(8.13) $\boxed{(W_{t_1}, \dots, W_{t_n})}$ $\quad Cov(W_{t_i}, W_{t_j})$

$$\boxed{(\underbrace{W_{t_1}}_{X_1}, \underbrace{W_{t_2} - W_{t_1}}_{X_2}, \underbrace{W_{t_3} - W_{t_2}}_{X_3}, \dots, \underbrace{W_{t_n} - W_{t_{n-1}}}_{X_n})}$$

$Y_1 = X_1, \quad Y_2 = X_1 + X_2, \quad \dots, \quad Y_n = X_1 + \dots + X_n$

$$E[f(Y_1, \dots, Y_n)] = \int_{R^n} f(y_1, \dots, y_n) \boxed{\psi(y_1, \dots, y_n)} \, dy_1 \dots dy_n$$

joint density

$\parallel$

$$E[f(X_1, X_1 + X_2, \dots, X_1 + \dots + X_n)]$$

$$\int_{R_n} f(\underbrace{x_1}_{y_1}, \underbrace{x_1 + x_2}_{y_2}, \dots, \underbrace{x_1 + \dots + x_n}_{y_n}) \boxed{\phi(x_1, \dots, x_n)} \, dx_1 \dots dx_n$$

$x_1 = y_1 \quad x_2 = y_2 - y_1 \quad x_3 = y_3 - x_1 - x_2$

$= y_3 - \cancel{y_1} - y_2 + \cancel{y_1}$

$x_4 = y_4 - y_3$

$\dots$

$\vdots$

$$\frac{1}{\sqrt{2\pi t_1}} e^{-\frac{y_1}{2t_1}} \frac{1}{\sqrt{2\pi(t_2 - t_1)}} e^{-\frac{(y_2 - y_1)^2}{2(t_2 - t_1)}}$$

(8.21) W_t L^2

$\underbrace{\left[\{X_\alpha : \alpha \in A\}\right]_{L^2}} \subseteq L^2$ $\quad \mathbb{E}[W_t] = 0$

$$\mathbb{E}[W_s W_t] = s \wedge t$$

(8.25) $\underline{(8.24)}$

$$\mathbb{E}\left[|\underbrace{W_{t+h} - W_t}| ^\alpha\right] \le C h^{\underline{1+\beta}}$$ $\forall t$ $\forall h$

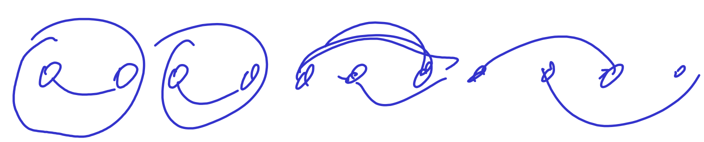
$\alpha, \beta, C \in \mathbb{R}_{++}$ $\quad \alpha = 2, 4, 6, 8 \ldots$

$$\mathbb{E}\left[(\underline{W_{t+h} - W_t})^4\right] = 3h^{1+1}$$ $\quad C = 3$, $\alpha = 4$, $\underline{\beta = 1}$

$W_{t_i} - W_{t_{i-1}}$ $\quad \underbrace{W_t - W_s}$ $\quad \underline{s < t}$

$\mathrm{var} = \underline{t-s}$

$$\mathbb{E}\left[(W_{t+h} - W_t)^4\right]$$



$\mathbb{E}[X_1 X_2 X_3 X_4]$ $\quad \mathbb{E}[X_i] = 0$

$$= \mathbb{E}[X_1 X_2]\mathbb{E}[X_3 X_4] + \mathbb{E}[X_1 X_3]\mathbb{E}[X_2 X_4] + \mathbb{E}[X_1 X_4]\mathbb{E}[X_2 X_3]$$

$$\mathbb{E}[X^4] = 3\mathbb{E}[X^2]^2 \qquad \mathbb{E}[X] = 0$$

$X(\omega, t)$ $\quad \omega \quad$ $\boxed{X(\omega, \cdot)}$ $\quad X_\omega$

contin. function of time

$\tilde{W}_t(\omega) = W_t(\omega)$ for a.e. ω $\quad \forall t$

$\tilde{W}_t$ $\quad W_t$ modification