1.

$X_0 = 0$

$$X_1 \begin{cases} \xrightarrow{p} \Delta_0 u S_0 + (1+r)(0-\Delta_0 S_0) = \Delta_0 S_0 (u-(1+r)) \\ \xrightarrow{(1-p)} \Delta_0 d S_0 + (1+r)(0-\Delta_0 S_0) = \Delta_0 S_0 (d-(1+r)) \end{cases}$$

$$p(u-(1+r)) + (1-p)(d-(1+r)) = 0$$

$$p(u-(1+r)) + (d-(1+r)) - p(d-(1+r)) = 0$$

$$p(u-\cancel{(1+r)}-d+\cancel{(1+r)}) + d-(1+r) = 0$$

$$p(u-d) + d - (1+r) = 0$$

$$p = \frac{1+r-d}{u-d}$$

$$0 < p < 1$$

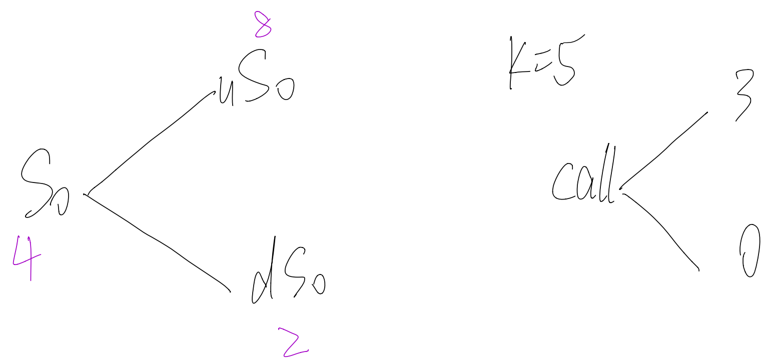
$$0 < \frac{1+r-d}{u-d} < 1$$

$$u-d > 0$$

$$0 < 1+r-d < u-d$$

$$\underbrace{0 < d}_{\text{Because no stock price can be negative.}} < 1+r < u$$

2.



$r = \frac{1}{4}$

(a) Δ shares in stock W dollar in bond at time 0

At time t:

$8\Delta + W^{1+r} = 3 \quad S_1 = S_1^u \quad ①$

$2\Delta + W^{1+r} = 0 \quad S_1 = S_1^d \quad ②$

$T = 1$, $r = 0.25$

Solve the equations:

$6\Delta = 3 \quad \Delta = \frac{1}{2}$

$-1 = W^{1.25}$

$W = 0.8$

$C = 4\Delta + W$

$= 4 \times \frac{1}{2} - 0.8 = 1.2$

(b) $X_1 = \Delta_0 S_1 + \Gamma_0 (S_1 - 5)_+ - \frac{5}{4}(4\Delta_0 + 1.20\Gamma_0)$

$$X_1 \begin{cases} \xrightarrow{p>0} 8\Delta_0 + 3\Gamma_0 - \frac{5}{4}(4\Delta_0 + 1.2\Gamma_0) = 8\Delta_0 + 3\Gamma_0 - 5\Delta_0 - \frac{3}{2}\Gamma_0 = 3\Delta_0 + \frac{3}{2}\Gamma_0 \\ \xrightarrow{q>0} 2\Delta_0 + 0 - \frac{5}{4}(4\Delta_0 + 1.2\Gamma_0) = 2\Delta_0 - 5\Delta_0 - \frac{3}{2}\Gamma_0 = -3\Delta_0 - \frac{3}{2}\Gamma_0 \end{cases}$$

If P is the probability of S_1^u occur, P is the probability of $X_1 = 3\Delta_0 + \frac{3}{2}\Gamma_0 > 0$

Then, $1 - P = q > 0$ is the probability of S_1^d occur, q is the probability of $X_1 = -3\Delta_0 - \frac{3}{2}\Gamma_0 < 0$

3.

$$
\begin{tikzcd}
 & \$100e^{(0.15)} \\
\$100 \arrow[ur, "p=0.5"] \arrow[dr, "q=0.5"'] & \\
 & \$100e^{(-0.05)}
\end{tikzcd}
$$

$K = 115$

$C = \$0.42$

(a)

$$\Delta_0 115 + We^r = 100e^{0.15} - 115$$

$$\Delta_0 95 + We^r = 0$$

$$20\Delta_0 = 100e^{0.15} - 115$$

$$\Delta_0 = 0.059171$$

$$0.059171 \times 100 + W = 0.42$$

$$W = -5.4971$$

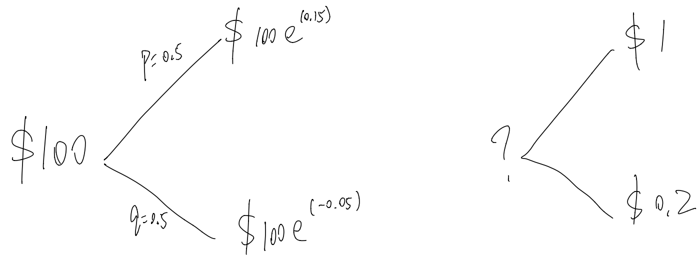
$$0.059171 \times 95 + (-5.4971)e^r = 0$$

$$r = 0.022332$$

$$1e^{0.022332 \times 1} = 1.022583$$

(b) $$\hat{p} = \frac{e^{rT} - e^{dT}}{e^{uT} - e^{dT}} = \frac{e^{0.022332} - e^{-0.05}}{e^{0.15} - e^{-0.05}}$$

$$\hat{Q} = 1 - \hat{p} = \frac{e^{0.15} - e^{0.022332}}{e^{0.15} - e^{-0.05}}$$



(c)

$$\Delta_0 115 + We^{0.022332} = 1$$

$$\Delta_0 95 + We^{0.022332} = 0.2$$

$$20\Delta_0 = 0.8$$

$$\Delta_0 = \frac{8}{200} = \frac{1}{25}$$

$$W = -3.520495$$

$$\frac{1}{25} \times 100 - 3.520495 \fallingdotseq 0.48$$

4.

put

$\$40$ (Stock price now), up: $40e^{0.3\sqrt{\frac{1}{2}}}$, down: $40e^{-0.3\sqrt{\frac{1}{2}}}$

$K=\$40$

4% / year

30% / year

6 months

(a) $30\% \cdot \frac{6}{12} = 15\%$

(b) $u = 15\%$

$d = -15\%$

(C) $\tilde{p} = \frac{e^{r\Delta t} - e^{d\Delta t}}{e^{u\Delta t} - e^{d\Delta t}} = \frac{e^{0.02} - e^{-0.15}}{e^{0.15} - e^{-0.15}} \approx 0.53$

(d)

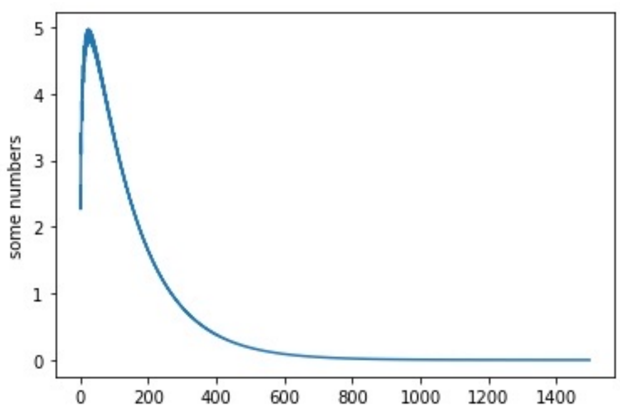
```
import math
import matplotlib.pyplot as plt

import numpy as np
def stock_matrix(N):
    stockM=np.zeros((1+2*N,N+1))
    stockM[N,0]=40
    for col in range(N):
        for row in range(1+2*N):
            if stockM[row,col]!=0:
                stockM[row-1,col+1]=stockM[row, col]*math.exp(0.15)
                stockM[row+1,col+1]=stockM[row,col]*math.exp(-0.15)
   # print(stockM)
    #print()
    return stockM
def payoff(matrix):
    matrix=matrix.T
    matrix=matrix[-1].T
    for i in range(len(matrix)):
        if matrix[i]!=0:
            matrix[i]=max(40-matrix[i],0)
    return matrix

def p0(N):
    p=np.zeros((1+2*N,N+1))
    p=p.T
    p[-1]=payoff(stock_matrix(N))
    p=p.T
    count=1
```

```
    last=2*N
    for col in range(N-1,-1,-1):
        #print(row)
        for row in range(count, last):
            p[row][col]=math.exp(-0.005)*(0.53*p[row-1][col+1]+0.47*p[row+1][col+1])
        count+=1
        last-=1
    return p

#print(p0(2))
list=[]
for i in range(1,1500):
    #print (p0(i))
    list.append(p0(i)[i][0])
    print(i)
print(list)
plt.plot(list)
plt.ylabel('some numbers')
plt.show()
```



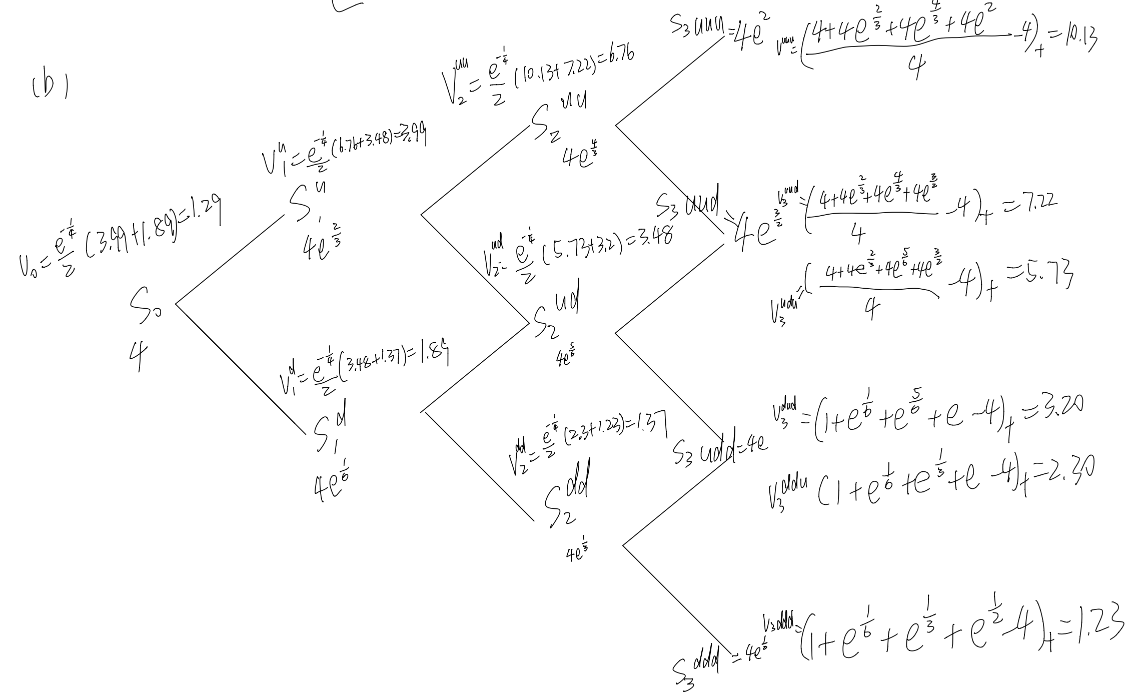
e. The put option price converge to 0.

5. (a) $V_n(s,y) = e^{-r\Delta t}\left[\tilde{p}\, V_{n+1}(s,y) + (1-\tilde{p})\, V_{n+1}(s,y)\right]$

$$= e^{-\frac{1}{4}}\left[\tfrac{1}{2} V_{n+1}(w_1 \cdots w_n u) + \tfrac{1}{2} V_{n+1}(w_1 \cdots w_n d)\right]$$

pay off

(b)



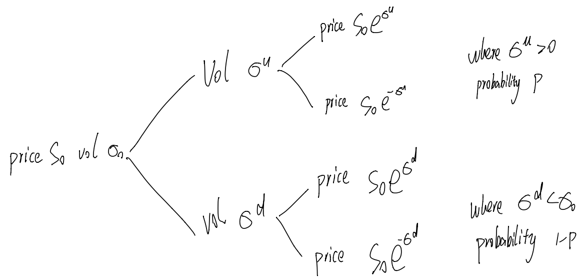
$V_0(4,4) = 1.29$

(c)

$$\delta_n(s,y) = \frac{\tilde{C}^u_{n+1}(s,y) - \tilde{C}^d_{n+1}(s,y)}{\tilde{S}^u_{n+1} - \tilde{S}^d_{n+1}}$$

6.

$S_0 = 100$



(a) $\tilde{P}(S_1 > S_0) = q$

$\tilde{P}(S_1 \leq S_0) = 1-q$

Define $q_\sigma = \tilde{P}(\sigma_1 = \sigma^u)$, $q^u = \tilde{P}(S_1 > S_0 \mid \sigma_1 = \sigma^u)$, $q^d = \tilde{P}(S_1 > S_0 \mid \sigma_1 = \sigma^d)$

$$S_0 = e^{-r}\left(q^u S_0 e^{\sigma u} + (q_\sigma - q^u) S_0 e^{-\sigma u} + q^d S_0 e^{\sigma d} + (1 - q_\sigma - q^d) S_0 e^{-\sigma d}\right)$$

$$e^r = q^u e^{\sigma u} + (q_\sigma - q^u) e^{-\sigma u} + q^d e^{\sigma d} + (1 - q_\sigma - q^d) e^{-\sigma d}$$

This equation has more than one solution since there are more than one variable in one equation.

(b) There are 4 different states at time 1, and it doesn't match the number of different values of a replication portfolio at time 1, which is 2.

So the market is not complete.