

## MF702 PROBLEM SET 1

1. Shell will need to buy 100,000 barrels of oil in 10 days and it is worried about fuel costs. Suppose Shell goes long 100 oil futures contracts, each for 1,000 barrels of oil, at the current futures price of \$60 per barrel. Suppose futures prices change each day as indicated in the following table

TABLE 1. Futures prices

Day	0	1	2	3	4	5	6	7	8	9	10
Prices	60	59.50	57.50	57.75	58.00	59.50	60.50	60.75	69.75	61.75	62.50

- (a) What is the mark-to-market profit or loss (in dollars) that Shell will have on each date?
  - (b) What is Shell's total profit or loss after 10 days?
2. Options on futures are exchange-traded instruments that are often written on a variety of indices, such as the S& P 500 index. An option on a futures gives you the right to enter a futures contract at maturity for a predetermined future price. This works as follows: The maturity of the option is  $T'$ , the strike is  $K$ , and the call (put) option gives the holder of option the right to enter a long (short) position on a futures at time  $T'$  with maturity  $T$  with  $T \geq T'$ . Options on futures are typically American style. Suppose that  $C_t(F)$  is the price of the call option at time  $t$  if the futures price is  $F$ , while  $P_t(F)$  is the price of the put option. Assume also that  $(F_{t,T})_{0 \leq t \leq T}$  is the process that describes the futures prices with maturity  $T$ .

For this problem, assume that the options are European instead of American, assume, moreover, we don't take into account the mark to margin practice for the future (i.e. treat the future as a forward.) Later we will revisit the pricing of futures options after we learn the risk neutral pricing.

- (a) Assume that  $T' \leq T$ , derive the put-call parity for options on a futures.
  - (b) Assume that  $T' < T$ , is your put-call parity the same as (a)?

Hint: When the holder of the European call on a futures exercises at time  $T'$ , the holder get a futures which allows the holder to buy an asset with price  $K$  at time  $T$ . The holder will only exercise this European call when  $F_{T',T}$  is larger than  $K$ , because the option allows the holder to buy an asset with price  $K$  at time  $T$ , which is cheaper than entering a long position of a futures at time  $T'$  with the futures price  $F_{T',T}$ . Calculate the payoff of this European call at time  $T$ . For part (b), think about a portfolio A which has a long

position of call option, a portfolio B which has a long position of a put option, a long position of a futures entered at time 0 and a short position of a futures entered at  $T'$ .

3. The current price of a stock is \$94 and a 3-months European call with a strike price of \$95 currently sell for \$4.70. An investor who feels that the price of the stock will increase is trying to decide between buying 100 shares and buying 2,000 options (= 20 contracts). Both strategies involve an investment of \$9,400. What advice would you give? How high does the stock price need to rise for the option strategy to be more profitable? Assume that the interest rate is zero.
4. Suppose a corporate treasurer asks you to construct the following derivative: “I will have 1 million GBP to sell in 6 months. If the exchange rate less than 1.25, I want you to give me 1.25. If it is greater than 1.35, I will accept 1.35. If the exchange rate is between 1.25 and 1.35, I will sell the British Pound for the exchange rate.” Plot the payoff of this derivative as a function of USD/GBP exchange rate in 6 months. How could you use options to construct the derivative?
5. On September 8, 2020, an investor owns 100 Alphabet shares. The share price on that day was about \$1,537 and a December European put option with a strike price of \$1,500 costs \$27.30. The investor is comparing two alternatives to limit downside risk. The first involves buying one December European put option contract with a strike price of \$1,500. The second involves instructing a broker to sell the 100 shares as soon as Alphabet’s price reaches 1,500. Discuss the advantages and disadvantages of the two strategies.