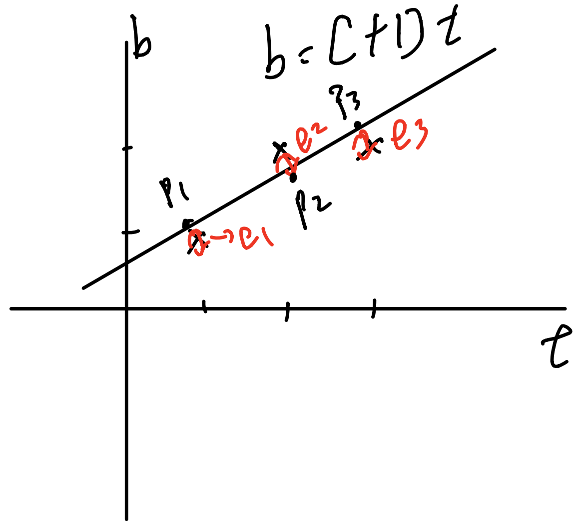
projection matrix $P = \frac{A \cdot A^T}{A^T \cdot A}$

If b in the column space: $Pb = b$ $(b = Ax)$

if $b \perp$ column space: $Pb = 0$

Application: Lease squares. Fitting by a line

最小二乘法拟合直线



(1,1) (2,2) (3,2)

$$\begin{cases} C + D = 1 \\ C + 2D = 2 \\ C + 3D = 2 \end{cases}$$

$$\underset{A}{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}} \underset{x}{\begin{bmatrix} C \\ D \end{bmatrix}} = \underset{b}{\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}$$

To get best solution, there is a minimize error: $\|Ax-b\|^2$

$= e_1^2 + e_2^2 + e_3^2$

solve i.e.: $\hat{x} = \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix}$, p

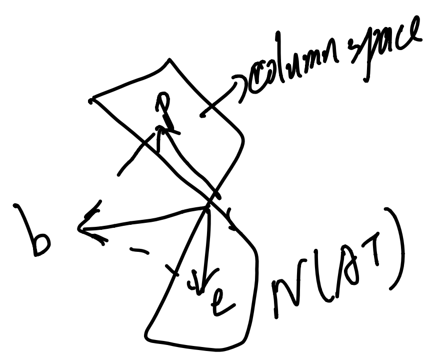
$$A^T A \hat{x} = A^T b$$

$$\begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix} \hat{x} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$$

$$3C + 6D = 5$$
$$6C + 14D = 11$$

$$\boxed{\begin{cases} C = \frac{2}{3} \\ D = \frac{1}{2} \end{cases}}$$

So the best line $b = \frac{2}{3} + \frac{1}{2}t$



$$b = p + e$$

$$\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{7}{6} \\ \frac{5}{3} \\ 13/6 \end{bmatrix}}_{} + \underbrace{\begin{bmatrix} -\frac{1}{6} \\ 2/6 \\ -1/6 \end{bmatrix}}_{}$$

$$p \perp e$$

To prove: If A has indepedent columns, then $A^T \cdot A$ is invertible

suppose $A^T \cdot A \cdot x = 0$

$$x^T A^T A \cdot x = 0$$

$$\underbrace{(Ax)^T \cdot Ax}_{} = 0$$

代表 Ax 的长度为0. 则 $Ax = 0 \Rightarrow x = 0$