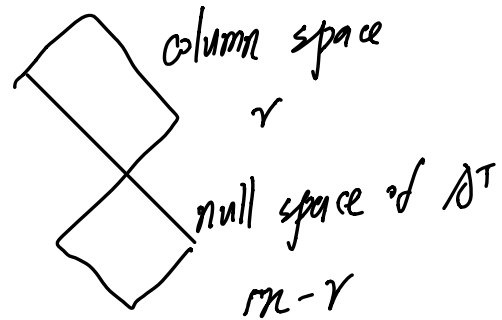
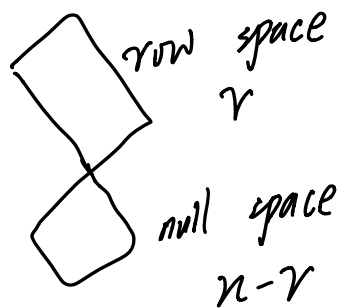


Orthogonal: 正交



Orthogonal vectors: $\perp \quad x^T \cdot y = 0$

Subspace S is orthogonal to subspace $T \Rightarrow$ every vector in S is orthogonal to vector in T .

Conclusion: Row space is orthogonal to null space
Column space is orthogonal to null space of A^T

Why? if x in null space, $Ax = 0$

$$\begin{bmatrix} \text{row} \\ \vdots \\ \text{row} \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

So, x is orthogonal to every row of A

\Downarrow
any combination of row A is orthogonal to x .

Solve $Ax=b$, if it has no solution

$$\downarrow$$
$$A^T A \hat{x} = A^T b$$

$$N(A^T A) = N(A)$$

$A^T A$ is invertible exactly A has independent columns.