IMPORTANT INEQUALITIES

1. Arithmetic-Geometric-Harmonic Means.

Arithmetic Mean of n numbers. $a_1, a_2, \ldots, a_n$ are positive real numbers; their *arithmetic mean* is

$$\frac{a_1 + a_2 + \ldots + a_n}{n}$$

Geometric Mean of n numbers. $a_1, a_2, \ldots, a_n$ are positive real numbers; their *geometric mean* is

$$\sqrt[n]{a_1 a_2 \ldots a_n}$$

Harmonic Mean of n numbers. $a_1, a_2, \ldots, a_n$ are positive real numbers; their *harmonic mean* is

$$\frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}}$$

2. AM-GM-HM

AM-GM-HM. $a_1, \ldots, a_n$ are positive real gumbers. Then

$$\frac{a_1 + a_2 + \ldots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \ldots a_n} \geq \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}} .$$

3. Generalized Means Inequality

Again, let $a_1, \ldots, a_n$ be positive real numbers, and let p be real and non-zero. Let

$$M_p(a_1, \ldots, a_n) = \left(\frac{a_1^p + a_2^p + \ldots + a_n^p}{n} \right)^{\frac{1}{p}} .$$

Note that M_1 is the AM and M_{-1} is the HM; also, $\lim_{p \to 0} M_p = GM$.

Then $M_p(a_1, \ldots, a_n) \leq M_q(a_1, \ldots, a_n)$ for all $p < q$, with equality if and only if $a_1 = a_2 = \ldots = a_n$.

4. Cauchy-Buniakowsky-Schwarz

As before, $a_1, \ldots, a_n$ are real positive numbers. Then

$$(a_1^2 + a_2^2 + \ldots + a_n^2)(b_1^2 + b_2^2 + \ldots + b_n^2) \geq (a_1 b_1 + a_2 b_2 + \ldots a_n b_n)^2 ,$$

with equality if and only if $a_1/b_1 = a_2/b_2 = \ldots = a_n/b_n$.

5. Chebyshev

Let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ be two sequences which are monotonic in the same direction (either both increasing or both decreasing). Then

$$\frac{a_1 b_1 + a_2 b_2 + \ldots + a_n b_n}{n} \geq \left(\frac{a_1 + a_2 + \ldots + a_n}{n} \right) \left(\frac{b_1 + b_2 + \ldots + b_n}{n} \right) .$$