MF703 Problem Set 2 Solutions Outline

October 15, 2020

Problem 1: Sector ETF Factor Modeling

(a)

https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html Note that the file comes with data all the way back to 1926 or so. We do not need all that data and are only interested in the data starting in 2010 since this is the time period that the ETF analysis was done on in HW1. Additionally, and this is very important, you need to divide the factor returns by 100 since what corresponds to a return of 1% in the data is given as 1, and not 0.01. If you don't divide, your betas in part (e) will not make sense and will not be comparable to the ones from HW1.

(b)

You should find that these correlations are significantly lower than those of the sector ETFs. Keep in mind that comparing covariance matrices does not actually give us any information about whether the correlations are large of small.

Note: If these factors were highly correlated, we would be introducing multicollinearity to our regression in part (e). This means that small changes in the data would cause very large changes in our estimated coefficients, so the regression becomes much less reliable.

(c)

Rolling correlations are not particularly stable over time, but they are (on average) much closer to 0 than those from HW1. You could calculate variance to compare.

(d)

Most common tests would be a Q-Q Plot or a KS Test (Kolmogorov - Smirnov). In either case, you need to know what distribution you are comparing your sample to. For example, if you are comparing to a standard normal distribution, then you need to standardize your data (subtract the mean and divide by the standard deviation). Some Q-Q plot libraries will do that for you if you specify it as a parameter, but as far as I know, you need to manually do that for a KS test.

(e)

In theory, these rolling betas should be more consistent that with the one-factor model from HW1. You can calculate variance of the rolling betas for each ETF to compare.

Note: We are only interest in β_{mkt} here as that is the coefficient that tells us how the ETF returns are related to the market returns.

(f)

The residuals are not normally distributed for a 3 factor model. You can check this again with either Q-Q plots or KS tests. This is somewhat of an issue because we have a large sample size and the CLT states that the residuals should converge to a normal distribution. Possible solutions include adding more factors, or running the same regression on monthly data.

Problem 2: Exotic Option Pricing via Simulation

(a)-(b)

The terminal values should have mean around 100 and standard deviation around 10. The histogram should look like a bell curve. Once again, check using Q-Q plot or KS test.

(c)

Using the given parameters, you should get something close to \$7.60. To compare to HW1, you need to use a different value of σ. Recall that in HW1, σ was given as a percentage and it was 0.25 . In the Bachelier model, σ is a dollar amount, so to compare you need to run this simulation with $\sigma = 25$. When you do this, you should get approximately \$19. This price for a lookback put is higher under the Bachelier model than under Black-Scholes, why? Would the price of a lookback call be higher under Black-Scholes or Bachelier? Think about these questions!

(d)

The value of Δ is approximately -1. Very small values of ϵ, will not work very well due to the randomness of the simulation. I found that $\epsilon = 1$ works fairly well (provided you are pricing with a sufficient number of paths). Alternatively, you can price the two options using the same random variables and get a much more accurate value for very small values of ϵ (i.e. 0.001 or less).
Note: You can get very accurate values for Δ if you compute the two prices in the numerator using the same random variables. The simplest way to do that would be to just set the seed to the same number before computing each price.