**Boston University Questrom School of Business**

**MF840 – Fall 2019**

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**Problem Set 1**

**Due Thursday February 7th in class**

Problems turned in after the beginning of student section have a notch deduction.

**Teams “*across sections*” turn in their homework at the beginning the morning class**

Problems turned in after class get a zero.

* Do the Problem Set in groups of two
* Turn in one paper copy in class with two names, no electronic submission accepted.
* **To get a check, you need to answer all the questions.**
* **All discussion and theoretical questions must be hand written with a pen to count**.

**Problem 1:** MLE and the Δ method

Consider the regression of Y on X, T observations and k variables including the intercept, as in the notes. Y = X β + ϵ . The errors are i.i.d. normal with mean zero and **precision h.** The precision is simply h = 1/ σ2, the inverse of the variance. It is more and more often used in modern econometrics.

a) Write the Log likelihood of the parameters (β , h) given the data (Y, X). To be clear, there can not be any σ or σ2 in this density, only h! Then write the two first-order conditions with respect to β and h. Find the MLE estimates of β and h.

b) Compute the 2nd order derivatives, then take the expectation, then compute the asymptotic covariance matrix of . What is variance of ?

c) Now use the Δ method to compute the asymptotic variance of the estimator of variance , where is the variance of the regression noise.

d) Now use the Δ method to compute the asymptotic variance of the estimator of **σ =** , the **standard deviation** of the regression noise.

**Problem 2:** MLE estimation with heteroskedasticity

Stock returns are independently distributed: rt ~ N(μ ,σt). You think that the standard deviation σt is best approximated as a simple function of an observable strictly positive variable xt: = σ2 xt.

a) You collect T independent returns rt’s and corresponding xt’s Write the log-likelihood of the parameters given the data: l(μ , σ | R , X).

b) Maximize it to compute , and then. Show your proof and result

c) Explain in a few words how this is like a weighted least squares estimator. What are the weights? What observations are down- or over- weighted in the estimator, relative to the basic sample average?

d) Use the Cramer-Rao lower bound to compute the asymptotic variance of .

e) Compute the variance of the basic equal-weighted sample mean account for heteroskedasticity.

f)Compare the variances of the MLE and the equal-weighted naïve sample mean. You can use the results in the handout AM-GM-HM-inequalities.pdf to conclude.

**Problem 3 is now made more precise**

**Problem 3:** Feasible GLS

The file size-day-0918.csv contains the retuns on the 10 size-decile portfolios of the US stock market. For this exercise, you will use Decile 2. The goal is to estimate the portfolio beta and alpha. Use only the first 3 years of data for this. If you think you need more data, explain what it is and get it on KF’s web site. Don’t forget that KF’s data are in %.

a) Estimate the portfolio’s β and α, in a table put the estimates and OLS standard errors. In a second row, add (both types of) HAC standard errors. On the view of the OLS and HAC standard errors do you suspect a potential GLS situation?

b)

* Plot the autocorrelations and partial autocorrelations of the residuals. Conclude as to the likeliest model (no need for a formal model search).
* Report in a table the first four autocorrelations and their standard errors.
* Do a feasible GLS allowing for autocorrelation. Use an MA(3). That is, use directly the autocorrelations you just estimated to compute a TxT correlation matrix C. T

Tip: To get the covariance matrix, you also need to estimate the standard deviation of the noise and put it in a diagonal matrix D, then you can use the relation:  Ω = D C D.

Tip: You can fill up C in one line of code with the command "toeplitz". What is a Toeplitz matrix??

xx <-c( values of the first row of the matrix )

cc <-toeplitz(xx) # Voila! Check that it work, read the manual

* Add the β estimates and standard errors to your OLS results table

c)

* For your diagnostic of heteroskedasticity, do these two things: Plot absolute values of residuals vs Xt. Conclusion? For a diagnostic of potential GARCH effects, plot the acf and pacf of the absolute values of the residuals. Conclusion?
* Estimate a GARCH(1,1) on the residuals of the OLS regression. Report the parameter estimates. Plot the time varying standard deviations of the residuals.
* Use these {σt}s to construct the Ω matrix and a feasible GLS with GARCH(1,1) errors.
* Add the β estimates and standard errors to the OLS results table.

Tip: Do not worry if you get a message of "failed convergence". Recall that the GARCH(1,1) can't estimate a variance for the first observation since it is an ARMA(1,1) in the squares. For the first observation, use the unconditional variance instead. That allows you to have a TxT matrix. Since GARCH gives you standard deviation estimates, you directly have the diagonal TxT covariance matrix.

d) Starting from c). Implement a simple loop to iterate your GLS. Save the estimate at each iteration. Stop the iteration when the % change in the estimates is less than 0.5%. Is the final estimate very different from the initial one?

e) Explain but do not do it, how you would implement a feasible GLS accounting for both autocorrelation and heteroskedasticity.