1. 正交 $Q^T \cdot Q = I$

2. projection G-S → orthonormal basis

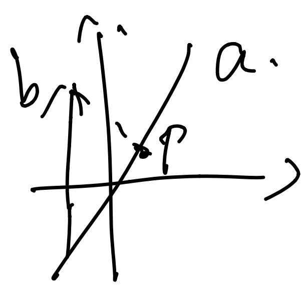
3. determinant, cofactor formula

4. eigenvalues and eigenvectors

$Ax = \lambda x$ $|A - \lambda I| = 0$

Questions:

1. projections: $a = \begin{bmatrix}2\\1\\2\end{bmatrix}$ P:



$P_b = A \cdot \frac{A^T \cdot b}{A^T \cdot A}$

$P = \frac{1}{9}\begin{bmatrix}2\\1\\2\end{bmatrix}\begin{bmatrix}2 & 1 & 2\end{bmatrix} = \frac{1}{9}\begin{bmatrix}4 & 2 & 4\\2 & 1 & 2\\4 & 2 & 4\end{bmatrix}$

rank(P) = 1 $\lambda = \boxed{0, 0}, 1$ → ∵ rank(A) = 1 < 3

so |A| = 0

$x = \begin{bmatrix}2\\1\\2\end{bmatrix}$ ∵ $\lambda_1 \cdot \lambda_2 \cdots \lambda_n = |A| = 0$

∴ 有λ为0

solve: $u_{k+1} = P u_k$ $u_0 = \begin{bmatrix} 9 \\ 9 \\ 0 \end{bmatrix}$, find u_k

$$u_1 = \frac{1}{9} \cdot \begin{bmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 9 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$$

$$u_0 = C_1 x_1 + \cdots + C_n x_n$$

$$u_1 = P u_0 \quad u_2 = P^2 u_0 \cdots \quad u_k = P^k u_0$$

$$u_k = C_1 \lambda_1^k x_1 + \cdots + C_n \lambda_n^k x_n$$

$$u_k = C_3 1^k \cdot \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = C_3 \cdot \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$$u_1 = \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix} = C_3 \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \qquad C_3 = 3$$

so $u_k = \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$

2、拟合. $t=1, y=4$, $t=2, y=5$ $t=3, y=8$

要通过原点. $y = Dt$

$$\underset{A}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} \cdot D = \underset{b}{\begin{bmatrix} 4 \\ 5 \\ 8 \end{bmatrix}}$$

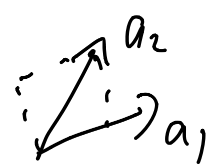
$$A^T \cdot (b - \hat{D} A) = 0 \qquad A^T \cdot A \cdot \hat{D} = A^T \cdot b$$

$$14 \cdot \hat{D} = 4 + 10 + 24 = 38$$

$$\hat{D} = \frac{38}{14} = \frac{19}{7}$$

3. orthogonality: G-S

$$a_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad a_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$



$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad B = a_1 \cdot \frac{a_1^T \cdot a_2}{a_1^T \cdot a_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \times \frac{6}{14} = \frac{3}{7} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

4、 4×4 matrix, $\lambda_1, \lambda_2, \lambda_3, \lambda_4$

(a) what condition of λ make A invertible

$\Rightarrow \lambda \neq 0$

$\det A = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4$

(b) $\det A^{-1} = \frac{1}{\lambda_1} \cdot \frac{1}{\lambda_2} \cdot \frac{1}{\lambda_3} \cdot \frac{1}{\lambda_4}$

(c) trace of $A+I$. : $\lambda_1+\lambda_2+\lambda_3+\lambda_4+4$

5. determiane

$$A_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$D_n = \det A_n$

$D_n = D_{n-1} - D_{n-2}$

$D_1 = 1 \quad D_2 = 0 \quad D_3 = -1$

$$\begin{bmatrix} D_n \\ D_{n-1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} D_{n-1} \\ D_{n-2} \end{bmatrix}$$

$$|A-\lambda I| = (1-\lambda)\cdot(-\lambda)+1 = \lambda^2-\lambda+1 = 0$$

$$\lambda = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}i}{2} \qquad e^{i\theta}$$

$$= e^{i\frac{\pi}{3}},\ e^{-i\frac{\pi}{3}}$$

6. $A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 3 & 0 \end{bmatrix} = A_4^T$

$A_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$ $\qquad P = \dfrac{A \cdot A^T}{A^T \cdot A}$

eigenvalues and eigenvectors.

$$|A_3 - \lambda I| = \begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 2 \\ 0 & 2 & -\lambda \end{vmatrix} = -\lambda \cdot (\lambda^2 - 4) - (-\lambda)$$
$$= -\lambda^3 + 5\lambda$$

$\lambda = 0, \sqrt{5}, -\sqrt{5}$

$\det A_4 = -(1 \times -9 - 0) = 9$