Boston University Questrom School of Business
MF840 – Fall 2019

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Problem Set 3
Due Thursday April 25th in class

Problems turned in after the beginning of student section have a notch deduction.
Teams "*across sections*" turn in their homework at the beginning the morning class
Problems turned in after class get a zero.

- Do the Problem Set in groups of two
- Turn in one paper copy in class with two names, no electronic submission accepted.
- **To get a check, you need to answer <u>all</u> the questions.**
- **All discussion and theoretical questions must be hand written with a pen to count**.

<u>Problem 1:</u> <u>Odds Ratios</u>

Use the file vix-sp-week.csv for this. Volatility is baaack! same story every time there is a little bump in the stock market. You discuss with a colleague that you don't even think one could reject the null hypothesis that the VIX mean is 15% from January 2018 to April 15th 2019. (The mean was way below that in 2015-2017)

a) Your colleague computes the t-stat. What does he find? What is the p-value (aka marginal significance level) for a one-sided test of the null $H_0 : \mu_{VIX} = 15$.

b) You know the p-value likely favors the alternative. On the other hand the sample size, 68, is not large. You wonder whether a Bayes Factor would find that $p(H_0|D)>0.5$.
You estimate the mean model with a conjugate prior, $p(\mu \mid \sigma)$ centered on H_0 of course, with the usual prior variance σ^2/T_0. You use $T_0 = T/10$ To reflect a fair lack of prior precision. You use the usual diffuse prior for σ.

What are the parameters of $p(\sigma \mid D)$?
What is the exact distribution of $p(\mu \mid \sigma , D)$, its parameters?
What is the exact distribution of $p(\mu \mid D)$, its parameters?

c) What is the Bayes Factor if you use the framework in section 6.5 of the notes.

d) Compute the Bayes Factor using the Savage Density ratio. Give the exact value of the prior and posterior ordinates as well as the ratio. Make sure to use the prior and posterior **marginal** densities of μ.

e) Now check the robustness of this answer to the choice of prior. Plot $P(H_0 |D)$ (from d), prior odds of 1 of course) for $T_0 \in [1, 20]$. For what range of T_0 is the conclusion robust?

SEE CODE and class discussion

Problem 2: Monte Carlo Precision (Use the lecture note on MCMC precision to answer this)

You simulate the posterior density of $\beta|D$ with a preliminary 1000 draws.
The sample mean and sample standard deviation of your draws are 0.67 and 0.35.
You will report the 0.67 and 0.35 as your MC estimates of $E(\beta|D)$ and $V(\beta|D)$. How many MC draws do you need to report 0.67 with total accuracy, to report 0.35 with total accuracy?

You just find that your MC simulation scheme has autocorrelation, $\rho_1 = 0.7$, $\rho_2 = 0.4$. The rest are zero. What is your new estimate of the standard deviation of your MC estimate of the posterior mean? How many MC draws should you have for the mean?

See results in Code

Problem 3: Prediction by simulation

You want to forecast the VIX up to two weeks ahead. For this, only use data from the first week of 2013 until 12/17/2018.
You estimate an AR(1): $\log(VIX_t) = \phi_0 + \phi_1 \log(VIX_{t-1}) + \epsilon_t,\ \epsilon \sim N(0, \sigma)$
with a Bayesian regression. Construct the X and the Y variable (you lose an observation).

a) In Table 1, write the posterior means and posterior standard deviations of ϕ_1, σ, computed analytically, using diffuse priors. Use AZ appendix as needed. Report two digits as in .xx

b) You boss reminds you that σ has a skewed distribution, so its standard deviation is useless. She asks you to add two columns for σ: the 5% and 95% quantiles. You will have to simulate.

You will use a flat prior on ϕ,σ but you believe that the VIX must be stationary and positively autocorrelated. You can enforce this belief now that you are simulating from $(\phi\,|\sigma, D)$.

You will make 2000 draws, report the number of rejected draws.

Simulate from $p(\sigma|D)$, add your MC estimates of the 5% and 95% quantiles to Table 1.

c) Now build on your simulation as in the notes to simulate the predictive density of the VIX for 12/31/2018 and 1/7/2019.
Of course, you don't care about log(VIX), you care about VIX. But for each draw of log(VIX), you have a draw of VIX by transformation.

On Figure 1, plot VIX from1/1/2018 to 12/24/2018, then the two predicted densities posterior means and 25%, 75% quantiles.
Add the actual realizations of VIX for these two weeks as points.

On Figure 2. Plot the predictive densities of the two weeks (density plot not histogram).

Report in Table 2, the 25%, 50%, 75% quantiles of the predictive density for these two weeks. On the third row, report those quantiles for the unconditional distribution of VIX.

See Code and class discussion

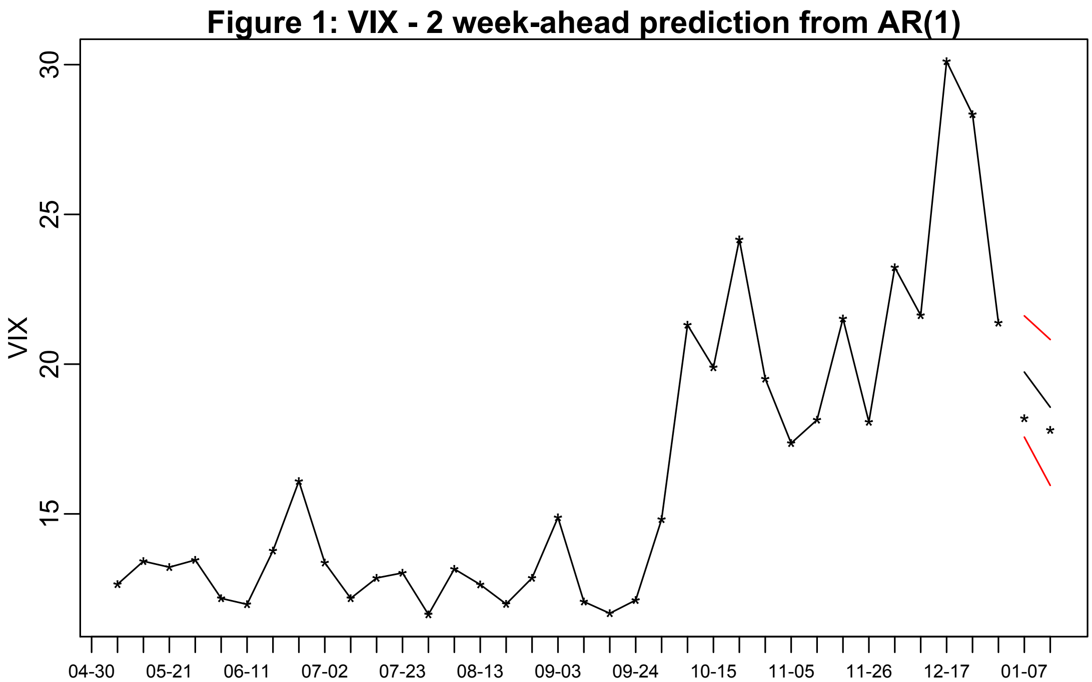

Figure 2