Yield Curve Construction

Goals:

- Describe the most common linear products that trade in interest rate markets.
- Define yield curves and discuss the most common yield curve construction techniques in interest rate markets.
- Discuss the empirical structure of the yield curve and the role of the Fed on the yield curve.

Interest Rates: Zero Coupon Bonds

- The simplest instrument in interest rate markets is a **zero coupon bond**, which pays its principal balance at maturity with no intermediate cashflows.
- Buying a zero coupon bond is equivalent to lending money today and being re-paid, with interest at maturity.
- Mathematically, the price of a zero coupon bond today can be written as:

$$Z(0,T) = exp\left(-\int_0^T r(s)ds\right) \tag{1}$$

- Basic present value concept suggest a dollar today is worth more than a dollar in the future. As a result, the price of a ZCB should be less than its principal value.

Interest Rates: Coupon Bonds

- Coupon bonds not only re-pay their principal at maturity P but pay a periodic, regular coupon C.
- Mathematically, the price of a coupon paying bond today can be written as:

$$B(0,T) = \left\{ \sum_i C \exp\left(-\int_0^{T_i} r(s)ds \right) \right\} + P \exp\left(-\int_0^{T} r(s)ds \right) \tag{2}$$

- Coupon bonds can be thought of as weighted sums of Zero Coupon Bonds, where the weights are determined by C and P respectively.

Interest Rates: Yield to Maturity

- In practice we are not given information on the discount process $r(s)$ directly. Instead we observe a set of bond prices.
- In this context, a natural first assumption is to use a constant discount rate, which is referred to as yield to maturity.
- For a zero-coupon bond, the pricing equation then becomes:

$$Z(0,T) = \exp\left(-yT\right) \tag{3}$$

- In this framework, we can easily extract y from a series of bond prices with different maturities.

Interest Rates: Duration & Convexity

- The risk to holders of zero-coupon or coupon paying bonds is that interest rates, or yields to maturity change.
- Note that in this portion of the course we are assuming that all cashflows will be paid in full. If this were not the case it would introduce another type of risk, **default risk**.
- Duration and convexity measure the sensitivity of a bond to changes in underlying yields.
- **Duration**, in particular, measures the first derivative with respect to a change in yield to maturity.
- **Convexity**, measures the second order change in bond price with respect to a change in yield to maturity.

Interest Rates: Daycount Conventions

- Revisiting the equation for pricing a zero-coupon bond, there is one more set of conventions that we need to know in order to value successfully, **daycount conventions**.
- In particular, we need to know how to measure T correctly.
- This may sound fairly unambiguous, but there are a set of conventions used in different markets and it is important to use the right one.
- Most common Daycount Conventions:
  - Actual / Actual
  - Actual / 360
  - 30 / 360

Interest Rates: Yield to Maturity vs. Forward Rates

- One challenge of using a yield to maturity driven approach is that for different bonds cashflows paid at the same time may be discounted at different rates.
- This clearly is not an appealing feature as the cashflows are economically equivalent.
- Forward rates overcome this problem by breaking the curve into different usually equally spaced segments.
- Each segment would then have a different rate that could be applied to all bonds with a cashflow linked to this rate.
- Later in this lecture we will see how we can use this set of forward rates to price bonds and construct a single consistent yield curve.

Interest Rate Markets: Traded Products

- **Treasuries**: Coupon paying bonds backed by the US Government.
- **Libor**: Libor is the instantaneous rate at which banks offer to lend to each other.
- **OIS**: Fed funds effective rate. The average rate at which banks lend to each other over a given day.
- **FRA's / Eurodollar Futures**: agreements to pay or receive Libor at a future date in exchange for a fixed rate set at trade initiation.
- **Swaps**: A swap is an agreement to exchange cash-flows at a set of pre-determined dates. The most common type of swap is a fixed for floating rate swap.
- **Basis Swaps**: A basis swap is a floating for floating rate swap where Libor is exchanged for Fed Funds / OIS.

Interest Rate Markets: Main Market Participants

- **Corporates** use interest rate derivatives markets to hedge interest rate risk.
- **Mortgage issuers** with inherent prepayment risk related to interest rates may also use interest rate products as hedging vehicles.
- **Banks** play the role of **market maker**, matching buyers and sellers and providing a liquid market for OTC products.
- **Hedge funds** also use interest rate products to express their macroeconomic views and express relative value positions.

Cash vs. Synthetic Instruments

- Bonds, including treasuries and other sovereign bonds, as well as corporate bonds, are cash instruments.
- Derivatives, such as swaps, futures and credit default swaps are synthetic instruments.
- In a cash instrument, a principal amount is received at maturity and there is an upfront cost to purchase the bond.
- In a synthetic instrument, no cashflow changes hands at trade initiation and there is no principal payment at maturity. The trade instead references a set notional value for calculation of each of its cashflows.

Cash vs. Synthetic Trading Example: Swap Spreads

- Swaps and treasuries provide similar exposure, therefore one might naturally think they would have the same yields.
  - In particular, consider a portfolio that borrows at Libor to invest in treasuries. How does this compare to a swap?
- However, because one is a cash instrument, requiring posting of upfront capital, and the other is a synthetic, they in practice trade at different yields.
- Swap spreads measure the difference in yields between vanilla swaps and treasuries.
- NOTE: Because of this spread between treasury and swap rates, treasuries are generally not used when constructing a yield curve.

Cash vs. Synthetic Trading Example: Swap Spreads

- Generally speaking, the level of the swap spread is a function of the following
  - Value of Collateral
  - Liquidity
  - Supply and Demand Dynamics
- Swap spreads, and other such bases, tend to have a risk-on feel to them, as collateral is most valuable in times of stress.
- Many investors, such as hedge funds try to harvest this spread.
- This requires a significant amount of leverage in order to make the potential return attractive.
- We will see a similar basis in the credit markets later in the course.

Forward Rate Agreements & Eurodollar Futures

- Forward Rate Agreements (FRA's) are agreements to exchange Libor for some fixed rate at some specified future date.
- FRA's are quoted in terms of the annualized rate. The actual payment will be multiplied by the trade **notional** as well as the **year fraction** of the rate exchanged.
- For example, if 3-month Libor is exchanged, a multiplier of ≈ 0.25 will be applied to the annualized rate.
- Eurodollar futures are futures contracts linked to 3 month Libor.
- They are exchange traded and have standardized terms.
- Eurodollar futures are quoted in price: $P = 100(1 - L)$

Difference between FRAs and Eurodollar Futures

- The difference between a FRA and a Eurodollar future is that a FRA is an OTC contract and a Eurodollar future is exchange traded.
- Exchange traded products require adjustments to their posted collateral daily, whereas this is not the case for OTC instruments.
- Because of the negative correlation between the P&L of a Eurodollar future and discount rates, a convexity correction emerges in Eurodollar futures.
- Specifically, if we are required to post collateral when rates are higher, we are posting collateral when the value we could earn on our deposits is highest.

Credit Risk Embedded in Libor

- Prior to the Financial Crisis, Libor was the sole rate used as the risk-free discount rate in interest rate markets.
- Many issues with Libor arised in the Financial crisis, including alleged manipulation.
- Additionally, Libor is an unsecured rate and thus includes a component of credit risk embedded in banks.
- Having said that, trillions of dollars of swaps and other derivatives have been creating that are linked to Libor, making it difficult to retire.
- This creates an added complexity when creating a curve, as we need to model both Libor and OIS rates.

Swaps

- A swap is an agreement to exchange cashflows periodically at predetermined future dates.
- A vanilla swap is a *fixed* for *floating rate* swap.
- The most common type of swaps have Libor as their floating rate and are fixed vs. floating.
- Each swap payment is calculated by multiplying the fixed or floating reference rate by the appropriate time period, using the appropriate daycount convention.
- Swaps have a specified **reference notional**
- Swaps can either be spot starting or forward starting.

Valuation of Swaps

- Valuing a swap requires the present value of the two legs:
  - Fixed Leg:

$$\text{PV(fixed)} = \sum_{i=1}^{N} \delta_{t_i} C D(0, t_i) \tag{4}$$

where C is the set fixed coupon and $D(0, t_i)$ is the discount factor from today until time t_i and δ_{t_i} is the time between payments.

  - Floating Leg

$$\text{PV(float)} = \sum_{i=1}^{N} \delta_{t_i} L_{t_i} D(0, t_i) \tag{5}$$

where L_{t_i} is the set fixed coupon and $D(0, t_i)$ is the discount factor from today until time t_i and δ_{t_i} is the time between payments.

- A break-even, or fair swap rate is the rate that equates the present values of the two legs.

- Equating these legs and solving for the unknown, the fixed coupon rate, we are left with:

$$\hat{C} = \frac{\sum_{i=1}^{N} \delta_{t_i} L_{t_i} D(0, t_i)}{\sum_{i=1}^{N} \delta_{t_i} D(0, t_i)} \tag{6}$$

Annuity Function

- The denominator in (6) is the value of a constant stream of payments and is often referred to as the annuity function:

$$A(t,T) = \sum_{i=1}^{N} \delta_{t_i} D(0,t_i) \tag{7}$$

- The annuity function, or PV01, tells us the present value of a one basis point annuity between two dates.
- It can be proven that the swap rate is a martingale under the annuity numeraire.
- Therefore when we price a swaption, today's annuity value plays the role that a discount factor plays in standard risk neutral valuation for other asset classes.

Basis Swaps

- A Basis swap is a floating for floating rate swap where the Libor rate is exchanged for the OIS rate.
- In a multi-curve paradigm, we must create a term structure of both Libor and OIS rates.
- Basis swaps provide us the link between the two, and enable us to extract the Libor-OIS basis as a function of time.

Valuation of Basis Swaps

- Valuing a swap requires the present value of the two legs:
  - Floating Libor Leg:

$$\text{PV(float\_libor)} = \sum_{i=1}^{N} \delta_{t_i} L_{t_i} D(0, t_i) \tag{8}$$

    where C is the set fixed coupon and $D(0, t_i)$ is the discount factor from today until time t_i and δ_{t_i} is the time between payments.

  - Floating OIS Leg

$$\text{PV(float\_OIS)} = \sum_{i=1}^{N} \delta_{t_i} (F_{t_i} + B) D(0, t_i) \tag{9}$$

    where L_{t_i} is the set fixed coupon and $D(0, t_i)$ is the discount factor from today until time t_i and δ_{t_i} is the time between payments.

- The breakeven basis swap rate can be written as:

$$\hat{B} = \frac{\sum_{i=1}^{N} \delta_{t_i}(L_{t_i} - F_{t_i})D(0,t_i)}{\sum_{i=1}^{N} \delta_{t_i} D(0,t_i)} \tag{10}$$

where L_{t_i} is the libor rate and F_{t_i} is the fed funds, or OIS rate.

Yield Curve / Term Structure of Interest Rates

- Our goal is to extract a consistent, coherent term structure, or yield curve from the set of quoted products, including swaps, FRA's and Eurodollar futures.
- We also want to overcome the issues of using yield to maturity and having the same cashflow valued using a different rate for different instruments.
- We also want to be able to incorporate all traded market information, including swaps, FRA's, basis swaps, etc.

Yield Curve / Term Structure of Interest Rates

- To do this correctly, we really need two term structures, one for Libor and one for the basis between Libor and OIS.
- The term structure is a representation of the interest rate surface that enables us to compute any instantaneous, forward or swap rate.
- The standard representation is a set of underlying **forward rates**.

Term Structure: Types of Rates

- **Spot Rates**: The rate at which we can instantaneously borrow or lend today for a specified amount of time.
- **Forward Rates**: The rate at which we can lock in to borrow or lend at some future time, for some predefined period.
- **Zero Rates**: Discount rates for zero coupon bonds.
- **Swap Rates**: Breakeven / Par rates on vanilla fixed for floating swaps.

Term Structure: Traded Market Components

- When calibrating a term structure, we have the following set of data to use:
  - Current values of Libor and Fed Funds
  - Eurodollar contracts corresponding to the first 8 quarterly contracts.
  - Swaps with maturities spanning 1 to 30 years
  - Basis Swaps with maturities spanning 3 months to 30 years.
- Note that some of the rates used in these instrument will be overlapping, creating a challenge when calibrating.

Components of a Term Structure

- Zero Coupon Bonds (Discount Factors):

$$D(S,T) = \exp\left(-\int_S^T f(s)ds\right) \tag{11}$$

- Forward Rates

$$F(S,T) = \frac{1}{\delta}\left[\exp\left(\int_S^T f(s)ds\right) - 1\right] \tag{12}$$

- Spot Rates

$$\text{Spot}(T) = F(0,T) \tag{13}$$

Components of a Term Structure

- Swap Rates

$$S(S,T) = \frac{\sum_{i=1}^{N} \delta_{t_i} L_{t_i} D(S,t_i)}{\sum_{i=1}^{N} \delta_{t_i} D(S,t_i)} \tag{14}$$

Extracting a Yield Curve from Market Data

There are two main methods for extracting a yield curve:

- **Bootstrapping**: Bootstrapping works iteratively beginning with the nearest expiry instrument. For this instrument, we find the constant rate that enables us to match the market observed price. The algorithm then moves onto the next instrument, fixes the part of the term structure that was used to match the previous set of instruments, and solves for the rate over the remaining period that matches the next market price.
- **Optimization**: Alternatively, we can use optimization to minimize the squared distance from our market data.
- For more information, see: Curve Fitting Lecture Notes

Yield Curve: Bootstrapping Procedure

- The exact details of bootstrapping a yield curve in the presence of Libor & OIS rates and a basis between them is beyond the scope of this course.
- Having said that, here's a summary of a simple bootstrapping procedure to give you the idea:
  - Order the securities that you have market prices for by maturity.
  - Begin with the closest maturity security. Invert the formula for the FRA / Eurodollar / Swap rate to find the **constant forward rate** that allows you to match the market traded FRA / Eurodollar / Swap Rate.
  - Fix this **constant forward rate** for this time interval.

  - Move to the next traded instrument. Holding the forward rate constant for the cashflows expiring prior to the last instrument, find the **constant forward rate** of the remaining dates that enable us to match market price.
  - Continue to do this until all market instruments have been matched.

- You will have a chance to work through an example of this on your homework.

Yield Curve: Bootstrapping Procedure in Practice

- In practice, the first contracts in our bootstrapping algorithm are Eurodollar futures.
- Eurodollars futures are quoted in price by convention, that is $P = 100 - R$ where R is the annualized Eurodollar rate.
- Eurodollar futures are processed sequentially by their maturity, and for each we would extract a distinct piecewise constant forward rate using equation (12).
- We would then proceed to process swaps sequentially by maturity, with the forward rates obtained at the front of the curve using Eurodollars fixed. These rates can be extracted using equation (13).

Yield Curve: Multi-Curve Bootstrapping in Practice

- Lastly, if we were working in a multi-curve framework, we would also need to back out Libor-OIS basis and OIS discount rates.
- For purposes of this course we will assume for simplicity that this basis is negligible and will focus on single-curve calibration.
- That said, in practice this is an important added step...

Yield Curve: Strengths and Weaknesses of Bootstrapping

- Bootstrapping is a simple, easy to understand technique.
- It will also generally fit all of the available market data exactly.
- Conversely, bootstrapping leads to jagged and unrealistic forward rates and does not provide us with a mechanism for incorporating smoothness of the curve as an input in the calibration.
- This issue can be overcome by using an optimization procedure, however this does add complexity, creating a natural trade-off.

Optimization: Background & Terminology

- Optimization problems that we face will have the following form:

$$\max_{x} \left\{ f(x) \,\middle|\, g(x) \leq c \right\} \tag{15}$$

- We refer to $f(x)$ as the **objective function** in our optimization.
- We refer to $g(x) \leq c$ as the **constraints** in our optimization.
  - We generally differentiate between *equality* and *inequality* constraints
- Remember that maximizing a function $f(x)$ is the same as minimizing the function $-f(x)$.
- We refer to x^* as a maximizer of the function $f(x)$ if: $f(x) \leq f(x^*) \quad \forall x$

First & Second Order Conditions: Single Variable

- Necessary, but not sufficient conditions for x^* to be a global maximizer of $f(x)$ are:

$$\frac{\partial f(x^*)}{\partial x^*} = 0 \tag{16}$$

$$\frac{\partial^2 f(x^*)}{\partial x^{*2}} < 0 \tag{17}$$

- Similarly, necessary but not sufficient conditions for x^* to be a global minimizer of $f(x)$ are:

$$\frac{\partial f(x^*)}{\partial x^*} = 0 \tag{18}$$

$$\frac{\partial^2 f(x^*)}{\partial x^{*2}} > 0 \tag{19}$$

First & Second Order Conditions: Many Variables

- We refer to the first derivative of the objective function as the **Gradient** and the second derivative as the **Hessian**.
- In multi-dimensional problems with N parameters:
  - The Gradient will be a vector of length N.
  - The F.O.C. is: $\nabla f(x^*) = 0$
  - The Hessian will be an $N \times N$ matrix.
  - The S.O.C. is that the Hessian is *negative definite* for a maximum and that it is *positive definite* for a minimum.

Unconstrained Optimization

- Let's start by walking through a very simple unconstrained minimization example.
- Consider the following problem:

$$\min_{x} \quad f(x) \tag{20}$$

$$f(x) = x^2 + 3x + 12 \tag{21}$$

- The F.O.C is: $\frac{\partial f(x)}{\partial x} = 2x + 3 = 0.$
- The S.O.C. is: $\frac{\partial^2 f(x)}{\partial x^2} = 2 > 0.$
- $x^* = -1.5$ is a minimum of f(x).
- Note that since the function is convex we can say that this local minimum is also a global minimum.

Lagrange Multipliers: Two Variables

- For constrained optimization problems we use Lagrange Multipliers.
- The key insight behind Lagrange Multipliers is that the function $f(x, y)$ is **tangent** to the constraint $g(x, y) = c$ at optimal points.
- Recall that two functions are tangent when their gradients are parallel. Therefore, we have:

$$\nabla f(x, y) = \lambda \nabla g(x, y) \tag{22}$$

when the constraint is $g(x, y) = c$.

Lagrange Multipliers: Two Variables

Lagrange re-wrote this as a single function, referred to as a Lagrangian, which incorporates the constraint. Note the extra variable λ:

$$\begin{aligned}
\mathcal{L}(x, y, \lambda) &= f(x, y) - \lambda(g(x, y) - c) \\
\nabla \mathcal{L}(x, y, \lambda) &= \nabla f(x, y) - \lambda \nabla g(x, y) \\
\partial_\lambda \mathcal{L}(x, y, \lambda) &= -(g(x, y) - c) \qquad (23)
\end{aligned}$$

Constrained Optimization Example

- To see this, let's consider a 2D objective function with a single equality constraint:

$$\max_{x,y} \left\{ f(x,y) \,\middle|\, g(x,y) = c \right\} \tag{24}$$

where

$$\begin{aligned} f(x,y) &= xy^2 \\ g(x,y) &= 3x^2 + 2y^2 = 6 \end{aligned} \tag{25}$$

- To solve this problem, we can use **Lagrange Multipliers**. That is, we can re-write (25) in terms of the following Lagrangian:

$$\begin{aligned} \max_{x,y,\lambda} & \quad \mathcal{L}(x,y,\lambda) \\ \mathcal{L}(x,y,\lambda) &= xy^2 - \lambda\left(3x^2 + 2y^2 - 6\right) \end{aligned} \tag{26}$$

Constrained Optimization with Lagrange Multipliers

- Notice that we have transformed our problem into an unconstrained optimization.
- As a result, we can use the same procedure for finding a local maximum as we did in the unconstrained case.
- The First Order Conditions are:

$$\frac{\partial \mathcal{L}(x,y,\lambda)}{\partial x} = y^2 - 6x\lambda = 0 \tag{27}$$

$$\frac{\partial \mathcal{L}(x,y,\lambda)}{\partial y} = 2xy - 4y\lambda = 0 \tag{28}$$

$$\frac{\partial \mathcal{L}(x,y,\lambda)}{\partial \lambda} = 3x^2 + 2y^2 - 6 = 0 \tag{29}$$

- We could similarly make a Hessian with second order derivatives.

Yield Curve: Optimization Procedure

- In the contexnt of yield curve construction, our goal is to apply these optimization techniques to find the best representation of the yield curve for a given set of market data.
- We may choose to try to match all the traded quantities exactly, or we may choose to use a least squares type approach.
- The process begins by specifying a parameterization of the yield curve. As an example, we could break the term structure into distinct, non-overalpping segments and assume that forward rates are piecewise constant in each interval.

Yield Curve: First Optimization Framework

- Let R_j denote the market traded rate for the jth benchmark instrument.
- Further, let $\hat{R}_j$ denote the model rate for the corresponding benchmark instrument.
- Also assume that our forward rate proces, $f(t)$, is a piecewise constant function.
- Remember that this benchmark instrument may be a swap, Eurodollar, etc.
- Our goal is to choose a set of piecewise constant rates that minimize the distance between market and model rates.
- To do this, we need to specify an appropriate objective function in

our optimization, such as least squared distance:

$$Q(f) = \sum_{j=1}^{m} (R_j - \hat{R}_j)^2 \tag{30}$$

- where m is the number of benchmark instruments.
- Clearly, we want to minimize $Q(f)$.

Yield Curve: Optimization Procedure

- While this would work, it would lead to discontinuities in the forward rate process on the edges of each interval.
- We might prefer a method that gives us a continuous forward rate process.
- Further, we might want certain derivatives also to be continuous along the process.
- It turns out this is possible via spline and b-spline functions, which have some properties making them attractive here.

Yield Curve: Splines

- Splines are piecewise polynomials of degree d that have d-1 continous derivatives.
- The most common type of Spline is a cubic spline, where $d = 3$:

$$f(i, t) = a_i + b_i t + c_i t^2 + d_i t^3 \tag{31}$$

- But we can't just choose the $f(i, t)$ independently, because we also want process to be continuous with continuous first and second order derivatives.
- Most programming languages, such as R and Python have built-in function for splines

Yield Curve: Optimization Framework with Splines & B-Splines

- The optimization framework, including the objective function continues to be the same as we incorporate splines/b-splines.
- The difference would come in how we formulate the forward rate process $f(t)$.
- For more details on the implementation of yield curve construction using splines and B-Splines, please see: Curve Fitting Lecture Notes

Yield Curve: Regularization of Optimization Procedure

- In addition to fitting to a set of market data, we also may want to add a term to our objective function ensuring smoothness of our curve (eliminating the jaggedness of forward rates we might see otherwise).
- A common regularization term is a **Tikhonov Regularizer**, which minimizes the curvature of the function.
- The following is an example of a Tikhanov regularization term:

$$Q(\lambda, f) = \lambda \int_0^T \frac{\partial^2 f(t)}{\partial t^2} dt \tag{32}$$

where λ is a constant denoted the weight on the smoothness term and Q is the value of the regularization term.

Yield Curve Construction: Comparison of Optimization to Bootstrapping

- Optimization techniques give us greater control over the yield curve construction process.
- Importantly, they enable us to balance between finding a good fit to the data and having intuitive, smooth, forward rates curves.
- They do however add some overhead as one needs to formulate and solve the optimization problem.
- R and Python have built in optimization packages that can handle solving these types of problems, such as nlopt in R and scipy & cvxpy in Python.

Yield Curve Construction: Economic Decomposition of a Yield Curve

- The following are the main economic drivers of a yield curve:
  - **Fed Policy Rate(s)**: Short term rate set by the Fed aimed at managing economic growth and inflation.
  - **Inflation**: (Nominal) Bonds lose value in the presence of inflation and thus investors will require higher levels of interest when inflation is higher.
  - **Term Premia**: Premium investors are compensated for locking up their capital for longer periods.
  - **Policy Uncertainty**: Expected future Fed policy decisions and uncertainty around these decisions

Yield Curve Construction: The Role of Fed Policy

- The Fed is responsible for setting short term interest rates via the Fed Funds Rate.
- They use this rate as a tool to accomplish their dual mandate of:
  - Maximum Employment
  - Stable inflation (2% target)
- There is a tradeoff between these two. If the economy is allowed to run too hot, it is likely to generate high levels of inflation.
- Conversely, if policy is tightened to rein in inflation, it is likely to led to lower levels of employment.
- Increases in the Fed Funds rate are used to tighten monetary conditions and are usually done during periods of high growth to

ensure the expansion is sustainable.

- Decreases in the Fed Funds rate are used to loosen monetary conditions and are usually done during recessions when the economy needs to be stimulated and jobs need to be created.

Yield Curve Construction: Extracting Implied Fed Policy from Market Data

- It is of great interest to market participants when the Fed will raise or lower rates.
- Under certain simplified assumptions, we can use a Binomial Tree type model to extract a set of hike/cut probabilities from the market traded futures contracts on the Fed Funds rate.
- When doing this exercise, we often assume a standardized size of all hikes/cuts.
  - As a simple example, lets assume that the current Fed Funds rate is 25 bps.
  - Lets also assume that the Fed Funds rate is 37.5 bps in one month.

  - If we assume that the only two actions the Fed can take are to leave the rate unchanged and hike by 25bps, what is the implied hike probability?

- More general models that incorporate options can also be created.
- These probabilities are watched closely by hedge funds and other fixed income investors, and tend to pretty accurately forecast the Fed's actions.