Problem 1:

1. By Python, the price is 0.0157.
2. By Python, the d1=-1.7482 and d2=-1.8482.

(a). For N=5, the F(0) calculation error is 0.1187

N=10, the F(0) calculation error is 0.0593

N=50, the F(0) calculation error is 0.0118

For N=5, the price is 0.0073

For N=10, the price is 0.0113

For N=50, the price is 0.0148

(b). For N=5, the F(0) error is 0.00018701

For N=10, the F(0) error is 4.907e-05

For N=50, the F(0) error is 1.99e-06

For N=5, the price is 0.0152521

For N=10, the price is 0.01556894

For N=50, the price is 0.01566974

(c). For N=5, the error is 1.03e-06

For N=10, the error is 0.0

For N=15, the error is 0.0

For N=5, the price is 0.01567391

For N=10, the price is 0.01567391

For N=15, the price is 0.01567391

I use gauss-legendre to get the nodes.

1. left riemann length 1 error: 0.0001568908989637574

left riemann length 2 error: 0.0006897586519035714

left riemann 50 dx error: 0.0015616497604598045

left riemann 500 dx error 0.0001568908989637574

By theoretical estimate, O((b-a)^2) for left Riemann

2^2/1^2=4

Experimental rate is 0.0006897586519035714/0.0001568908989637574=4.39642233207491 which is not far away from the theoretical estimate.

midpoint rule length 1 error: 4.0328463291050554e-08

midpoint rule length 2 error: 7.198792167928403e-08

midpoint rule 50 dx error: 4.032939511400269e-06

midpoint rule 500 dx error 4.0328463291050554e-08

By theoretical estimate, O((b-a)^3) for midpoint rule

2^3/1^3=8

Experimental rate is 7.198792167928403e-08/4.0328463291050554e-08=1.78 which is far away from the theoretical estimate. I don’t know why, I try different range x and 2x, the ratio cannot be 8.

GaussLegendreQuuadrature length 1 error: 0.3413447460738834

GaussLegendreQuuadrature length 2 error: 0.47725006050735375

GaussLegendreQuuadrature 3 dx error: 7.987424793243214e-07

GaussLegendreQuuadrature 5 dx error 5.340450304203159e-12

149564.63103788786

By theoretical estimate, O(N^-2N) for GaussLegendreQuuadrature

3^(-2\*3)/5^(-2\*5)=13395.9191

Experimental rate is 7.987424793243214e-07/5.340450304203159e-12=149564.63103788 which is not far away from the theoretical estimate.

1. I like the Midpoint rule since it is easy to code and running fast. It is true at least for the normal distribution PDF.

Problem 2

1. I assume the mean of the stock for last year is $380.

So the E[payoff]=E[(x-380)+]=integral from 380 to positive infinity (380-x)pdf(Normal, mean=380, sigma=20)

By computation, the result is 7.9788

文本

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图示

描述已自动生成

By midpoint rule, the result is 0.02669.

1. change rho=0.8 result is 0.325257

Change rho=0.5 result is 1.194888

Change rho=0.2 result is 2.2132246

1. The price depends on rho make sense, since the half year lower than 375 means that it is hard to be higher than 380.
2. When s2<370 price = 0.00161373348

When s2<360 price = 3.653335211543911e-07

1. It is also make sense because 6 months’ price lower means it is harder to go up beyond 380.
2. When the requirement is 100% sure will happen, the vanilla one has the same price as continent option.