MF 796: Computational Methods of Mathematical Finance

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Problem set # 1

Due: Monday, February 1, by 2 pm

1. **What areas of finance** are of most interest to you (e.g. big data, trading, portfolio management, risk management, derivative pricing, analyzing complex derivatives)?

2. **What is the primary reason for your interest in this course?**

3. **List all programming languages you have used** and your level of familiarity with each.

4. **Option traders often say that when buying options we get gamma at the expense of theta.** What do you think they mean?

5. **Consider the CEV Model:**

$$dS_t = rS_t\,dt + \sigma S_t^{\beta}\,dW_t$$

Assume a CEV model that is defined by the following parameters:

$$S_0 = 100 \tag{1}$$

$$r = 0.0 \tag{2}$$

$$\beta = 1.0 \tag{3}$$

$$\sigma = 0.25 \tag{4}$$

(a) Describe what each model parameter does.

(b) Price an at-the-money one year European call option via Monte Carlo simulation. (HINT: Increments of Brownian Motion are distributed normally with mean 0 and variance dt. One approach is to construct a set of paths by generating random normal variables to use as the increments of the Brownian Motion)

(c) Calculate the price of the same European call option via the Black-Scholes formula. Is this price the same as you obtained via simulation? Should it be? Why or why not?

(d) In the Black-Scholes model, we know that the delta of a European call option is:

$$\Delta = \Phi(d_1)$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left(\ln\frac{S_0}{K} + \left(r + \frac{\sigma^2}{2}\right)T\right),$$

Calculate the delta of an at-the-money European call option with one year to expiry.

(e) Using the delta obtained above, calculate how many shares of stock you need to construct a delta neutral portfolio that is long one unit of the call option?

(f) Use simulation to estimate the payoff of the delta neutral portfolio obtained above. How does the payoff compare to the Black-Scholes model price of the option you obtained earlier? What conditions cause this portfolio to make money? Lose money?

(g) Modify the model dynamics so that $\beta = 0.5$ and all other parameters are the same as in the original question. Using the same hedging portfolio perform another simulation to estimate the payoffs of the delta-neutral portfolio under these dynamics. Are the payoffs higher or lower? Why?

(h) Modify the model dynamics so that $\sigma = 0.4$ and all other parameters are the same as in the original question. Using the same hedging portfolio perform another simulation to estimate the payoffs of the delta-neutral portfolio under these dynamics. Explain the relationship between a delta-neutral portfolio and the σ parameter.