Bond Markets

- T bill price
  $P = \$100 \times \left[1 - d \times \frac{t}{360}\right]$
- T note and T bond price
  Invoice Price = Flat Price + Accrued Interest
- Repo interest
  Interest = loan amount × repo rate × 1/360
- Repo gain/loss
  capital gain/loss on <u>entire</u> bond + carry

Bond Valuation

- Annual effective rate
  AER = $(1 + \text{APR}/m)^m - 1$
- Continuous compounding
  $m \to \infty \Rightarrow \text{AER} \to e^{\text{APR}} - 1$
- General bond pricing formula
  $$P = \sum_{i=1}^{n} \frac{CF_i}{(1+x)^i}$$
- General bond pricing formula with ann. APR
  $$P = \sum_{i=1}^{n} \frac{CF_i}{(1+x)^i} = \sum_{i=1}^{T\times m} \frac{CF_i}{(1+y/m)^i}$$
- Zero coupon bond price and yield
  $$P = \frac{F}{(1+y/m)^{T\times m}} \qquad y = m \times \left[\left(\frac{F}{P}\right)^{1/(T\times m)} - 1\right]$$
- Perpetuity price and yield
  $$P = \sum_{i=1}^{\infty} \frac{c/m}{(1+y/m)^i} = \frac{c}{y} \qquad y = \frac{c}{P}$$
- Annuity price
  $$P = \underbrace{\frac{c}{y}}_{\text{perpetuity}} - \underbrace{\frac{1}{(1+y/m)^{T\times m}}}_{\text{delayed}} \underbrace{\frac{c}{y}}_{\text{perpetuity}}$$
  $$= \frac{c}{y} \times \left[1 - \frac{1}{(1+y/m)^{T\times m}}\right]$$
- Coupon bond price
  $$P = \frac{\frac{c}{m}}{\left(1+\frac{y}{m}\right)^1} + \frac{\frac{c}{m}}{\left(1+\frac{y}{m}\right)^2} + \cdots \frac{\frac{c}{m}}{\left(1+\frac{y}{m}\right)^{n-1}} + \frac{\frac{c}{m}+F}{\left(1+\frac{y}{m}\right)^n}$$
  $$P = \frac{c}{y}\left[1 - \frac{1}{(1+y/m)^{T\times m}}\right] + \frac{F}{(1+y/m)^{T\times m}}$$

Term Structure of Interest Rates

- Brandt's preferred yield model
  $$r(t) = \alpha_0 + \alpha_1 t + \alpha_2 \ln(1+t) + \alpha_3\left(\frac{1}{1+t} - 1\right) + \epsilon(t)$$
- Brandt's preferred discount function model
  $$P(t) = 100 \times e^{\alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \epsilon(t)}$$

Forward rates implied by spot rates

$$f(0,1) = r(1)$$
$$f(1,2) = m \times \left[\frac{(1+r(2)/m)^2}{(1+r(1)/m)^1} - 1\right]$$
$$f(2,3) = m \times \left[\frac{(1+r(3)/m)^3}{(1+r(2)/m)^2} - 1\right]$$
$$f(3,4) = m \times \left[\frac{(1+r(4)/m)^4}{(1+r(3)/m)^3} - 1\right]$$
$$\ldots$$
$$f(n-1,n) = m \times \left[\frac{(1+r(n)/m)^n}{(1+r(n-1)/m)^{n-1}} - 1\right]$$

- Spot rates implied by forward rates
  $$r(1) = f(0,1)$$
  $$r(2) = m \times \left(\left[(1+f(1,2)/m)\times(1+r(1)/m)^1\right]^{1/2} - 1\right)$$
  $$r(3) = m \times \left(\left[(1+f(2,3)/m)\times(1+r(2)/m)^2\right]^{1/3} - 1\right)$$
  $$r(4) = m \times \left(\left[(1+f(3,4)/m)\times(1+r(3)/m)^3\right]^{1/4} - 1\right)$$
  $$\ldots$$
  $$r(n) = m \times \left(\left[(1+f(n-1,n)/m)\times(1+r(n-1)/m)^{n-1}\right]^{1/n} - 1\right)$$

Price Sensitity and Hedging

- Dollar value of a basis point
  $$\text{DV01} = -\frac{1}{10,000} \times \frac{dP}{dy}$$
- Duration
  $$D_{\text{mod}} = -\frac{1}{P} \times \frac{dP}{dY} \qquad D_{\text{mac}} = \left(1 + \frac{y}{m}\right) D_{\text{mod}}$$
- Macaulay duration of zero coupon bond
  $D_{\text{mac}} = T$
- Macaulay duration of coupon bond
  $$D_{\text{mac}} = \sum_{i=1}^{m\times T} w_i \times \frac{i}{m} \qquad w_i = \frac{CF_i}{(1+y/m)^i} \times \frac{1}{P}$$
- 1st-order approximation of bond price change
  $$\frac{\Delta P}{P} \simeq -D_{\text{mod}} \times \Delta y$$
- 1st-order approximation of DV01
  $$\text{DV01} \simeq -\frac{\Delta P}{10,000 \times \Delta y} = -\frac{P \times \Delta P/P}{10,000 \times \Delta y}$$
- Convexity
  $$C = \frac{1}{P}\frac{d^2P}{dy^2}$$
- Convexity of zero-coupon bond
  $$C = \frac{T \times (T + 1/m)}{(1+y/m)^2}$$
- Convexity of coupon bond
  $$C = \sum_{i=1}^{m\times T} w_i \times \frac{i}{m} \times \left[\frac{i}{m} + \frac{1}{m}\right] \times \frac{1}{(1+y/m)^2} \qquad w_i = \frac{CF_i}{(1+y/m)^i} \times \frac{1}{P}$$
- 1st-order approximation of duration change
  $$\Delta D_{\text{mod}} \simeq -C \times \Delta y$$
- 2nd-order approximation of bond price change
  $$\frac{\Delta P}{P} \simeq -D_{\text{mod}} \times \Delta y + \frac{1}{2} \times C \times (\Delta y)^2$$
- Duration of portfolio
  $$D_{\text{mod},p} = w_A \times D_{\text{mod},A} + w_B \times D_{\text{mod},B}$$
  $$w_A = \frac{n_A \times P_A}{n_A \times P_A + n_B \times P_B} \quad w_B = \frac{n_B \times P_B}{n_A \times P_A + n_B \times P_B} = (1 - w_A)$$
- Duration neutral portfolio
  $$n_B = -n_A \times \frac{P_A \times D_{\text{mod},A}}{P_B \times D_{\text{mod},B}} = -n_A \times \frac{\text{DV01}_A}{\text{DV01}_B}$$
- Volatility weighted duration neutral portfolio
  $$n_B = -n_A \times \frac{\text{Std}[\Delta y_A]}{\text{Std}[\Delta y_B]} \times \frac{\text{DV01}_A}{\text{DV01}_B}$$
- Regression-based duration neutral portfolio
  $$n_B = -n_A \times \text{Corr}[\Delta y_A, \Delta y_B] \times \frac{\text{Std}[\Delta y_A]}{\text{Std}[\Delta y_B]} \times \frac{DV01_A}{DV01_B}$$