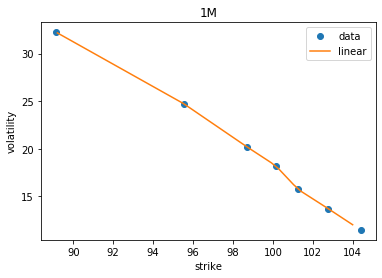
1. implementation of breeden-Litzenberger

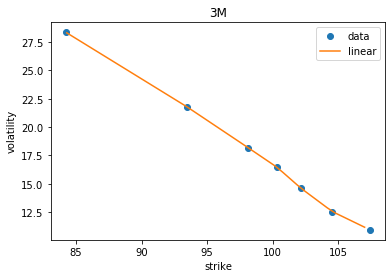
(a).

文本

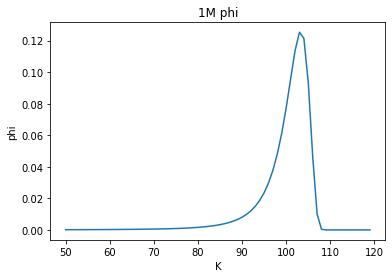
描述已自动生成

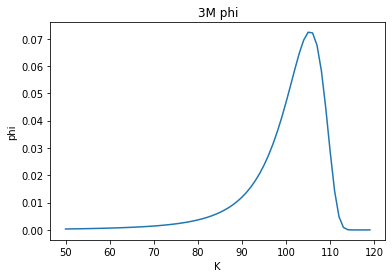
(b).





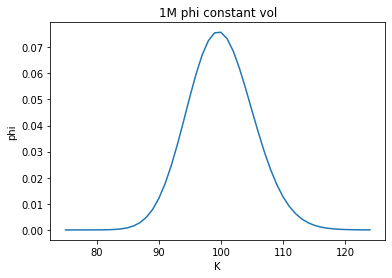
(c).

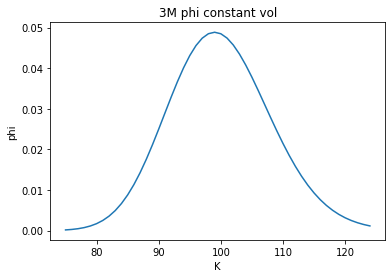




It seems that 3 month has bigger variance than 1 month which make sense. Longer time interval leads more change.

(D)





When use constant volatility, the distribution seems no skewness. For the same T, constant volatility leads to a bigger variance distribution.

(e)

(1)

Digital put=Φ（K）

Price=0.9984998779842045

(2)

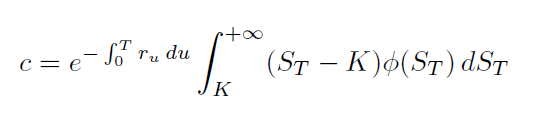
Digital call

文本

描述已自动生成

Price = 0.3308543479578533

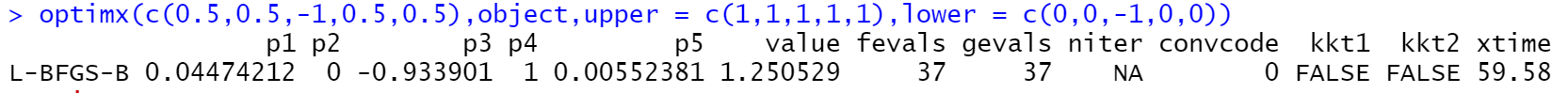
(3)



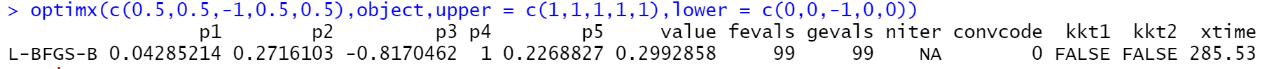
2M call 100 price = 2.646949186641023

1. calibration of Heston model
2. after checking all the type of arbitrage, there is no arbitrage in the spread sheet.

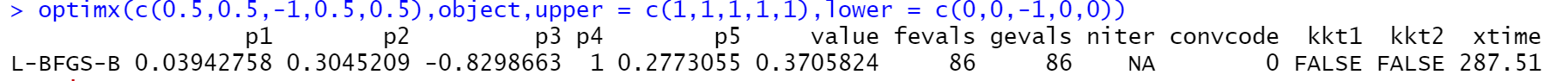
expT = 0.13



expT = 0.38

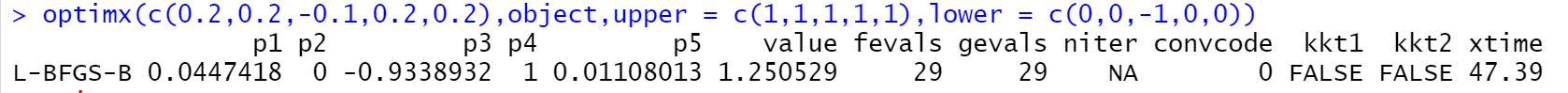


expT = 0.56



1. Change starting point

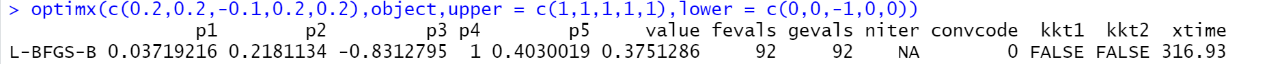
expT = 0.13



expT = 0.38



expT = 0.56

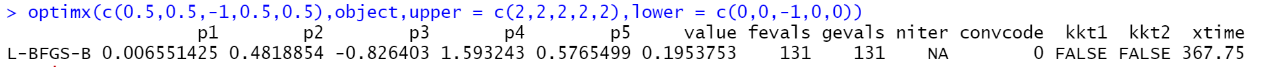


Change range

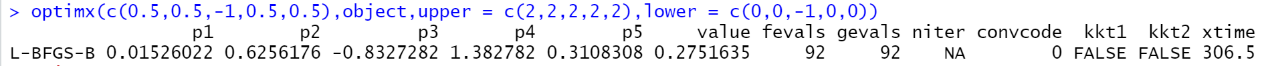
expT = 0.13



expT = 0.38



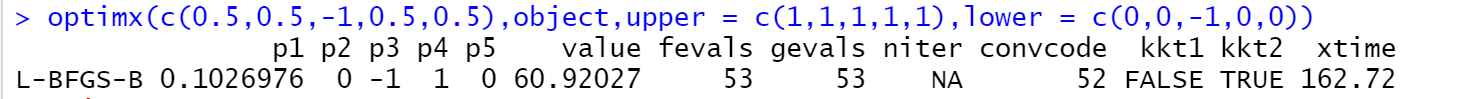
expT = 0.56



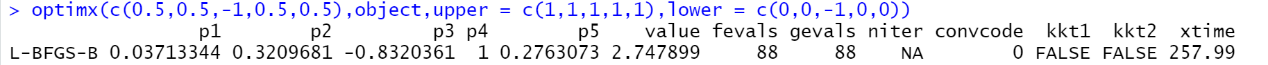
Change starting point seems doesn’t affect the result a lot, but change the range seems change result significantly.

1. Add weight

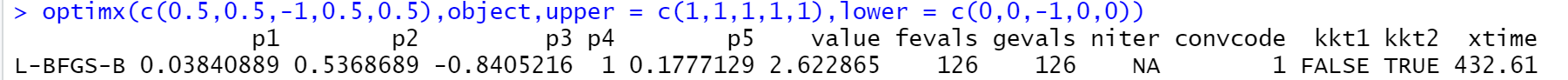
expT = 0.13



expT = 0.38



expT = 0.56



1. Hedging Under Heston Model

(a)

Delta for expT = 0.13 for the K=275 call is C(expT=0.13, S0=267.65)-C(ExpT=0.13, S0=266.65) = 3.12902- 2.573991=0.555029

Delta for expT = 0.38 for the K=275 call is C(expT=0.38, S0=267.65)-C(ExpT=0.38, S0=266.65)= 6.964781-6.358499=0.606282

So delta for expT=3/12 K=275 is (0.555029+0.606282)/2=0.5806555

By FFT, C(S0=267.15,K=275,expT=3/12)= (C(expT=0.13,K=275,S0=267.15)+ C(expT=0.13,K=275,S0=267.15))/2=(2.851506+6.66164)/2=4.756573

C(S0=268.15,K=275)= C(S0=267.15,K=275)+delta=4.756573+0.5806555=5.337229

C(S0=266.15,K=275)= C(S0=267.15,K=275)-delta=4.756573-0.5806555=4.175918

(i)

Delta under black scholes model

Sigma(expT=0.13)=0.1500416

Sigma(expT=0.38)=0.1547563

Sigma(expT=3/12)=( Sigma(expT=0.13)+ Sigma(expT=0.38))/2= 0.152399

Delta = C(sigma=0.152399,expT=3/12,S0=267.65)-C(sigma=0.152399,expT=3/12,S0=266.65)=5.258232-5.068147=0.190085

They are different one is 0.58 the other one is 0.19.

I think the Heston Model’s delta is better because it includes random volatility.

I would like to use Heston delta to do hedging.

(ii) I need to short 0.58 units of the underlying is delta neutral.

(b)

(i)

expT=0.13

C(vol=0.154245)=2.851506

C(vol=0.2670309)=6.898197

Vega=(6.898197-2.851506)/(0.2670309-0.154245)= 35.87941

expT=0.38

C(vol=0.1709933)=3.425767

C(vol=0.2746863)=7.183395

Vega=(7.183395-3.425767)/(0.2746863-0.1709933)= 36.23801

expT=3/12

vega= (35.87941+36.23801)/2=36.05871

FFT based Heston model

C(theta=(0.2268827+0.00552381)/2; nu=(0.04474212+0.04285214)/2;expT=3/12)=8.342524 where vol=0.3057197

C(theta=(0.2268827+0.00552381)/2+0.05; nu=(0.04474212+0.04285214)/2+0.05;expT=3/12)= 8.342524+vega=8.342524+36.05871=44.40123

C(theta=(0.2268827+0.00552381)/2-0.05; nu=(0.04474212+0.04285214)/2-0.05;expT=3/12)= 8.342524-vega=8.342524-36.05871=-27.71619

The negative call price is meaningless. I think I move nu and theta too much.

(ii)

BlackScholes(267.15,275,0.015,3/12,0.3057197,'C')

b=BlackScholes(267.15,275,0.015,3/12,0.3057197+(0.2746863-0.1709933)/2,'C')

a=BlackScholes(267.15,275,0.015,3/12,0.3057197-(0.2746863-0.1709933)/2,'C')

b-a

vega=5.501714

They are quite different, I think the Heston model has a better result since it consider the changeable volatility