Bond Markets

• T bill price

$$P = \$100 \times \left[1 - d \times \frac{t}{360}\right]$$

- T note and T bond price Invoice Price = Flat Price + Accrued Interest
- Repo interest Interest = loan amount \times repo rate \times 1/360
- Repo gain/loss capital gain/loss on entire bond + carry

Bond Valuation

- · Annual effective rate $AER = (1 + APR/m)^m - 1$
- · Continuous compounding $m \to \infty \Rightarrow AER \to e^{APR} - 1$
- General bond pricing formula $P = \sum_{i=1}^{n} \frac{CF_i}{(1+x)^i}$
- General bond pricing formula with ann. APR $P = \sum_{i=1}^{n} \frac{CF_i}{(1+x)^i} = \sum_{i=1}^{T\times m} \frac{CF_i}{(1+y/m)^i}$

$$P = \frac{F}{(1 + y/m)^{T \times m}} \qquad y = m \times \left[\left(\frac{F}{P} \right)^{1/(T \times m)} - 1 \right]$$

$$P = \sum_{i=1}^{\infty} \frac{c/m}{(1+y/m)^i} = \frac{c}{y}$$
 $y = \frac{c}{P}$

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$$P = \underbrace{\frac{c}{y}}_{\text{perpetuity}} - \underbrace{\frac{1}{(1+y/m)^{T \times m}}}_{\text{delayed}} \underbrace{\frac{c}{y}}_{\text{perpetuity}}$$
$$= \frac{c}{y} \times \left[1 - \frac{1}{(1+y/m)^{T \times m}}\right]$$

Coupon bond price

$$P = \frac{\frac{c}{m}}{\left(1 + \frac{y}{m}\right)^{1}} + \frac{\frac{c}{m}}{\left(1 + \frac{y}{m}\right)^{2}} + \dots + \frac{\frac{c}{m}}{\left(1 + \frac{y}{m}\right)^{n-1}} + \frac{\frac{c}{m} + F}{\left(1 + \frac{y}{m}\right)^{n}}$$

$$P = \frac{c}{y} \left[1 - \frac{1}{(1 + y/m)^{T \times m}}\right] + \frac{F}{(1 + y/m)^{T \times m}}$$

Term Structure of Interest Rates

• Brandt's preferred yield model

$$r(t) = \alpha_0 + \alpha_1 t + \alpha_2 \ln(1+t) + \alpha_3 \left(\frac{1}{1+t} - 1\right) + \epsilon(t)$$

· Brandt's preferred discount function model $P(t) = 100 \times e^{\alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \epsilon(t)}$

Forward rates implied by spot rates

$$f(0,1) = r(1)$$

$$f(1,2) = m \times \left[\frac{(1+r(2)/m)^2}{(1+r(1)/m)^1} - 1 \right]$$

$$f(2,3) = m \times \left[\frac{(1+r(3)/m)^3}{(1+r(2)/m)^2} - 1 \right]$$

$$f(3,4) = m \times \left[\frac{(1+r(4)/m)^4}{(1+r(3)/m)^3} - 1 \right]$$

 $f(n-1,n) = m \times \left[\frac{(1+r(n)/m)^n}{(1+r(n-1)/m)^{n-1}} - 1 \right]$

· Spot rates implied by forward rates

$$r(2) = m \times ([(1 + f(1,2)/m) \times (1 + r(1)/m)^{1}]^{1/2} - 1)$$

$$r(3) = m \times ([(1 + f(2,3)/m) \times (1 + r(2)/m)^2]^{1/3} - 1)$$

$$r(4) = m \times \left(\left[(1 + f(3,4)/m) \times (1 + r(3)/m)^3 \right]^{1/4} - 1 \right)$$

$$r(n) = m \times \left(\left[(1 + f(n-1, n)/m) \times (1 + r(n-1)/m)^{n-1} \right]^{1/n} - 1 \right)$$

Price Sensitity and Hedging

• Dollar value of a basis point $DV01 = -\frac{1}{10,000} \times \frac{dP}{dy}$

$$D_{\text{mod}} = -\frac{1}{P} \times \frac{dP}{dY} \qquad D_{\text{mac}} = \left(1 + \frac{y}{m}\right) D_{\text{mod}}$$

- · Macaulay duration of zero coupon bond $D_{\text{mac}} = T$
- · Macaulay duration of coupon bond

$$D_{\text{mac}} = \sum_{i=1}^{m \times T} w_i \times \frac{i}{m} \qquad w_i = \frac{CF_i}{(1 + y/m)^i} \times \frac{1}{P}$$

 1st-order approximation of bond price change $\frac{\Delta P}{P} \simeq -D_{\mathsf{mod}} \times \Delta y$

• 1st-order approximation of DV01
DV01
$$\simeq -\frac{\Delta P}{10,000 \times \Delta y} = -\frac{P \times \Delta P/P}{10,000 \times \Delta y}$$

Convexity

$$C = \frac{1}{P} \frac{d^2 P}{dv^2}$$

• Convexity of zero-coupon bond $C = \frac{T \times (T + 1/m)}{(1 + y/m)^2}$

$$C = \frac{T \times (T + 1/m)}{(1 + y/m)^2}$$

• Convexity of coupon bond

$$C = \sum_{i=1}^{m \times T} w_i \times \frac{i}{m} \times \left[\frac{i}{m} + \frac{1}{m} \right] \times \frac{1}{(1 + y/m)^2} \qquad w_i = \frac{CF_i}{(1 + y/m)^i} \times \frac{1}{P}$$

• 1st-order approximation of duration change

$$\Delta D_{\text{mod}} \simeq -C \times \Delta y$$

• 2nd-order approximation of bond price change $\frac{\Delta P}{P} \simeq -D_{\text{mod}} \times \Delta y + \frac{1}{2} \times C \times (\Delta y)^2$

• Duration of portfolio

$$D_{\text{mod},p} = w_A \times D_{\text{mod},A} + w_B \times D_{\text{mod},B}$$

$$w_A = \frac{n_A \times P_A}{n_A \times P_A + n_B \times P_B} \quad w_B = \frac{n_B \times P_B}{n_A \times P_A + n_B \times P_B} = (1 - w_A)$$
• Duration neutral portfolio
$$n_B = -n_A \times \frac{P_A \times D_{\text{mod},A}}{P_B \times D_{\text{mod},B}} = -n_A \times \frac{DV01_A}{DV01_B}$$

$$n_B = -n_A \times \frac{P_A \times D_{\text{mod},A}}{P_B \times D_{\text{mod},B}} = -n_A \times \frac{\text{DV01}_A}{\text{DV01}_B}$$

• Volatility weighted duration neutral portfolio $n_B = -n_A \times \frac{\operatorname{Std}[\Delta y_A]}{\operatorname{Std}[\Delta y_B]} \times \frac{\mathsf{DV01}_A}{\mathsf{DV01}_B}$

$$n_B = -n_A \times \frac{\text{Std}[\Delta y_A]}{\text{Std}[\Delta v_B]} \times \frac{\text{DV01}_A}{\text{DV01}_B}$$

• Regression-based duration neutral portfolio

$$n_B = -n_A \times \text{Corr}[\Delta y_A, \Delta y_B] \times \frac{\text{Std}[\Delta y_A]}{\text{Std}[\Delta y_B]} \times \frac{DV01_A}{DV01_B}$$