1. 5×3 matrix U $\quad r = 3$

① what is null space?

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

②. 10×3 matrix B, $B = \begin{bmatrix} U \\ 2U \end{bmatrix} \to \begin{bmatrix} U \\ 0 \end{bmatrix}$

$r = 3$.

③ $C = \begin{bmatrix} U & U \\ U & 0 \end{bmatrix} \to \begin{bmatrix} U & U \\ 0 & -U \end{bmatrix} \to \begin{bmatrix} U & 0 \\ 0 & U \end{bmatrix}$

$r = 6$

④ $\dim N(C^T)$

$C: 10 \times 6 \quad C^T: 6 \times 10$

$= 10 - 6 = 4$

$\dim N(C^T) = m - r = 10 -$ [illegible]

2. $Ax = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$ $\qquad x = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

① $A: 3 \times 3$ $\quad r = 1$ (has 2 special)

$\dim N(A) = 2 = n - r = 3 - 1 = 2$

So: $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -2 & 0 \\ 1 & -1 & 0 \end{bmatrix}$

② $Ax = b$ can be solved if ?

b has the form $b = c \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

3. if $B^2 = 0 \Rightarrow B = 0$?   False

$$B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow B^2 = 0$$

4. $$B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

① 求 basis for $N(B) \subseteq \mathbb{R}^4$.

$$\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

② solve $Bx = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$x = x_p + x_{null} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

5. $V = \begin{bmatrix}1\\2\\3\end{bmatrix}$ can't be in null space and a row of A

Why not?

假设 $A \cdot \begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$$\begin{bmatrix} - & - & - \\ 1 & 2 & 3 \\ - & - & - \end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$?? 明显不可能

Intersection of null space and column space only has zero vector