Determinant 行列式：$\det A = |A|$

$$\boxed{\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc}$$

properties

① $\det I = 1$

② exchange rows, reverse the sign of det

③ $\begin{cases} \text{用}t\text{乘一行，其他行不变} \quad \begin{vmatrix} ta & tb \\ c & d \end{vmatrix} = t\begin{vmatrix} a & b \\ c & d \end{vmatrix} \\ \begin{vmatrix} a+a' & b+b' \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix} \end{cases}$

basic properties

④ two rows equal, $\det A = 0$

⑤ row k − $l \cdot$ row i, det don't change

⑥ 有 0 行 则 $\det = 0$

⑦ $U = \begin{bmatrix} d_1 & & * \\ & \ddots & \\ 0 & & d_n \end{bmatrix}$ $\quad \det U = d_1 \cdot d_2 \cdots d_{n-1} \cdot d_n$

⑧ $\det A = 0$, exactly when A is singular

奇异矩阵：不是满秩

$\det A \neq 0$, A 可逆

⑨ $\det AB = \det A * \det B$

$\det A^{-1}A = \det A^{-1} \cdot \det A = 1$

$\det A^2 = (\det A)^2$

$\det 2A = 2^n * \det A \quad (A: n \times n)$

⑩ $\det A^T = \det A$

So. 交换列. det 也变符号.

有全0列, $\det A = 0$

Prove: $A = LU$

$|A^T| = |A|$

$|(LU)^T| = |LU|$

$|U^T| \cdot |L^T| = |L| \cdot |U|$

显然 $|U| = |U^T|$, $|L^T| = |L|$

So we can get $|A^T| = |A|$