.

1. Formula of A^{-1}

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Formula of A^{-1}

So $\boxed{A^{-1} = \frac{1}{|A|}C^T}$ (C为A的代数余子式)

prove $\frac{C^T}{|A|}\cdot A = I$ 即 $A\cdot C^T = |A|$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ \vdots & & a_{nn} \end{bmatrix}\begin{bmatrix} C_{11} & \cdots & C_{n1} \\ \vdots & & \vdots \\ C_{1n} & \cdots & C_{nn} \end{bmatrix} = \begin{bmatrix} |A| & & \\ & \ddots & 0 \\ & |A| & \\ 0 & & \ddots \\ & & |A| \end{bmatrix}$$

2. $Ax=b$ $\quad x = A^{-1}b = \frac{C^T}{|A|}\cdot b$

CRAMER'S RULE:

$$x_1 = \frac{|B_1|}{|A|}$$

$$\vdots$$

$$x_n = \frac{|B_n|}{|A|}$$

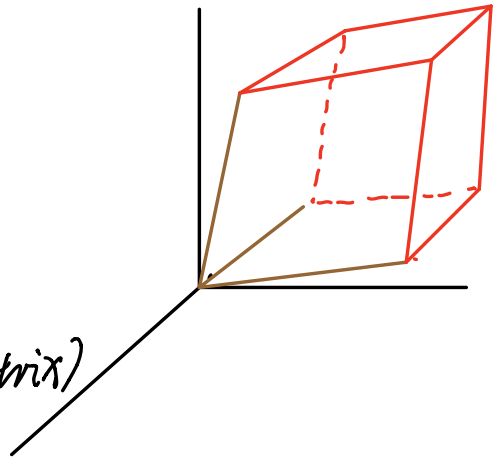
where it's B_i,

$$B = \left[\begin{array}{c:c} b & \begin{array}{c} n-1 \\ \text{columns} \\ \text{of } A \end{array} \end{array}\right]$$

$$|B| = b_{11} \cdot C_{11} + b_{21} C_{21} + \cdots + b_{n1} \cdot C_{n1}$$

3. Volumn.

$|\det A|$ = volumn of a box



eg: 3x3 matrix

when $A = I$, it is true

when $A = Q$ (Q is orthogonal matrix)

$$V = 1, \quad Q^T \cdot Q = I$$

$$\text{so } (\det Q)^2 = 1$$

$$|Q| = \pm 1 \quad \text{it is true}$$