4 fundamental subspace

$A: m \times n$

- Column space : $C(A)$ in R^m
- Null space : $N(A)$ in R^n
- row space $= C(A^T)$ in R^n
- nullspace of A^T : $N(A^T)$ in R^m

① dimension of $C(A) = R(A)$, a basis of $C(A)$ is the pivot columns, first r columns of R

② dimension of $C(A^T) = R(A) = r$, a basis of $C(A^T)$ is first r rows of R

③ dimension of $N(A) = n - R(A) = n - r$, a basis of $N(A)$ is a group special solutions.

④ dim of $N(A^T) = m - R(A) = m - r$

4th space $N(A^T)$.

$A^T \cdot y = 0 \iff y^T \cdot A = 0$

$E \cdot [A_{m\times n} \quad I_{m\times m}] \to [R_{m\times n} \quad E_{m\times m}]$

$EA = R$

其中使 R 中 row 为 0 的行则是 basis.