MF 728: Fixed Income

Professors Eugene Sorets and Christopher Kelliher
Boston University Questrom School of Business, 595 Comm Ave

Lecture: Wed, 7:30am - 10:15am in HAR 208 and on zoom
Recitation: Wed, 10:30am - 1:15mm in HAR 208 and on zoom
Office hours: Fri, 11am - 12pm on zoom
Email: sorets@bu.edu & cmk44@bu.edu

Teaching Assistants:

- Zhe Fei. Email: zhefei@bu.edu
- Fanwei Wei. Email: fanwei@bu.edu
- Xinyang "Fiona" Li. Email: fionalxy@bu.edu

Course Goals

- This course will introduce you to the most common concepts, instruments, and computational methods used in fixed income.
- It will provide you with hands-on experience implementing these methods as well as commonly used financial models.
- It will help you to understand the strengths and weaknesses of each of the methods and enable you to choose optimally when faced with these problems in your career.
- It will provide you with tools that you can use to evaluate approximations in practical financial applications.

Questrom Covid Statement of Norms

- Use the designated paths and doors to enter and exit the classrooms and move around the Hariri building.
- Use the wipes provided in classrooms to clean desktops and other spaces before and after use.
- Wear an appropriate face covering when in the Hariri building, including classrooms and offices. Students not wearing a face covering will be asked to leave and are expected to comply with the request.
- Be willing to display your "green screen" compliance app upon request (e.g., in class or for a meeting).
- Eating and drinking are NOT permitted in the classroom.
- Update your LfA location and class status (in-person or 100% remote) on the StudentLink as changes occur throughout the

semester.

- Attend class remotely if your behavior outside of the classroom might put others in the community at risk.

Requirements for this course

- This course assumes knowledge and fluency in linear algebra, stochastic calculus, statistics, and probability.
- This includes: random variables, probability distributions and densities, characteristic functions, Fourier Inversions, measure changes, Taylor series and Ito's formula.
- This course will also require a significant amount of programming and we expect you to be familiar with either R, Matlab, or Python.
- Additionally, we assume that you have been exposed to the concepts of derivative pricing, risk neutral valuation, and basics of fixed income such as covered in MF702.

Grading

- Your final grade for this course will be assigned based on your performance on:
  1. The Exam (35% of your final grade)
  2. The final project (35% of your final grade)
  3. Homework (30% of your final grade)

Exam

- There will be one take-home exam on March 24th.
- You will have 24 hours to complete the exam. You will be allowed to use any books or other resources you choose, but you must cite your sources.
- Collaboration on the exam, however, is forbidden and considered cheating.

Final Project

- You will be required to build teams of 4-6 people to work with on a project of practical relevance.
- Your team will be asked to give a 10-minute presentation of your results. The presentations will be held during the last two sessions.
- In addition, your team will be required to submit a written summary of at most 10 pages containing your method and results. You will have to submit your summary paper two days after you give your presentation.
- We expect all group members to contribute to their team's final project. We also expect all group members to be able to discuss all aspect's of their teams work. We reserve the right to ask any group or individual member about any aspect of their project and incorporate these answers into the individual's project grade.

Final Project

- For your final project we expect you to:
  1. Choose a topic/technique directly related to computational methods in fixed income.
  2. Research this topic/technique thoroughly, highlighting its applications in finance and potential uses.
  3. Apply this topic to a real-world finance problem. This includes implementing the topic/technique in R/Matlab/Python and obtaining substantive results.
  4. Analyze the strengths and weaknesses of the topic/technique based on your results as well as external research.

Final Project Proposal

- You will be required to choose a topic and submit a project proposal to us by February 24th.
- We will give you a significant amount of flexibility in choosing your topic for the final project; however, it must be directly related to the topics that we discuss in class.
- Your proposal should describe the topic that you are planning to research, and include at least 3 references that you plan to use in your research.

Homework

- Homework will be assigned every week.
- The assignments will be a combination of theoretical and computational questions, leaning toward computation.

  Some assignments will require a significant amount of coding.

  Students may choose to program in R, Matlab or Python.
- Students are allowed to discuss their homework assignments; however, all solutions and any code must be unique and written by the student submitting the assignment.

Homework

- Homework will usually be due at 2pm on Wednesdays.
- Please upload your homework solutions to your class website.
- Late homework submissions will result in a loss of 10 points for each day past the due date.

Instrument Kingdoms

The world of finance is generally divided into

- Equity Securities (.e.g. common stock, preferred stock)
- Debt Securities, also called Fixed Income (e.g. bonds)

Guess which one is bigger? (Hint: The other one is much more discussed in the press.)

Sometimes people separate derivatives as a third kingdom.

Examples of Debt

- loan to a friend
- credit card balance
- payday loan (loan to a worker to get to the next paycheck)
- commercial paper (short-term loans by companies to pay workers)
- Annuities
- Mortgages

Examples of Securitized Debt (aka Fixed Income Securities)

- corporate bond (a securitized loan to a company)
- municipal bond (a securitized loan to a city)
- Treasury bills, notes, and bonds (loans to countries)
- Inflation Protected Bonds
- Callable and Putable Bonds
- Floaters and Inverse Floaters

More examples of Fixed Income Securities

- Bond Futures and Options
- Options on Bond Futures
- Residential Mortgage-Backed Security (RMBS)
- Commercial Mortgage-Backed Security (CMBS)
- Credit Default Swap (CDS)
- Collateralized Debt Obligation (CDO)
- Synthetic CDO
- Synthetic CDO tranche

More examples of Fixed Income Securities

- Interest Rate Swap
- Forward Rate Agreement (FRA)
- Interest Rate Cap and Floor
- Interest Rate Swaption

Debt Markets

- Fixed income instruments are mostly traded over-the-counter (OTC), i.e., via dealers.
- There have been numerous attempts to set up bond exchanges, but they never quite gained traction.
- A popular way for retail investors to trade fixed income products is via ETFs, but they are used more and more by professionals as well.

We hope that by the end of the course, you will be able to price many of the instruments on the last few slides and will have an idea how to approach the others.

Pricing a Bond: warmup

Let's start with trying to price a simple bond. A bond is a promise to repay the principal at maturity and make coupon payments in between. Usually, the coupon payments are twice a year and are constant.

So, let t_i be the time from now (in years) of the coupon payments and T the maturity. Also, let's assume that the principal payment is $\$1$, even though, usually, bonds are quoted per $\$100$ face value.

Let's denote by $B(t, T; c)$ the value of the bond with maturity T and coupon c at the time t. Then the present value of the bond

$$B(t, T; c) = \sum_{i=1}^{N} c\, e^{-r(t_i - t)} + 1 \cdot e^{-r(T-t)} \tag{1}$$

where r is the interest rate that we assume for the moment to be constant. N, of course, is the number of coupon payments.

Pricing a Bond: yield curve

Well, we assumed a lot in (1)!

Is the interest rate r really constant?

Let's think where the interest rates come from.

- If I'm going to lend to you, I'm giving up the use of my money (and incurring opportunity cost) for a period of time, say, one year.
- But if I agree to lend to you for two years I have more uncertainty about the second year so I will want to charge you more per year.
- The excess yield that the investors demand for longer-term bonds is called *term premium*, and it is usually, but not always, positive.

In other words, r should be an increasing function of maturity.

Pricing a Bond: yield curve

- We can generalize r to the function $r(t, T)$, which is the interest rate measured at time t for maturity T.
- The function of maturity $r(0, T)$ is called the yield curve and it is normally upward-sloping as mentioned on the previous slide.
- Note that the value of the bond in (1) depends on all of the yield curve up to maturity T.

Pricing a Bond: deterministic rates

- But we are still treating our rate $r(t, T)$ as deterministic entity.
- But we know that changes every day and it has no reason to stop changing in the future.
- Therefore, it really is a random variable and the yield curve that we observe today is a best guess at the average of what the rates will do in the future.

It turns out that for pricing simple bonds such as the one above, taking randomness into account does not really change the answer because it only feels the expectation of the curve, not its movement.

It does matter when we start considering bonds with embedded options and it matters a lot for outright derivatives, which are instruments with asymmetric payoffs.

Pricing a Bond: stochastic rates

Therefore, if one wants to work with fixed income derivatives, one has to be able to model stochastic yield curves.

First stochastic models tried a shortcut and tried to treat only the instantaneous spot rate as stochastic, but they proved insufficiently flexible to fit all the possible yield curves, to say nothing of the derivative prices.

Therefore, people had to extend the models that treated the whole yield curve as a stochastic process.

We will study both kinds of models and see when each kind is appropriate.

Back to Basics

We need to set up some notation.

A zero coupon bond (ZCB) is an instrument that comes to life at the settlement date S and pays $\$1$ at a specified future time T, called maturity. At evaluation time t, its value is

$$P(t, S, T) \tag{2}$$

for $t < S < T$. Zero coupon bonds are also called zeros or discount factors, and often denoted by Z or D.

Using no-arbitrage arguments, we can prove that

$$P(t, S, T) = \frac{P(t, t, S)}{P(t, t, T)} \tag{3}$$

Coupon Bonds

A coupon bond can be viewed as collection of zero coupon bonds:

$$B(t,T;c) = \sum_{i=1}^{N} c\,P(t,t,t_i) + 1 \cdot P(t,t,T) \tag{4}$$

Here we wrote it with face value of 1, which is a common notation in books and papers to reduce clutter. In practice, bonds are usually quoted with face value of 100.

If the price of the bond is greater than 1 (or 100), it is said to be trading *above par*. If the price is below, then the bond is *below par*, and, if equal, then *at par*.

Duration

Sensitivity of a bond to the interest rate is called *duration*.

To see where the name comes from, consider the case of constant interest rate as in (1)

$$B(0,T;c) = \sum_{i=1}^{N} c\, e^{-rt_i} + 1 \cdot e^{-rT} \tag{5}$$

Duration is defined as

$$\frac{1}{B(0,T)} \frac{\partial}{\partial r} B(0,T) = \frac{-1}{B(0,T)} \left\{ \sum_{i=1}^{N} c t_i e^{-rt_i} + 1 \cdot T e^{-rT} \right\} \tag{6}$$

and the right hand side can be viewed as a weighted sum of payment times. In particular, it has the units of time. Thus the name. However, when people talk about duration they are almost always thinking of it as sensitivity to the interest rate.

Duration

- Note that duration cannot exceed the maturity T.
- Definition of duration above makes sense when the interest rate is constant. But what to do when you have a whole yield curve of interest rates? The curve can move in many different ways!
- The main duration is then with respect to the parallel (up-and-down) moves of the curve. But people also routinely measure durations with respect to particular maturities as well as steepening/flattening of the curve.

Convexity

Convexity of a bond is

$$\frac{1}{B(0,T)} \frac{\partial^2}{\partial r^2} B(0,T) = \frac{1}{B(0,T)} \left\{ \sum_{i=1}^{N} c t_i^2 e^{-r t_i} + 1 \cdot T^2 e^{-rT} \right\} \quad (7)$$

which is always positive.

Both duration and convexity are important numbers in risk management of fixed income portfolios, because they are sensitivities to the movements of the interest rates.

It is a separate and a more difficult job to build distributions of the movements of the interest rates.