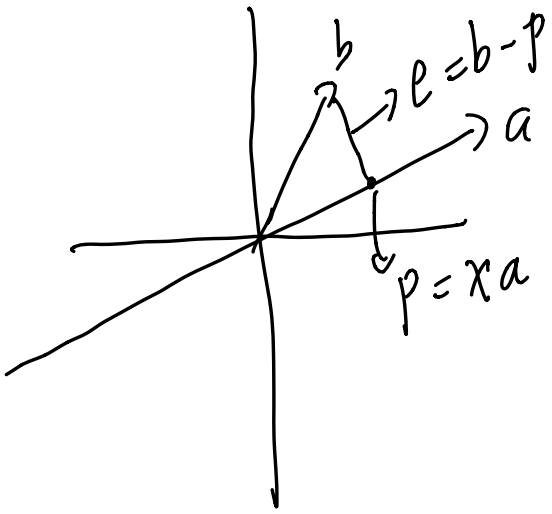
Projections: 投影



$$a^T \cdot (b - xa) = 0$$

$$a^T b = x a^T \cdot a$$

$$x = \frac{a^T b}{a^T \cdot a}$$

$$p = a \cdot \frac{a^T b}{a^T \cdot a} = P b$$

$$P = \frac{a \cdot a^T}{a^T \cdot a}$$

$P^T = P$. 对称.

$P^2 = P$. (投影两次仍相同)

Why projection??

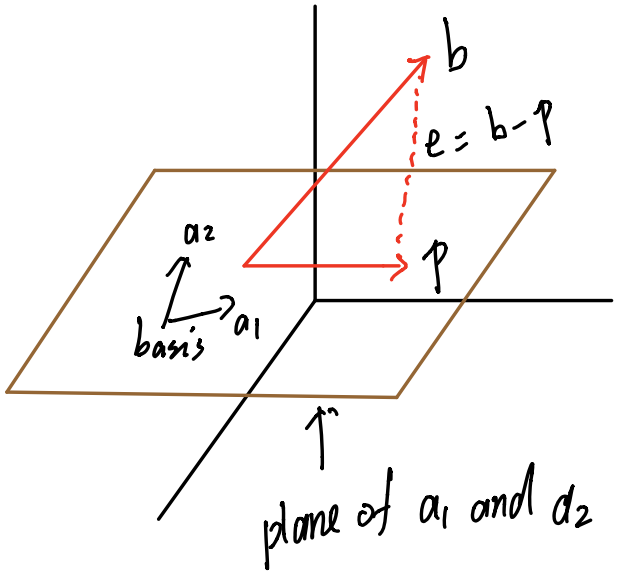
$Ax = b$. may have no solution.

↓ change

$Ax = p$. p is projection of b onto Col subspace.

Three dimension

$$p = \hat{x}_1 a_1 + \hat{x}_2 a_2 = A\hat{x}$$



$b - A\hat{x} \perp$ plane

So $\begin{cases} a_1^T (b - A\hat{x}) = 0 \\ a_2^T (b - A\hat{x}) = 0 \end{cases}$

$$\begin{bmatrix} a_1^T \\ a_2^T \end{bmatrix} (b - A\hat{x}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Downarrow$$

$$A^T (b - A\hat{x}) = 0 \qquad \hat{x} = \frac{A^T b}{A^T \cdot A}$$

$$\downarrow$$

e in $N(A^T)$

$e \perp C(A)$

$$p = \frac{A \cdot A^T}{A^T \cdot A} b \qquad P = \frac{A \cdot A^T}{A^T \cdot A}$$

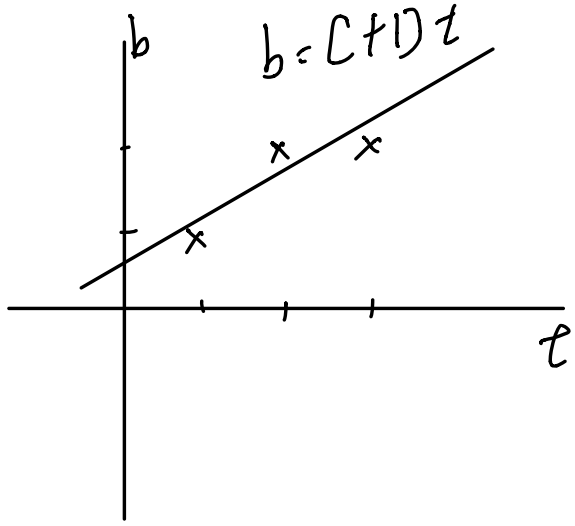
$P^T = P$ 对称, $P^2 = P$

Application: Lease squares. Fitting by a line

最小二乘法拟合直线

$(1,1)\ (2,2)\ (3,2)$

$$\begin{cases} C+D=1 \\ C+2D=2 \\ C+3D=2 \end{cases}$$



$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$