MF 728: Fixed Income

Professors Eugene Sorets and Chris Kelliher Spring 2021

Problem set # 3

Due: Thursday, February 18th, by 2pm.

1. **Caplet Pricing in Different Models:**

   Consider a 1Y expiry 1.25% strike put option on the 1Y Libor Rate. Recall that in the T-Forward measure the pricing equation for a caplet put option can be written as:

   $$P(K) = \delta D(0,T)\mathbb{E}\left[(K - F_T)^+\right] \tag{1}$$

   As Libor is a three-month rate you may assume $\delta = 0.25$.

   (a) Suppose you conducted a bootstrapping algorithm that identified the constant instantaneous forward rate over the first year to be 1.25%. Calculate the discount factor needed to obtain the caplet price.

   (b) Assume that the 1Y Libor rate follows a log-normal distribution:

   $$dF_t = \sigma F_t dW_t \tag{2}$$

   and that $\sigma = 0.15$. Calculate the price of the option on 1Y Libor by adapting the Black-Scholes formula.

   (c) Consider the Bachelier or Normal model:

   $$dF_t = \sigma_n dW_t \tag{3}$$

   Calculate what you believe to be the σ_n that will make the Normal model best approximate the Log-Normal model above.

   (d) Using this σ_n, calculate the price of the 1Y option on Libor under the Bachelier model. Compare to the Black-Scholes model price. Are they similar? Why or why not? Which one is higher? Why?

   (e) Using the adapted Black-Scholes formula, calculate the price of put options with the following strikes:

   0.5 %
   0.75 %
   1.0 %
   1.25%

In addition, calculate the standard set of Black-Scholes Greeks for each option: delta, gamma, vega and theta. Comment on which options have the highest gamma, vega and theta respectively. Why is this the case?

2. **Stripping Caplet Volatilities:**

   (a) Consider the following table of two-year at-the-money cap volatilities under the Black's model:

1Y	0.15
2Y	0.2
3Y	0.225
4Y	0.225
5Y	0.25

   Assume that caplets are paid quarterly on three-month Libor each with $\delta = 0.25$. Further assume a flat interest rate curve with 1% instantaneous forward rates along the entire curve. Calculate the price of each cap using Black's model.

   (b) Extract the Black-Scholes at-the-money implied volatilities for each caplet from the provided set of cap volatilities. Comment on the shape of the caplet vs. cap volatilities.