MF702 PROBLEM SET 5

1. Write an Excel spreadsheet which takes as input any one of the four quantities

(1) Discount Factors : $d(t_j)_{j=1,\dots,J}$
(2) Spot Rates : $\hat{r}(t_j)_{j=1,\dots,J}$
(3) Forward Rates : $f(t_j)_{j=1,\dots,J}$.
(4) Par Rates : $c(t_j)_{j=1,\dots,J}$.

and computes the other three. Above, we always assume that t_j is a multiple of $1/2$. Make sure your spreadsheet plots each of the four quantities.
For input data use the spreadsheet "Rates_Discount_Factors.xlsx". Using actual Treasury yield curve data, this file contains

(a) Spot rates from September 1, 1995.
(b) Discount factors from September 2, 2005.
(c) Par rates from September 3, 2015.

It will be interesting to compare the three sets of rates. Data is given in six month increments from .5 to 20 years. Note that Treasury data is for the 6 month, 1,2,3,5,7,10 and 20 year periods. To obtain the rest of the data, I have used a cubic-spline interpolation.
Note: Ideally, you should write a macro with a command button that automatically calculates the rates/discount factors. However, if you cannot write a macro, it is OK to hard code the formulas into your spreadsheet.

2. (semi-annual) Coupon bonds with a maturity $T = 10$ years, face value $F = \$10,000$, and annual coupon rate $q = 4\%$ are currently trading at $\$9,060$ per bond. Coupon bonds with maturity $T = 10$ years, face value $\$5,000$ and annual coupon rate $q = 8\%$ are currently trading at $\$6,146.50$ per bond. Determine the value of the following securities:

(1) A zero coupon bond with face value $\$20,000$ and maturity 10 years.
(2) An annuity that will make payments of $\$500$ twice per year for the next 10 years.
(3) A coupon bond with maturity 10 years, face value $\$7,500$ and annual coupon rate $q = 6\%$.

3. Let $T > \eta > 0$ be given and let $\hat{R}(\eta), \hat{R}(T)$ denote the effective spot rates for dates η and T. Let $R^{\text{for}}_{0,\eta,T}$ denote the effective forward rate agreed upon at time 0 for borrowing between time η and T. Recall that for every dollar borrowed at time η the amount to be repaid at time T is $\left(1+R^{\text{for}}_{0,\eta,T}\right)^{T-\eta}$. Assume that $\hat{R}(T) > \hat{R}(\eta)$. Show that $R^{\text{for}}_{0,\eta,T} > \hat{R}(T)$.

4. Assume that the spot curve is flat at 6%. A portfolio consists of three zero-coupon bonds each having a face value of $\$1,000,000$. The maturities of the bonds are 2, 5, and 10 years.

(a) Find the DV01 and duration of the portfolio of bonds.

(b) Suppose that you are asked to purchase a single par-coupon bond having maturity of 7 years such that the DV01 of the par-coupon bond will match the DV01 of portfolio of zero-coupon bonds. What should the face of the par-coupon bond be?

(c) Suppose that the 2 year spot rate increases by 50 basis points (bp), the 5 year rate increases by 42 bp and the 10 year rate increases by 38 bp. Compute the exact price change of portfolio of zero coupon bonds.

(d) Now assume a *parallel* shift in the (flat) spot curve from parts $(a),(b)$: i.e. all the spot rates move by the same amount. What parallel shift would explain the price change in part (c)? Try to find a shift which exactly explains the price change. Is this possible? What if you use the first order approximation implied by the portfolio's duration? What shift does this yield? Compare the results.

5. Consider a zero-coupon bond with face value $\$10,000$ and maturity 10 years. Assume that $\hat{r}(10) = 4.8736\%$.

(a) Compute the convexity of the bond.

(b) Compute the exact price change in the bond corresponding to a 35 bp increase in $\hat{r}(10)$.

(c) Compute the first-order approximation to the price change in the bond for a 35 bp increase in $\hat{r}(10)$.

(d) Compute the second-order approximation to the price change in the bond for a 35 bp increase in $\hat{r}(10)$.

6. Suppose that the zero-coupon yield curve is upward sloping. In particular we have $\hat{r}(t+.5) \geq \hat{r}(t)$ for $t = .5, 1, \ldots, 10$. Consider the following securities

- **A**: a zero-coupon bond with maturity 10 years.
- **B**: an annuity with maturity 5 years.
- **C**: an annuity with maturity 10 years.
- **D**: a coupon bond with maturity 10 years.

If possible, order these securities by yield to maturity from lowest to highest. Give a BRIEF explanation of your ordering. If it is not possible to order the yields (or if more information is needed), give an explanation why not (including what additional assumptions you may need to provide an ordering).

7. Assume that the spot curve is flat at some level $y > 0$. Let η denote the Macauley duration of a 10 year par-coupon bond (η is assumed to be a multiple of six months). Consider the following securities:

- **A**: a 10 year premium bond.
- **B**: a zero-coupon bond with maturity 10.
- **C**: a zero-coupon bond with maturity η.

If possible, order these securities by their Macauley duration from largest to smallest. Given a brief explanation of your reasoning. If it is not possible to order the securities by duration based upon the given information, please explain why not, including any additional assumptions you would need to provide the ordering.