AB 的 inverse 是 $B^{-1}A^{-1}$

Transpose · 转置矩阵

$$(AA^{-1})^T = \underbrace{(A^{-1})^T}_{= (A^T)^{-1}} \cdot A^T$$

$$\overset{E}{\begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}} \overset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \overset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}$$

$L = E^{-1}$

$$A = \underset{\text{(lower)}}{L} \; \underset{\text{(upper)}}{U}$$

$$\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

3X3 matrix:

$$E_{32} \cdot E_{31} \cdot E_{21} \cdot A = U$$

$$A = E_{21}^{-1} \cdot E_{31}^{-1} \cdot E_{32}^{-1} \cdot U$$

$A = \underbrace{E_{21}\, E_{31}\, E_{32}}_{L}\, U$

$n$阶matrix 消元需进行 $\underbrace{n^2+(n-1)^2+\cdots+1^2}_{\approx \frac{n^3}{3}}$ 次操作

## Transpose and Permutation

转置　　　置换

$A^T$

eg: $A = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 3 \end{bmatrix}$　　$A^T = \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$

permutation: 是一个方形二进制矩阵，它的每一行每一列只有一个1，其余都是0

$\underbrace{P^{-1} = P^T}$

$n$阶矩阵有 $n!$ 个置换矩阵