Formula for det A

Cofactor formula. 代数余子式

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \qquad A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$$

$$\det A = \sum_{n!} \pm a_{1\alpha} a_{2\beta} \cdots a_{n\omega}$$

Cofactor formula

$$\det A_{3\times 3} = a_{11}(\underline{a_{22}a_{33} - a_{23}a_{32}}) + a_{12}(-a_{21}a_{33} + a_{23}a_{31}) + a_{13}(\qquad)$$

$$\text{Cofactor}(a_{ij}) = \pm \det\begin{pmatrix} n-1 \text{ Matrix} \\ \text{without row } i \\ \text{column } j \end{pmatrix}$$

if $i+j$ is even $+$

$i+j$ is odd $-$

$$\det A = a_{11}\cdot C_{11} + a_{12}C_{12} + \cdots + a_{1n}\cdot C_{1n}$$

Ex:

$$A_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

三对角矩阵

$|A_1| = 1$

$|A_2| = 0$

$|A_3| = -1$

$|A_4| = |A_3| - |A_2|$

$\vdots$

$|A_n| = |A_{n-1}| - |A_{n-2}|$

6周期变化：$1 \quad 0 \quad -1 \quad -1 \quad 0 \quad 1$