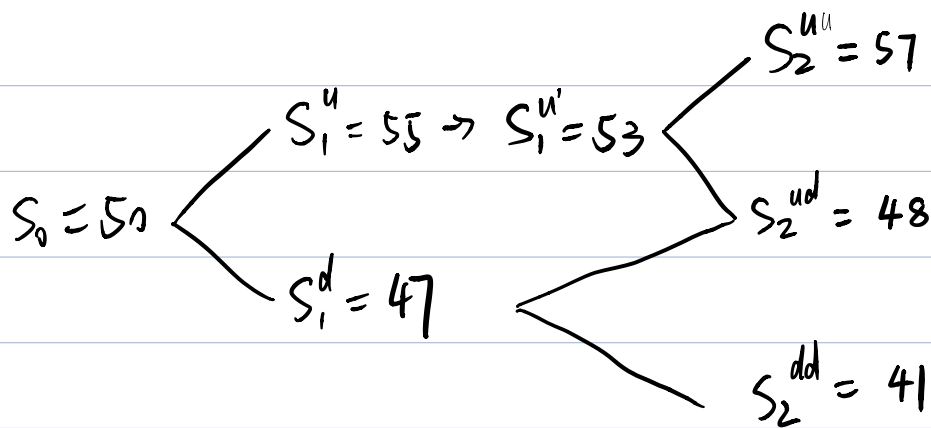


(1) (a)



Period 1. $50 = \check{p} \cdot (1+r)^{-1} \cdot 55 + (1-\check{p}) \cdot (1+r)^{-1} \cdot 47$

$$50 = \check{p} \cdot 55 / 1.02 + (1-\check{p}) \cdot 47 / 1.02$$

$$\check{p} = 0.5$$

$$1-\check{p} = 0.5$$

Period 2. $53 = \check{p} \cdot (1+r)^{-1} \cdot 57 + (1-\check{p}) \cdot (1+r)^{-1} \cdot 48$

$$S_1^u \rightarrow S_2^{uu} \quad 53 = \check{p} \cdot 57 / 1.02 + (1-\check{p}) \cdot 48 / 1.02$$

$$\check{p} = 0.67$$

$$1-\check{p} = 0.33$$

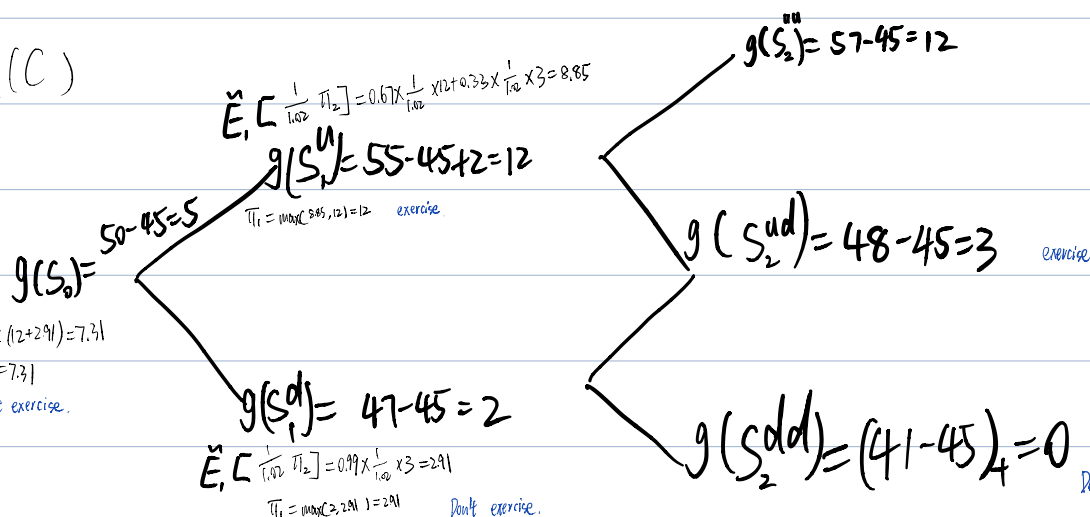
$$47 = \check{p} \cdot (1+r)^{-1} \cdot 48 + (1-\check{p}) \cdot (1+r)^{-1} \cdot 41$$

$$S_1^d \rightarrow S_2^{du} \quad 47 = \check{p} \cdot 48 / 1.02 + (1-\check{p}) \cdot 41 / 1.02$$

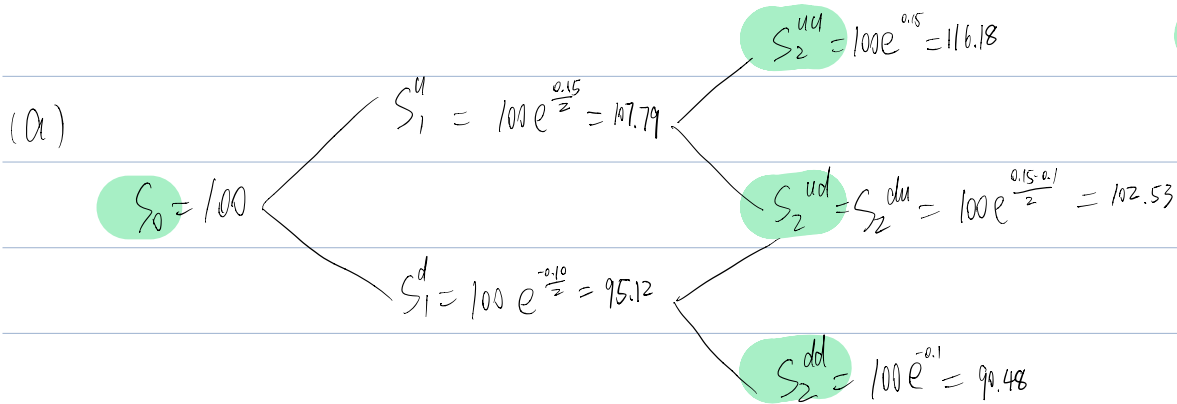
$$\check{p} = 0.99$$

$$1-\check{p} = 0.01$$

(b), (c)

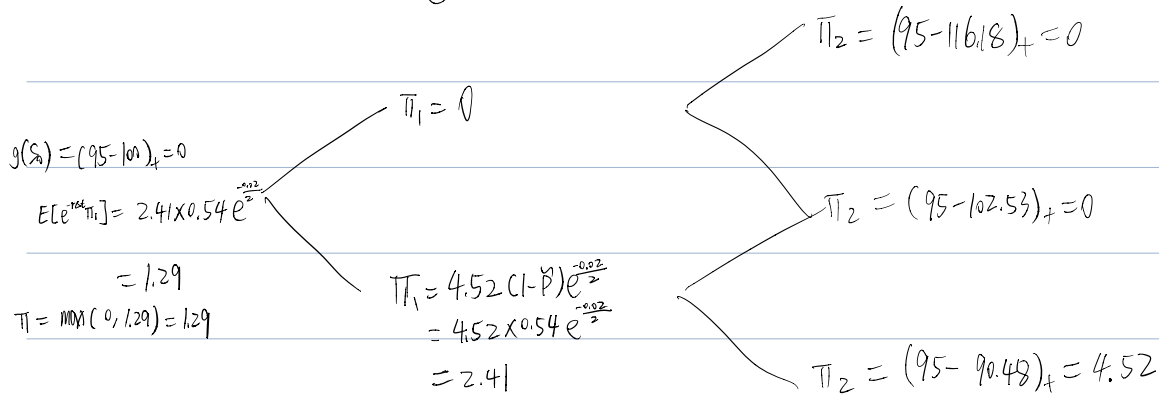


(2)



(b)

$$\tilde{p} = \frac{e^{\frac{0.02}{2}} - e^{\frac{-0.1}{2}}}{e^{\frac{0.15}{2}} - e^{\frac{-0.1}{2}}} = 0.46 \quad 1 - \tilde{p} = 0.54$$

(C) We should only exercise when S^{uu} .

(3). Need to prove $(S_n - K)_+ \leq \tilde{E}[e^{-r(m-n)}(S_m - K)_+]$

By Jensen's inequality, if f is convex, $f(E[X]) \leq E[f(X)]$

Let $f: x \rightarrow (x - K)_+$, f is convex.

Since $\{S_t e^{rt}\}_{t \geq 0}$ is a martingale, $E[S_t e^{rt} | \mathcal{F}_t] = S_t e^{rt}$

$$E[f(S_t e^{-rt})] = E[f(E[S_t e^{rt} | \mathcal{F}_t])] \leq E[E[f(S_t e^{rt}) | \mathcal{F}_t]] = E[f(S_t e^{rt})]$$

Payoff at $t = (S_t - K)_+$

$$\tilde{E}[e^{-rt}(S_t - K)_+] \leq \tilde{E}(e^{-rt}S_t - e^{-rt}K)_+$$

$$E[(S_t e^{-rt} - K)_+] \leq E[(S_t e^{rt} - K)_+]$$

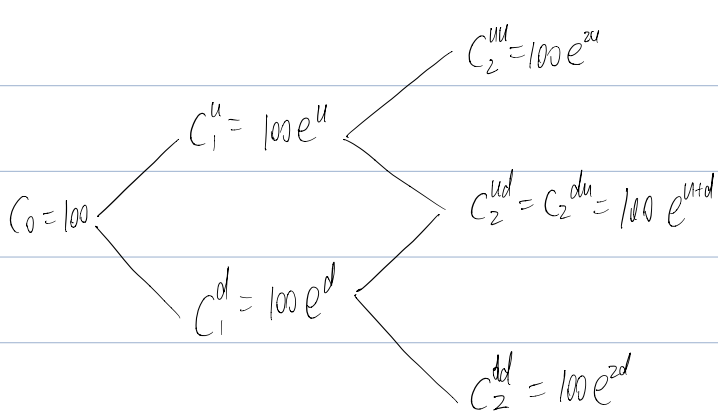
$$E[(S_t e^{-rt} - K e^{-rt})_+] \leq E[(S_t e^{rt} - K e^{-rt})_+]$$

$$(S_t - K) e^{-rn} \leq E[(S_m - K)_+ e^{-rm}] \quad 0 \leq n \leq m \leq T$$

$$(S_t - K) \leq E[(S_m - K)_+ e^{-r(m-n)}]$$

(4)

(a)



(b)

Payoff

