

$$A = LU$$

$$\downarrow$$

$$\textcircled{P}A = L\tilde{U} \Rightarrow \text{for any invertible matrix } A$$

permutation matrix

所有的  $P$  存在  $P^{-1}$ , 且  $P^{-1} = P^T$

$$P^T \cdot P = I$$

Transpose Matrix:

$$A_{ij} = (A^T)_{ji}$$

Symmetric Matrix

$$A_{ij} = A_{ji}, A = A^T$$

$A \cdot A^T$  是一个对称矩阵. why?

$$\text{eg: } A: m \times n \quad A^T: n \times m$$

$$B = A \cdot A^T: m \times m$$

$$\therefore B_{ij} = \underbrace{\text{row } i \text{ of } A} \times \underbrace{\text{col } j \text{ of } A^T}$$

$$\therefore B_{ji} = \underbrace{\text{row } j \text{ of } A} \times \underbrace{\text{col } i \text{ of } A^T}$$

$$\therefore B_{ij} = B_{ji}$$

so  $B$  is a symmetric matrix.

$$\underline{(A \cdot A^T)^T = A \cdot A^T}, \text{ so } A \cdot A^T \text{ is symmetric}$$

## Chapter 3 Vector space

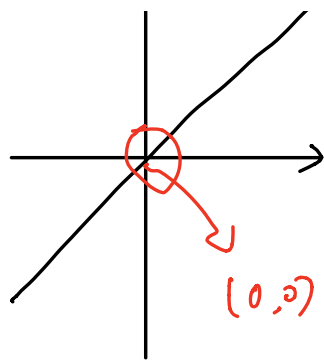
$\mathbb{R}^2$ : 2-dim vectors

$\mathbb{R}^3$ : 3-dim vectors.

x-y plane eg:  $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

↓  
两两相乘, 数乘, 线性组合  
仍在  $\mathbb{R}^2$  内

↑ ↘



a subspace of  $\mathbb{R}^2$

must add and multiply

$(0,0)$  must be in the space

subspace of  $\mathbb{R}^2$  {

- ①  $\mathbb{R}^2$  itself
- ② any line through  $(0,0)$
- ③ zero vector alone