MF 703: Programming for Mathematical Finance

Professor Chris Kelliher   Fall 2020

Problem set # 3

Due: Wednesday, October 22nd, by 4pm.

1. **Volatility Experiments:**

   (a) Download historical data for the S&P using the SPY ETF and for the VIX index, which we will use as a proxy for volatility.

   (b) Examine both the S&P and the VIX index data for autocorrelation. You may do this using the regression approach you used in the first two homeworks, or via stationarity tests and ARMA models. Do you find evidence of autocorrelation? Which series has more evidence of autocorrelation? Where would you expect more autocorrelation?

   (c) Calculate the correlation of the S&P and its implied volatility (using VIX as a proxy) on a daily and monthly basis. Is the correlation significant? What implications does this have for an options pricing model, such as the Black-Scholes model?

   (d) Calculate rolling 90-day correlatons of the S&P and its implied volatility as well. When does the correlation deviate the most from its long-run average?

   (e) Calculate rolling 90-day realized volatilities in the S&P and compare them to the implied volatility (again using VIX as a proxy). Plot the premium of implied vol. over realized vol. Is the premium generally positive or negative? When is the premium highest? Lowest?

   (f) Construct a portfolio that buys a 1M at-the-money straddle (long an at-the-money call and long an at-the-money put) every day in your historical period. Use the Black-Scholes model to compute the option prices and use the level of VIX as the implied vol input into the BS formula.

   (g) Calculate the payoffs of these 1M straddles at expiry (assuming they were held to expiry without any rebalances) by looking at the historical 1M changes in the S&P. Calculate and plot the P&L as well. What is the average P&L?

   (h) Make a scatter plot of this P&L against the premium between implied and realized volatility. Is there a strong relationship? Would you expect there to be a strong relationship? Why or why not?

   NOTE: All code for completing these exercises should be completed either in Python or C++ and should be written generically. You may end up using this code on future assignments so I encourage you to code thoughtfully.