Interest Rate Models

Goals:

- Describe the most common interest rate derivatives and the main approaches to modeling them.
- Describe volatility smiles in interest rate derivatives and the most common SDEs used

Review: Interest Rate Products

- As we saw in the last lecture, there are many linear products that we can trade, including:
  - FRA's
  - Eurodollar Futures
  - Swaps
  - Basis Swaps
- We also saw that we can express each of these products in terms of **forward rates** and saw how we can parameterize the yield curve by these forward rates.

Review: Approaches to Yield Curve Construction

- Recall from the last lecture that constructing a yield curve involves choosing the forward rates that best match market data.
- In particular, we discussed two approaches for handling this:
  - **Bootstrapping**: Iterate through the instruments from shortest to longest maturity fixing their forward rates as you proceed.
  - **Optimization**: Minimize the least squares distance between model and market rates via an optimization procedure.
- Optimization provided us with more flexibility to ensure a smooth, intuitive curve shape but adds complexity.

Review: Using Bootstrapping to Extract a Yield Curve

- We begin by using the current Libor to set the current spot rate
- Next we iterate through the Eurodollar futures. Each Eurodollar future is 3 months so at each point you will be setting the forward rate for a 3 month period.
- Finally, proceed to swaps beginning with the two-year swap and follow the same process.
- An astute observer might notice that the 8 Eurodollar contracts and two year swap rate span the same sections of the forward curve.
- Therefore, we cannot match both and must only use the first 7 Eurodollar futures or omit the two-year swap.

Review: Using Optimization to Extract a Yield Curve

- When using optimization we would begin with a parameterization of the forward rate process of the curve, $f(t)$.
- We could make $f(t)$ piecewise constant, or use spline/b-spline functions.
- Next, we choose an objective function that minimizes the distance between market and model interest rates.

$$Q(f) = \sum_{j=1}^{m} (R_j - \hat{R}_j)^2 \tag{1}$$

- Recall that we could also add a regularization term in order to try to control the jaggedness of the curve.

Empirical Observations of Yield Curves

- Generally speaking a yield curve can be described by:
  - The level of interest rates
  - The slope of the curve
  - The curvature of the curve
- Slope is of particular interest and watched by market participants.
- Upward sloping curves correspond to normal conditions with term premium in the curve and/or expected future rate increases.
- Inverted yield curves correspond to stressed market condition and usually signal market expectations of future rate cuts.
- The Fed and other researchers have found that in almost all cases an inverted yield curve portends a recession.

Principal Component Analysis

- We can gain insight into the empirical behavior of yield curves by examining the structure of the comovements of different rates.
- One way to do this is via **Principal Component Analysis**
- PCA is at heart a matrix decomposition that creates orthogonal factors.
- The underlying spectral decomposition of V can be written as:

$$V = CDC^{\top}$$

- PCA can help us reduce dimensionality of the movements of the yield curve and show which factors are driving the largest portions of the variance.

Yield Curve Construction: PCA of the Yield Curve

- PCA can help determine what drives changes in the yield curve.
- This can be done by performing PCA on a covariance matrix or correlation of empirical rate shifts.
- In practice, when we do this we generally find that the first few principal components have intuitive appeal and roughly correspond to chages in:
  - Level of Rates (i.e. Parallel Shifts)
  - Slope of the Curve (i.e. Steepening / Flattening)
  - Curvature
- These first 2-3 components generally explain $\approx 95\%$ of the variance of the yield curve.

Interest Rate Derivatives: Overview

- Vanilla Options
  - Eurodollar Options
  - Caps
  - Swaptions
- Exotic Options
  - Bermudan Swaptions
  - Spread Options
  - CMS Options

Linear vs. Non-Linear Payoffs

- Linear Payoffs (e.g. Swaps)

  Can be replicated statically

  Do not require use of stochastic models

- Vanilla Non-Linear Payoffs (e.g. Swaptions)

  Liquid market pricing exists

  Requires calibration of a stochastic model

  Can be valued via a simple, market standard model

  Requires dynamic replication

- Exotic Non-Linear Payoffs (e.g. Bermudan Swaptions)

  Illiquid products with no standardized market prices.

  No market standard model. Generally requires bigger, term structure model.

Numeraires in Interest Rate Modelling

- In traditional valuation of derivatives in most asset classes, we operate in the risk neutral measure.
- Consider the following risk neutral pricing equation in the risk neutral measure:

$$p_0 = \int_{-\infty}^{+\infty} e^{-\int_0^T r_u \, du} F(S_T) \phi(S_T) \, dS_T \tag{2}$$

- When doing this, we often assume that interest rates are deterministic (and sometimes constant)
- In interest rate markets, however, interest rates play a role of discounting and are also in the payoff.
- This adds another level of complexity and often means that the risk neutral measure will not be the most convenient.

What is a Numeraire?

- In this class we don't delve too deply into the derivations of numeraires but it is important to understand the basic concept/intuition.
- A numeraire is a **tradable asset** with a strictly positive prices process at all times.
- When solving options pricing problems, we compute the relative price of an asset or derivative discounted by the numeraire:

$$S^N(t) = \frac{S(t)}{N(t)} \tag{3}$$

- In standard pricing problems in other asset classes, the numeraire is a zero-coupond bond, that is: $N(t) = \exp(rT)$

What is a Numeraire?

- While this is a convenient numeraire for equities and other asset classes, it is not always convenient in rates modelling.
- Note that the choice of a numeraire is a matter of convenience. We will see in rates modelling this varies by the derivative we are working with.
- This means we can work in a so called **equivalent martingale measure** which may be different for **swaptions** than for **caps**.

Change of Numeraire

- Arbitrage free pricing dictates that the following pricing equation holds:

$$\frac{V(s)}{N(s)} = \mathbb{E}^Q \left[\frac{V(T)}{N(T)} \right] \tag{4}$$

- Of course we could choose another numeraire, and Girsanov's theorem tells us this will only change the drift term.
- It is most convenient for us not to model the drift at all, this is why we choose to work in a **martinagle measure**.

Change of Numeraire

- Returning to 4:
  - $N(s)$ is today's value of the chosen numeraire.
  - $N(T)$ is the value of the chosen numeraire at expiry. It will be convenient to choose numeraires where this term is equal to 1 (i.e. a Zero-Coupon Bond)
- Assuming we choose such a numeraire, that is $N(T) = 1$, we are left with:

$$V(s) = N(s)\mathbb{E}^{Q}\left[V(T)\right] \tag{5}$$

Common Numeraires in Rates Modeling

- **Spot Numeraire**: A deposit in a bank accruing the instantaneous risk-free rates.

$$N(T) = \exp\left(\int_0^T r(s)ds\right) \tag{6}$$

- Note that if we assume $r(s)$ is a constant then this is the familiar risk neutral measure we see in equity derivatives.
- **T Forward Numeraire**: The price of a zero coupon bond at time t that matures at time T.
- This arises naturally when pricing instruments with maturity T.
- The spot measure is a special case of the T Forward measure with $t = 0$.

Common Numeraires in Rates Modeling

- **Annuity Numeraire**: The value of a one basis point stream of payments beginning at time t and ending at time T.

$$A(t,T) = \sum_{i=1}^{N} \delta_{t_i} D(t, t_i) \tag{7}$$

- Recall that this annuity function plays a special role in determining the par swap rate.

Simple Example: Caplets

- The simplest interest rate derivative is a caplet, which is an option on a single LIBOR forward rate.
- We can model a caplet by assuming that its underlying LIBOR forward rate follows some stochastic process.
- For example we might assume that the LIBOR forward follows a *Normal* or *Lognormal* model.
- In order to incorporate caplet volatility skew, we may need a more realistic model, such as SABR.

Caplet Pricing Formulas

- The price of a caplet in the T-Forward measure can be written as:

$$C(K) = \delta P(0,T)\mathbb{E}\left[(F_T - K)^+\right] \tag{8}$$

- If we use Black's model, then there is a closed-form solution to the price of a caplet:

$$C = \delta P(0,T)\left[F_0 N(d_1) - KN(d_2)\right] \tag{9}$$

$$d_1 = \frac{\log\left(\frac{F_0}{K}\right) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \tag{10}$$

$$d_2 = \frac{\log\left(\frac{F_0}{K}\right) - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \tag{11}$$

Caplet Pricing Formulas: Black's Model Implied Volatility

- Black's formula tells us how to compute an option price given inputs T and σ.
- However in practice we observe bid and offer prices, and need to extract the value of σ_{implied} that matches the market price.
- This procedure of fitting an implied volatility to a market price requires a one-dimensional root-finding algorithm.
- Many modern programming languages such as Python have built-in functions for these types of calculations. In Python we can use the **optimize.root** function in the **scipy** module.
- A similar procedure would exist for the Bachelier model, which is also commonly applied in interest rate markets.

Caplets vs. Caps

- Caps are baskets of caplets consisting of multiple options each linked to a different LIBOR forward rate.
- Caps are OTC instruments.
- Caps are a function of strike, starting expiry and length
- As caps are baskets of caplets, their pricing equation can be written as:

$$C(K) = \sum_i \left\{ \delta_i P(0,T) \mathbb{E}\left[(F_T - K)^+ \right] \right\} \tag{12}$$

where i indicates that we are looping over all individual caplets.

Caps, Caplets & Stripping Caplet Volatilities

- In practice, the market trades caps rather than caplets and caps are quoted in terms of a single (Black) implied volatility.
- This single implied volatility is interpreted as the *constant volatility* that allows us to match the price of the entire string of caplets.
- In order to obtain volatilities from individual caplets we need to extract them from the constant cap vols. This process is referred to as **stripping caplet volatilities**.

Caps, Caplets & Stripping Caplet Volatilities

- There are two main approaches to stripping caplet volatilities from a series of cap prices or volatiliies:
  - **Bootstrapping**: start at the shortest expiry cap and move further away trying to match cap prices while fixing the individual caplet volatilities.
  - **Optimization**: start with a parameterized function for the caplet volatilities and perform and optimization that minimizes the pricing error over the set of parameters in the function.
- This procedure mimics the process we saw when constructing the yield curve with a different objective function and underlying pricing formulas.

Swaptions

- **Swaptions** are vanilla options on LIBOR based swaps.
- Payer (Receiver) swaptions are call (put) options on the underlying par swap rate.
- Swaptions are referred to by their expiry and tenor.
- For example, a 1y2y swaption has a 1 year expiry and the underlying is a 2 year swap that starts at option expiry.

Swaptions

- Just as we wanted to make the underlying LIBOR forwards the stochastic variable in a caplet / Eurodollar option, here the most convenient approach will be to model the underlying swap.
- It will also be convenient for us to work in a different numeraire (or measure) which is defined by the annuity function of a swap.
- Recall that the formula for a (par) swap rate is:

$$\widehat{S(t,T)} = \frac{\sum_{i=1}^{N} \delta_{t_i} L_{t_i} D(0,t_i)}{\sum_{i=1}^{N} \delta_{t_i} D(0,t_i)} \tag{13}$$

Annuity Numeraire for Swaptions

- The denominator in (13) is the value of a constant stream of payments and is often referred to as the annuity function:

$$A(t,T) = \sum_{i=1}^{N} \delta_{t_i} D(t, t_i) \tag{14}$$

- The annuity function, or PV01, tells us the present value of a one basis point annuity between two dates.
- It can be proven that the swap rate is a martingale under the annuity numeraire.
- Therefore when we price a swaption, today's annuity value plays the role that a discount factor plays in standard risk neutral valuation for other asset classes.

Swaption Pricing in Black / Normal Models

- The price of a payer swaption in the annuity measure can be written as:

$$P = A_0(t,T)\mathbb{E}\left[(F-K)^+\right] \tag{15}$$

  where in this case F refers to the current par swap rate.

- To price a swaption, we will need to specify some dynamics for the underlying swap rate S. For example, we could use:
  - Black's Model

$$dF_t = \sigma F_t dW_t \tag{16}$$

  - Normal Model

$$dF_t = \sigma dW_t \tag{17}$$

Swaption Pricing in Black / Normal Models

- If we use either the normal or lognormal model, then we will have a closed form solution for the price of a payer (or receiver) swaption in the annuity measure:
  - Black's Model

$$P = A_0(t,T)\left[F_0 N(d_1) - K N(d_2)\right] \tag{18}$$

$$d_1 = \frac{\log\left(\frac{F_0}{K}\right) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \tag{19}$$

$$d_2 = \frac{\log\left(\frac{F_0}{K}\right) - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \tag{20}$$

– Normal Model

$$P = A_0(t,T)\sigma\sqrt{T}\left[d_1 N(d_1) + N'(d_1)\right] \quad (21)$$

$$d_1 = \frac{F_0 - K}{\sigma\sqrt{T}} \quad (22)$$

$$d_2 = -d_1 \quad (23)$$

- Note that while these models are simple, they are unlikely to explain the entire volatility skew for a given underlying swap.
- To incorporate the skew in a robust way, we will need to resort to a stochastic volatility model, or a jump process.
- Further notice that incorporating skew using a stochastic volatility or jump process only helps us connect the volatility of different strikes on the same underlying swap..
- We will need then need to employ other tools to connect the volatilities of different underlying swaps.

Pros and Cons of Normal / Black's Model for Swaption Pricing

- The Black and Bachelier models are both simple models with closed form solutions for their pricing equations.
- Both models rely on a single constant volatility parameter, σ.
- As a result, neither model will enable us to match the volatility skews that we observe in markets.
- The Bachelier model permits negative rates, whereas Black's model does not.
- For swaptions, the market standard quoting convention is to use the Bachelier model rather than Black's model, unlike equity markets.

Swaption Pricing: CEV Model

$$dF_t = \sigma F_t^{\beta} \, dW \tag{24}$$

- The CEV model is a generalization of the log-normal and normal models.
- Model parameters σ and β
- The additional parameter, β can be used to account for some degree of skew in the volatility surface.
- Cases $\beta = 1$ and $\beta = 0$ reduce back to the log-normal/normal models respectively.
- CEV model prices can be obtained via an asymptotic approximation.

Swaption Pricing: Incorporating the Volatility Smile

- The two most common approaches for fitting the volatility skew in rates markets are via **stochastic volatility models** and **jump diffusion models**.
- **Stochastic Volatility Models**: add a second, and potentially correlated process to the model that controls volatility.
- **Jump Diffusion Models**: allows the process to jump rather than follow continuous paths.
- Both of these techniques help to put more mass in the tails of the distribution, either symmetrically, or asymmetrically.
- The market standard stochastic model in rates is SABR, which, as we will see next, is a stochastic volatility model.

Swaption Pricing: Incorporating the Volatility Smile

- In rates the standard way of incorporating skew is via the SABR model, whose dynamics can be written as:

$$
\begin{aligned}
dF_t &= \sigma_t S_t^{\beta}\, dW_t^1 \\
d\sigma_t &= \alpha\sigma\, dW_t^2 \\
Cov(dW_t^1, dW_t^2) &= \rho\, dt
\end{aligned}
\tag{25}
$$

- Unlike the Black and Normal models, there will be no closed form solution to the SABR model, however, there are well-documented implied volatility approximation formulas.

- These formulas give an implied volatility for a given combination of SABR parameters (α, β, ρ and σ_0), strike and expiry.

SABR Model: Overview

- The SABR model is defined by the following four parameters:
  - α: Volatility of volatility
  - β: CEV exponent
  - ρ: Correlation between interest rates and volatility.
  - σ_0 starting level of the volatility process.
- Generally speaking, ρ defines the slope of the volatility skew. Why?
- α is closely related to the kurtosis in the distribution, and larger α will lead to fatter tails.
- Unless β is set to zero, SABR does not permit negative rates.
- If we use $\beta = 1$ or $\beta = 0$ then the forward rate stochastic process simplifies to Log-Normal / Normal.

SABR Model: Asymptotic Approximation

- The following asymptotic formula can be used to compute the Bachelier or Normal volatility of an option under the SABR model:

$$\sigma_n(T, K, F_0, \sigma_0, \alpha, \beta, \rho) = \alpha \frac{F_0 - K}{\Delta(K, F_0, \sigma_0, \alpha, \beta)} \times \ldots \quad (26)$$

$$\{1 + \left[\frac{2\gamma_2 - \gamma_1^2}{24}(\frac{\sigma_0 C(F_{mid}))}{\alpha})^2 + \frac{\rho\gamma_1}{4}\frac{\sigma_0 C(F_{mid})}{\alpha} + \frac{2 - 3\rho^2}{24}\right]\epsilon\} \quad (27)$$

Where:

$$
\begin{aligned}
C(F) &= F^{\beta} \\
\gamma_1 &= \frac{C'(F_{mid})}{C(F_{mid})} \\
\gamma_2 &= \frac{C''(F_{mid})}{C(F_{mid})} \\
F_{mid} &= \frac{F_0 + K}{2} \\
\Delta(K, F_0, \sigma_0, \alpha, \beta) &= \log\left(\frac{\sqrt{1 - 2\rho\zeta + \zeta^2} + \zeta - \rho}{1 - \rho}\right) \\
\zeta &= \frac{\alpha}{\sigma_0(1 - \beta)}\left(F_0^{1-\beta} - K^{1-\beta}\right) \\
\epsilon &= T\alpha^2
\end{aligned}
$$

- A similar expansion exists for Black's model.

SABR Model: Asymptotic Formula for ATM Options

- When pricing an at-the-money option, $F_0 = K$, an astute observer might notice that both the numerator and the denominator are equal to zero.
- Clearly this is not ideal, as at-the-money options are usual of the most interest.
- Thankfully, we can apply L'Hopital's rule to handle these situations. That is:

$$\lim_{F_0 \to K} \alpha \frac{F_0 - K}{\Delta(K, F_0, \sigma_0, \alpha, \beta)} = \lim_{F_0 \to K} \alpha \frac{\frac{\partial F_0 - K}{\partial F_0}}{\frac{\partial \Delta(K, F_0, \sigma_0, \alpha, \beta)}{\partial F_0}} \quad (28)$$

SABR Model: Calibration

- The SABR model allows for a fairly good fit of the volatility surface for a **single expiry**.
- The SABR model relies on separate calibration results for each expiry.
- When calibrating a SABR model, we need to find the best parameters (α, β, ρ, σ_0) for our data.
- Although we have four distinct parameters, they are correlated. This presents challenges in calibration.
- As a result, we often fix one parameter, generally β, and calibrate the three remaining parameters.

SABR Model: Calibration

- A calibration process involves minimizing distance between market data and model prices.
- For example, we could use the following least squares approach:

$$\vec{p}_{\min} = \underset{\vec{p}}{\operatorname{argmin}} \left\{ \sum_{\tau, K} \left(\hat{c}(K, \vec{p}) - c_K \right)^2 \right\} \tag{29}$$

  where $\vec{p} = (\alpha, \beta, \rho, \sigma_0)$ and c_K and $\hat{c}(K, \vec{p})$ are market and model option prices respectively.
- In the case of the SABR model, $\hat{c}(K, \vec{p})$ would be obtained using the SABR approximation formula on the previous slide.

SABR Model: An Alternate Approach to Selecting Beta

- If we consider at-the-money options in Black's asymptotic SABR formula, we find the following relaionship:

$$\log \sigma_{ATM} \approx \log \alpha - (1-\beta) \log F \tag{30}$$

- This provides another way that we can estimate the β parameter
- We can use **linear regression** on a time series of the log of at-the-money implied vol against the log of the forward rate.
- Note that we need to apply L'Hopital's rule in order to use the asymptotic formula on at-the-money options.

SABR Model for Pricing Swaptions

- Recall that the price of a payer swaption in the annuity measure can be written as:

$$P = A_0(t,T)\mathbb{E}\left[(S-K)^+\right] \tag{31}$$

- Applying the SABR model means that we assume the par swap rate, S, follows the process in (25).
- To proceed with pricing we need to specify SABR parameters and apply the SABR approximation formula to obtain a normal / Bachelier volatility.
- Once we've done this, we can obtain a swaption price using (21).

Swaption Volatility Cube

- Earlier in this course, we saw that in equity markets the volatility surface was a two dimensional object (across strike and expiry).
- In rates, we have another dimension that arises from the tenor or length of a swap. This three dimensional object is referred to as the **volatility cube** and has dimensions:
  - Expiry
  - Tenor
  - Strike
- Clearly the volatilities for swaptions with different tenors but the same strike and expiry should be somewhat related.
- Because of the extra dimension, fitting an interest rate volatility model is a more complex endeavor than it is for other asset classes.

Modeling the Volatility Cube

- In order to model the volatility cube we must come up with a pricing model that can match prices (or implied volatilities) across all three dimension
- NOTE: If we are pricing a single swaption or cap, we will not need to rely on these "bigger" models for pricing, however, if we are interested in pricing exotics or computing consistent risk metrics, then we will need to apply one of these techniques.
- There are two main approaches to modeling the volatility cube:
  - Short Rate Models
  - Forward Rate (Libor) Market Models

Difference between Caps and Swaptions

- Recall that the pricing equation for a cap can be written as:

$$C(K) = \sum_i \{\delta P(0,T)\mathbb{E}\left[(F_T - K)^+\right]\} \tag{32}$$

Switching sum and expectation we have:

$$C(K) = \mathbb{E}\sum_i \{\delta P(0,T)\left[(F_T - K)^+\right]\} \tag{33}$$

- Similarly, the pricing equation for a swapion can be written as:

$$P = A_0(t,T)\mathbb{E}\left[(S - K)^+\right] \tag{34}$$

$$\widehat{S(t,T)} = \frac{\sum_{i=1}^{N} \delta_{t_i} L_{t_i} D(0,t_i)}{\sum_{i=1}^{N} \delta_{t_i} D(0,t_i)} \tag{35}$$

- Question: Are these trades the same? What is the difference?

Trading Caps vs. Swaptions: The Wedge Trade

- The difference between a cap and a swaption is akin to the difference between a basket of options and an option on a basket.
- Therefore, the trades are not the same, and the gap between them is driven by **forward rate correlations**
- In practice this is a commonly traded structure as it give investors a precise way to bet on these correlations and also harvest any dislocations between the two markets.

Mid-Curves

- Mid-curves are options on a forward starting swap.
- Because they are options on a forward starting swap, there is another variable that defines a mid-curve, namely how far forward the swap starts.
- Mid-curves are reference by their expiry / forwardness / tenor combination.
- For example, a 1y2y3y mid-curve refers to a one year expiry on a three year swap which starts 2 years after the option expiry.
- Pricing a mid-curve is conceptually equivalent to pricing a swaption, and is generally done under the annuity measure.

Mid-Curves

- Note that the exercise decision on a mid-curve must be made at expiry and the forwardness and tenor tell us which part of the curve the option is on.
- Clearly, the volatility between mid-curves and swaptions with the same or similar expiries must be related.
- Mid-curves provide a convenient instrument for trading *forward volatility*.
- In fact, it turns out that a portfolio of swaptions and mid-curves can be put together to form interesting correlation or dispersion trades.

Correlation Products: Basics of Vanilla vs. Mid-Curve Triangles

- Using a swap and a forward starting swap we can decompose a swap into two components.
- In the case of swaps this decomposition is generally not interesting.
- However, this decomposition can also be done with swaptions and mid-curves.
- In options space this is a way to isolate the correlation or dispersion of the two pieces vs. the whole. (Why?)

Correlation Products: Basics of Vanilla vs. Mid-Curve Triangles

- For example, consider the following portfolio:
  - 1y3y Swaption
  - 1y2y Swaption
  - 1y2y1y Mid-Curve
- Notice that the first item and the last two items have the same expiry and span the same underlying swap dates.
- This portfolio is essentially trading a basket of options vs. an option on a basket. The difference in valuation between these two quantities will depend on correlation.

Correlation Products: Spread Options

- Spread options, or options on the difference between two rates also are a relatively large piece of the OTC rate derivative market.
- Calls spread options are called caps, whereas put spread options are called floors.
- For example, a common spread option might be a 1y 2s30s option.
- In this case, the payout would be based on the 30 year swap rate less the two year swap rate, and the option expiry would be in 1 year.
- These products present an efficient way for an investor to place a bet on the steepness of the yield curve.

Correlation Products: Spread Options

- As spread options are options on the difference of two random variables, their valuation will clearly depend on correlation.
- To see the intuition behind pricing spread options, recall that for two random variables X and Y, we have:

$$\sigma_{x-y} = \sigma_x + \sigma_y - 2\rho\sigma_x\sigma_y \tag{36}$$

- This formula is exact under certain assumptions, but does not account for the volatility smile (among other things).
- In particular, we can see that the higher the correlation between the underlying swap rates, the cheaper the spread option.
- Spread options can either be priced via a SABR/LMM or other volatility cube model, or can be priced using a copula approach.