Recurrent nets

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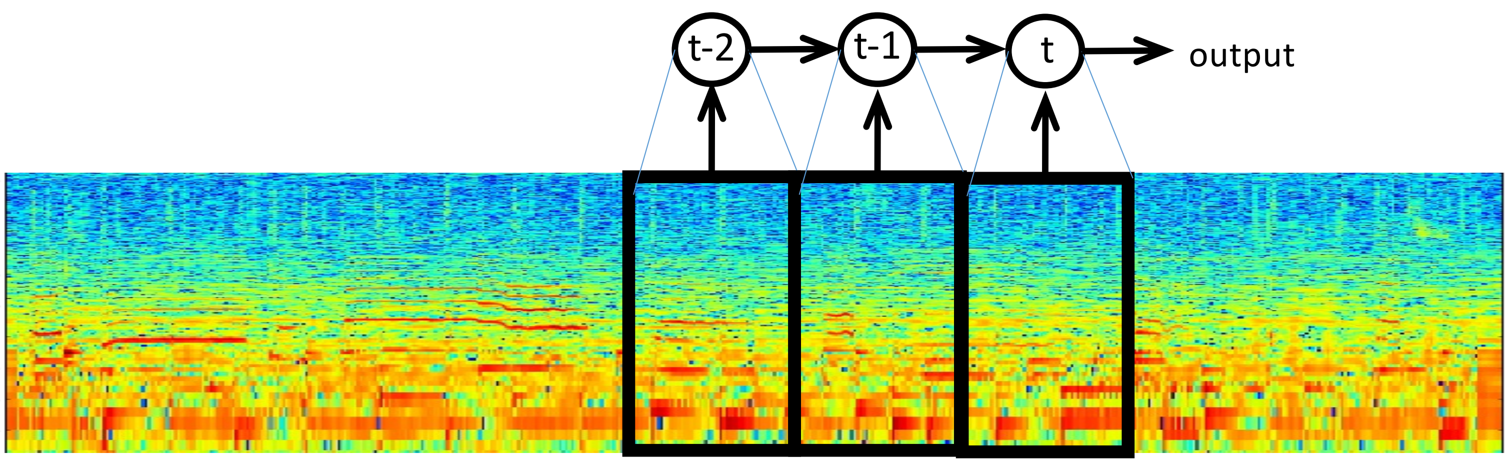
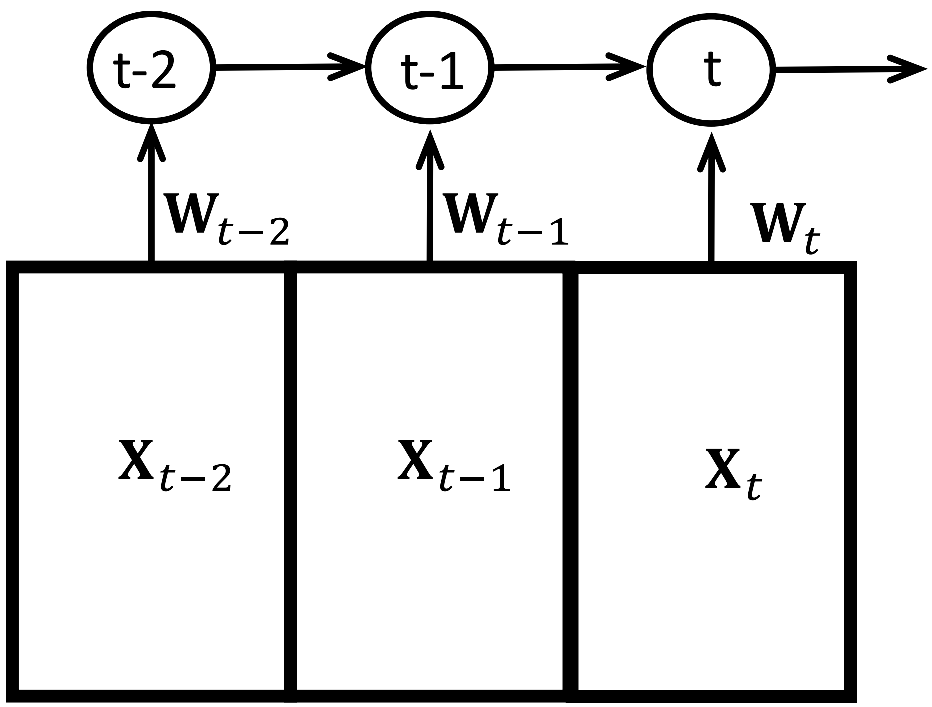
Dealing with time

- With a ”standard” feed-forward architecture, you process data from within a window, ignoring everything outside the window.
- To get influence from the processing of earlier time steps, add nodes and connections
- This doesn’t scale well

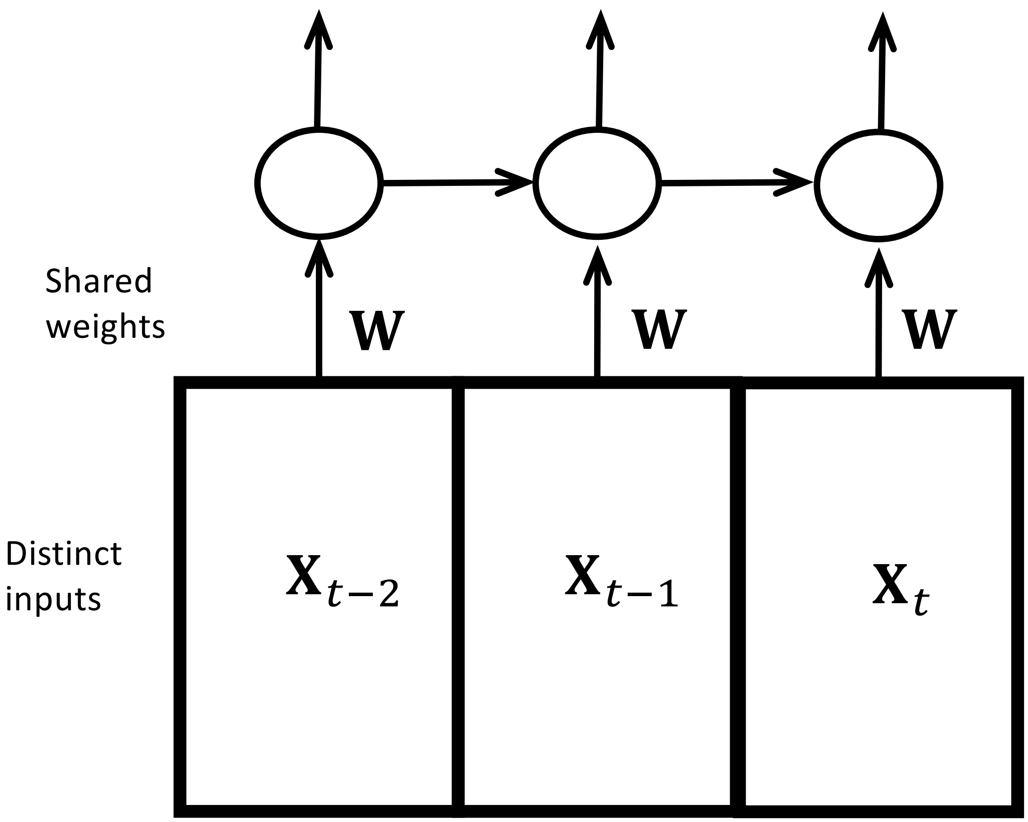


Let’s look at that net.



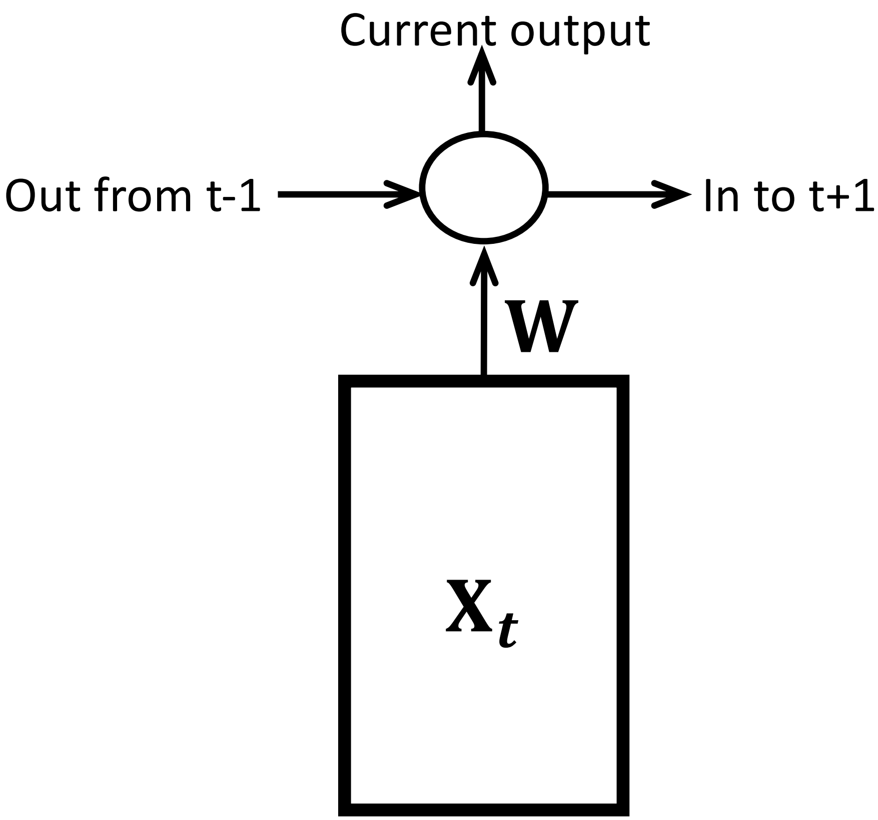
- An entire new set of weights for EACH time step.
- Audio is sampled at 44,100 times per second
- The number of past time steps you could consider is limited by the architecture.
- The number of weights to learn quickly gets out of control.

Take an idea from CNNs and HMMs



- Markov property: The state of the world can be captured by knowing current state + immediately previous state
- Markov models use recurrent connections
- CNNs use the same set of shared weights on different parts of the input

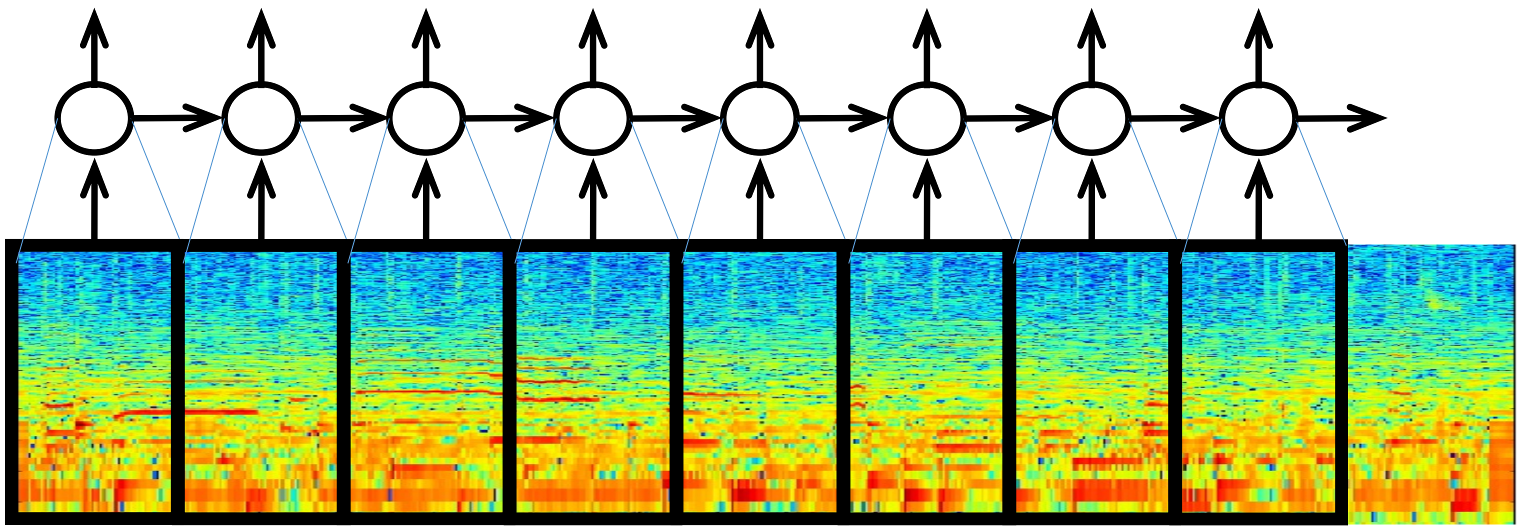
Take an idea from CNNs and HMMs



- If all the windows share the same input weights (like in a feature map), then we only have the same number of weights as if we had a single window.
- This is a recurrent net.
- How do you train this?
- Are there any obvious limitations?

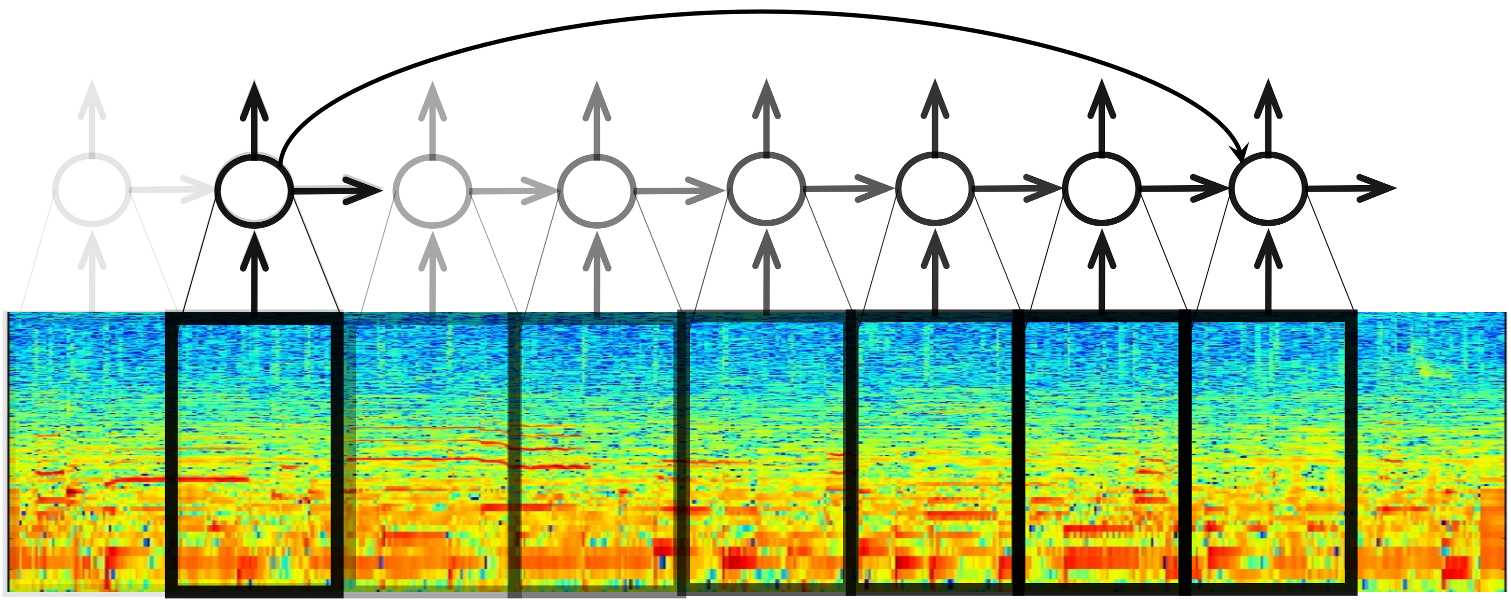
Backprop through time: “Unrolling”

- Pick a number of steps over which you’re going to “unroll” the net.
- Treat it like you’re training a convolutional neural net
- Pick the number of steps based on your frenemy: Exponential decay



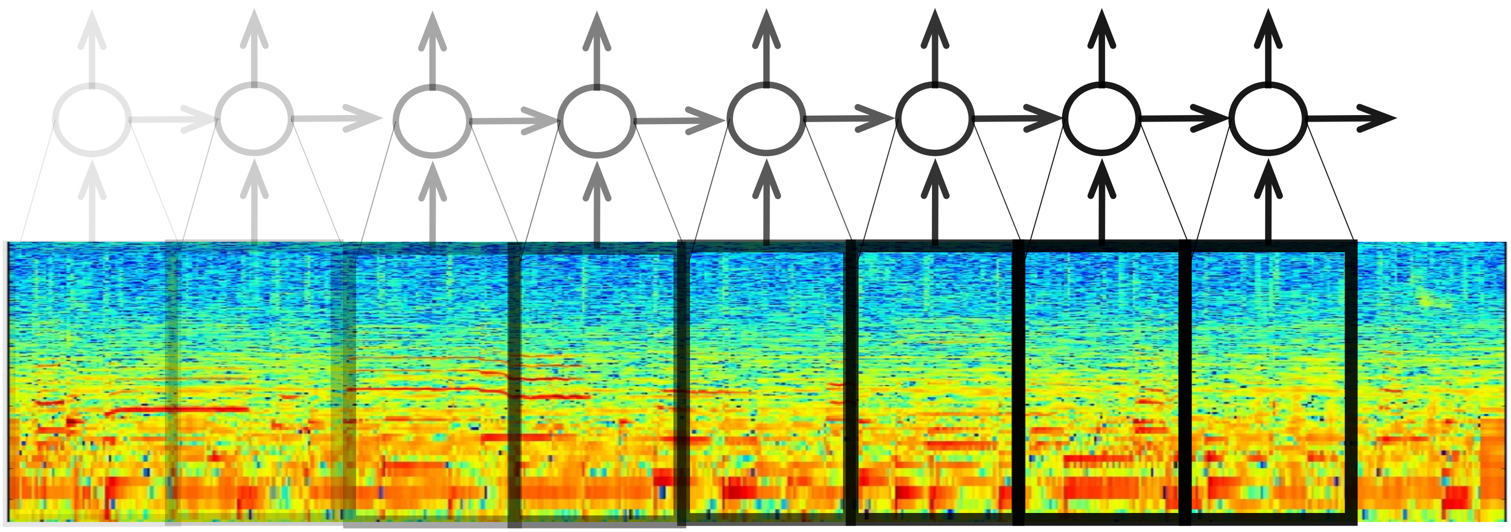
Getting influence from the past: Skip connections (used in Highway networks)

- Widely used
- Limited by the length of the skip

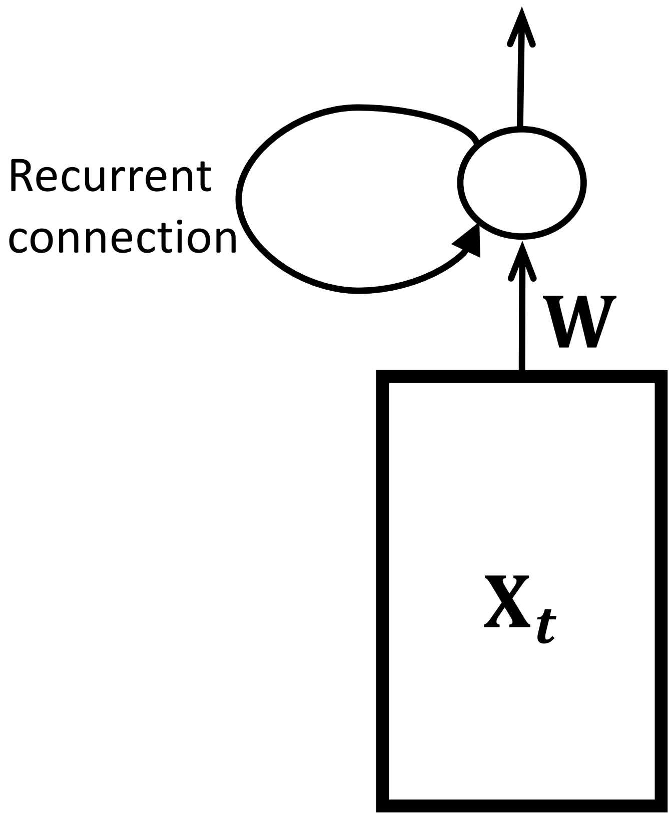


Exponentially decaying influence

- If your network needs to connect information from a distant timestep, the influence of the earlier one tends to get lost
- Why? Exponential decay.



Exploding and vanishing gradients



- What if the weight on the recurrent connection is greater than 1?
- What if the weight on the recurrent link is less than 1?
- What if it is exactly 1?

An RNN example: Language modeling

- In language modeling, the game is to be able to predict the next word, given the previous N words.

- Examples

  “Two plus two equals…”

  “A stitch in time saves….”

  “I never did…”

Our text encoding

- 1000 most common English words. + start + stop + other

- Encoding: 1003 element one-hot vector for each word in a sentence
  - Word index determined by popularity
  - Start = 1001
  - Stop = 1002
  - Other (any word not in the top 1000) = 1003

- Examples:

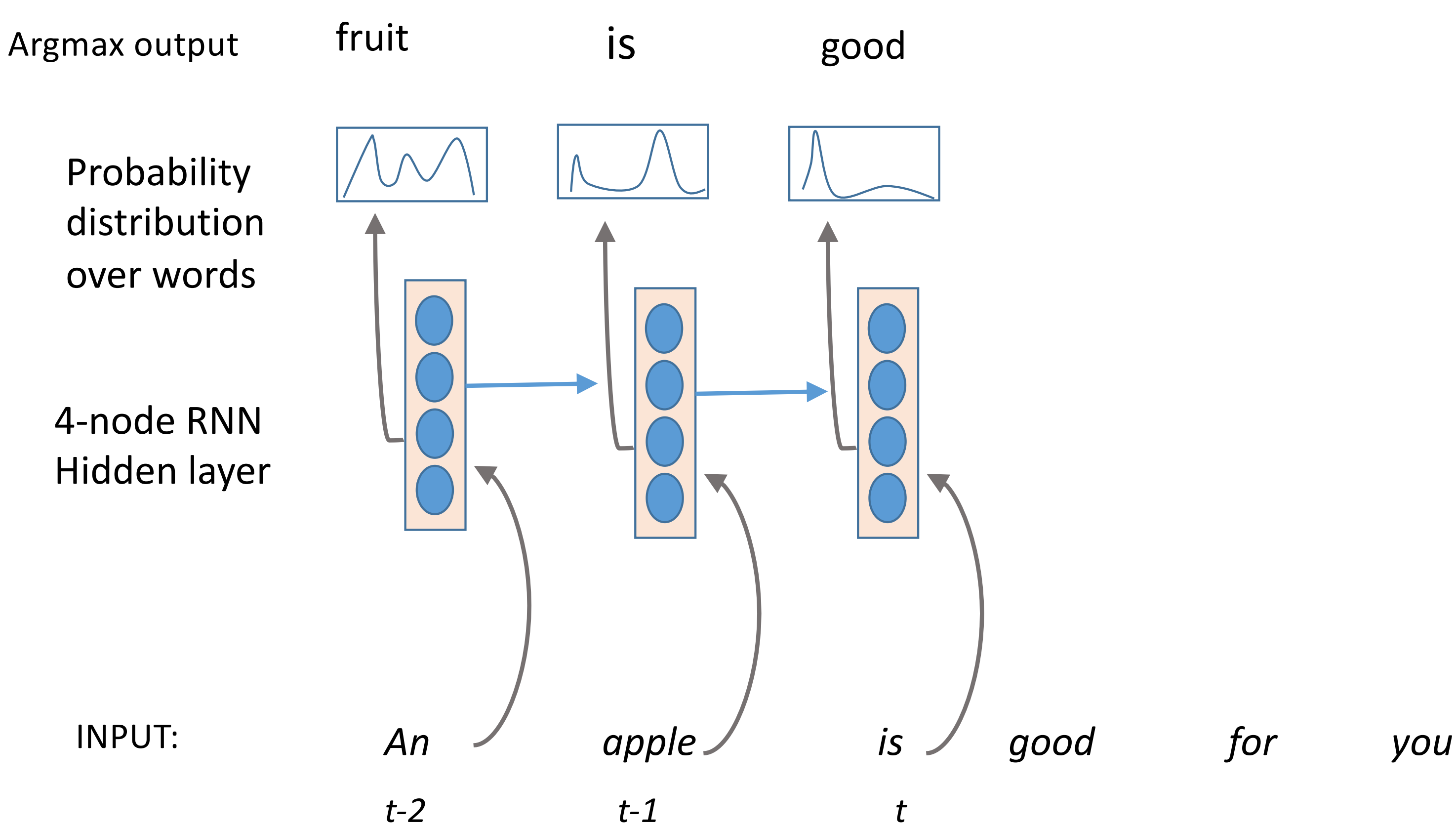
  An apple is good for you. -> [1001, 48, 927, 24, 121, 7, 26, 1002]

  Lilliputian dilatants prognosticate parsimoniously! -> [1001, 1003, 1003, 1003, 1003, 1002]

The goal: predict the next token

- Each sentence is its own label.
- Given “An apple is…”, predict “good” as the next word.
- Our model output will be a probability distribution over the 1003 element vector (top 1000 words + start + stop + other).
- We can use cross-entropy loss, comparing the one-hot vector to the probability vector output by the model.

Our network



Argmax output
fruit
is
good
Probability distribution over words
4-node RNN Hidden layer
INPUT:
An
apple
is
good
for
you
t-2
t-1
t

A RNN with 4 hidden nodes : how many weights?

OUTPUT a 1003 element probability distribution over the set of words.

Output layer $[0.01, .0000098, \ldots, .0023, \ldots, .001, 0, .000053]$ Softmax activation

ALL HIDDEN NODES ARE FULLY CONNECTED TO THE OUTPUT LAYER

Hidden layer Sigmoid activation

ALL HIDDEN NODES ARE FULLY CONNECTED TO THE INPUT LAYER

Input +
prev state

$[0, 0, 0, \ldots, 1, \ldots, 0, 0, 0]$

INPUT word w(t): a 1003 element one-hot vector encoding word *t.*

$[.2, 0, .001, .3]$

Previous state s(t-1): a vector of the output from each hidden unit from time *t-1*

RNN: the math

OUTPUT a 1003 element probability distribution over the set of words.

Output layer $[0.01, .0000098, \ldots, .0023, \ldots, .001, 0, .000053]$ Softmax activation

ALL HIDDEN NODES ARE FULLY CONNECTED TO THE OUTPUT LAYER

Hidden layer

Sigmoid activation

$$x(t) = [w(t), s(t-1)]$$ this vector has 1003 + *n* elements

Input + prev state

$$w(t)$$

INPUT word w(t): a 1003 element one-hot vector encoding word *t.*

$$s(t-1) = [s_1(t-1), \ldots, s_n(t-1)]$$

Previous state s(t-1): a vector of the output from each hidden unit from time *t-1*

What if we use a more realistically sized net?

- Dictionary size = 50,000
- Hidden states = 100

- 50,000*100*2 = 10,000,000

- It's just that easy to have 10 million weights.

- Adding a couple of extra hidden layers (even fully connected ones) doesn't cost you much, compared to the dictionary size.

RNN: the math

OUTPUT a 1003 element probability distribution over the set of words.

Output layer $[0.01, .0000098, \ldots, .0023, \ldots, .001, 0, .000053]$ Softmax activation

ALL HIDDEN NODES ARE FULLY CONNECTED TO THE OUTPUT LAYER

Hidden layer Sigmoid activation

$$x(t) = [w(t), s(t-1)]$$ this vector has 1003 + *n* elements

Input +
prev state

$$w(t)$$

INPUT word w(t): a 1003 element one-hot vector encoding word *t.*

$$s(t-1) = [s_1(t-1), \ldots, s_n(t-1)]$$

Previous state s(t-1): a vector of the output from each hidden unit from time *t-1*

RNN: the math

OUTPUT a 1003 element probability distribution over the set of words.

Output layer $[0.01, .0000098, \dots, .0023, \dots, .001, 0, .000053]$ Softmax activation

Hidden layer

Each node *j* $$s_j(t) = \sigma\left(\sum_i u_{ij}\, x_i(t)\right)$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

Hidden node activation function

$$x(t) = [w(t), s(t-1)]$$

this vector has 1003 + *n* elements

Input + prev state

$$w(t)$$

INPUT word w(t): a 1003 element one-hot vector encoding word *t.*

$$s(t-1) = [s_1(t-1), \dots, s_n(t-1)]$$

Previous state s(t-1): a vector of the output from each hidden unit from time *t-1*

RNN: the math

OUTPUT a 1003 element probability distribution over the set of words.

Output layer

Each output *k*

$$y_k(t) = g\left(\sum_j s_j(t) v_{jk}\right)$$

$$g(z_k) = \frac{e^{z_k}}{\sum_m e^{z_m}}$$

Softmax

Hidden layer

Each node *j*

$$s_j(t) = \sigma\left(\sum_i u_{ij}\, x_i(t)\right)$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

Hidden node activation function

$$x(t) = [w(t), s(t-1)]$$

this vector has 1003 + *n* elements

Input + prev state

$$w(t)$$

INPUT word w(t): a 1003 element one-hot vector encoding word *t.*

$$s(t-1) = [s_1(t-1), \ldots, s_n(t-1)]$$

PREVIOUS STATE: an *n* dimensional vector, where each element is the output of a hidden node at the previous time step *t*-1.

RNN: Predicting the next word

$$prediction = \hat{w}(t+1) = \underset{k}{\operatorname{argmax}}[y_1(t),\dots,y_k(t),\dots,y_m(t)]$$

Output layer

Each output k $$y_k(t) = g\left(\sum_j s_j(t) v_{jk}\right)$$

$$g(z_k) = \frac{e^{z_k}}{\sum_m e^{z_m}}$$

Softmax

Hidden layer

Each node j $$s_j(t) = \sigma\left(\sum_i u_{ij}\, x_i(t)\right)$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

Hidden node activation function

$$x(t) = [w(t), s(t-1)]$$ this vector has 1003 + *n* elements

Input + prev state

$$w(t)$$

INPUT word w(t): a 1003 element one-hot vector encoding word *t*.

$$s(t-1) = [s_1(t-1), \dots, s_n(t-1)]$$

PREVIOUS STATE: an *n* dimensional vector, where each element is the output of a hidden node at the previous time step *t*-1.

Recurrent neural network-based language model (Interspeech 2010)

This is an autoregressive model

- An autoregressive model forecasts the variable of interest using a linear combination of past values of the variable

- The term autoregression indicates that it is a regression of the variable against itself

A language model is a generative model

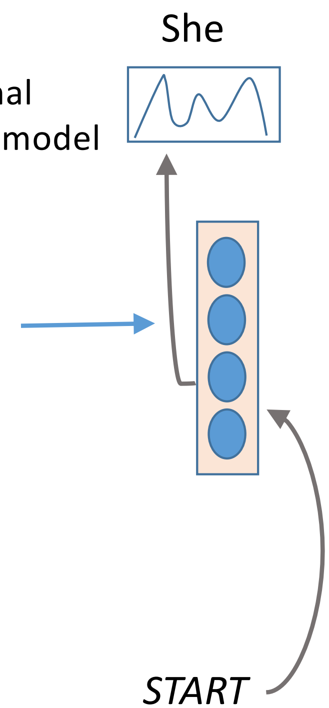
- If you have something that predicts the next word, you have something that can "generate" the next word.
- Sentence completion is possible
- Sentence generation is possible

Generating a new sentence with the model

Sample from the conditional distribution output by the model

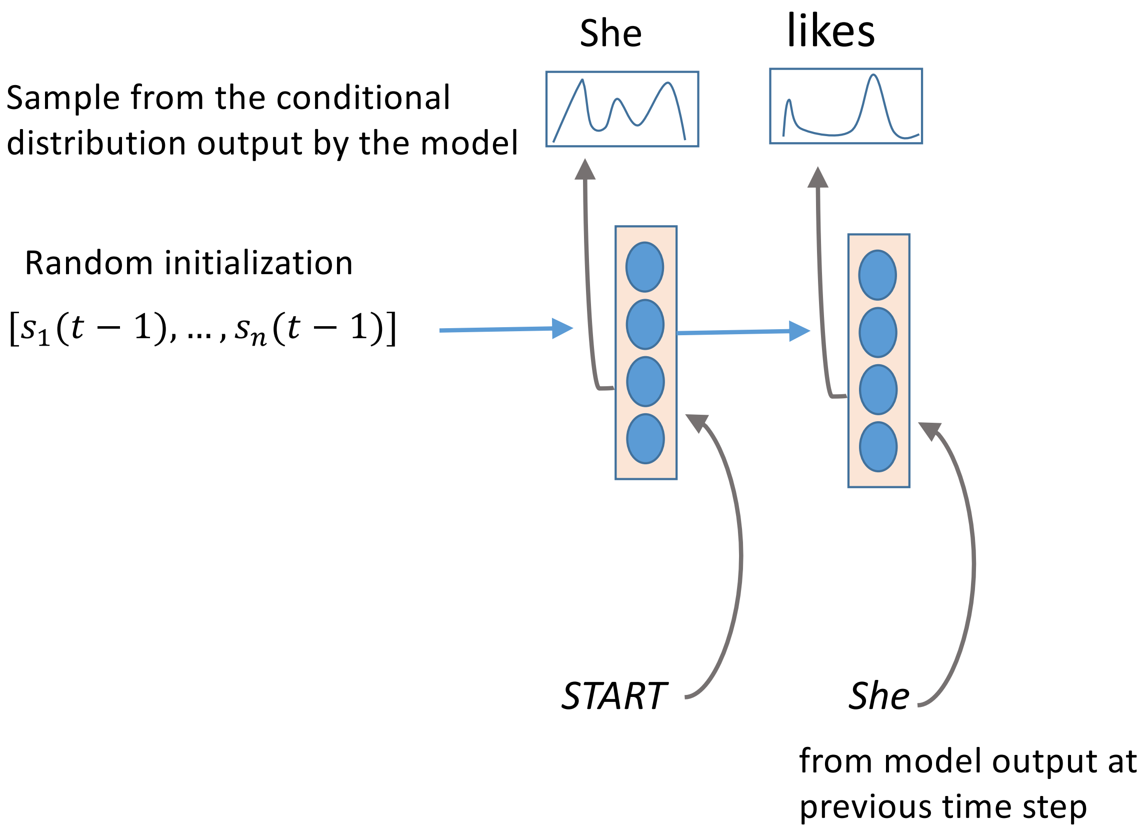
Random initialization

$[s_1(t-1), \ldots, s_n(t-1)]$

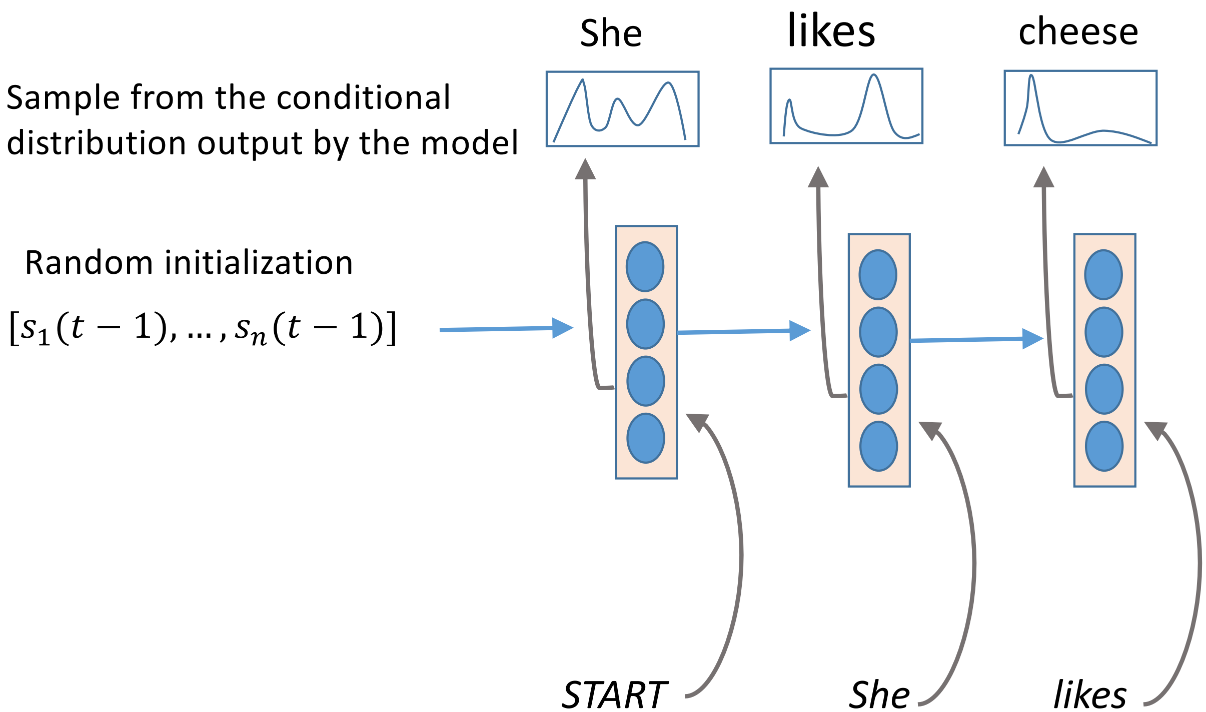


Pick an initialization token

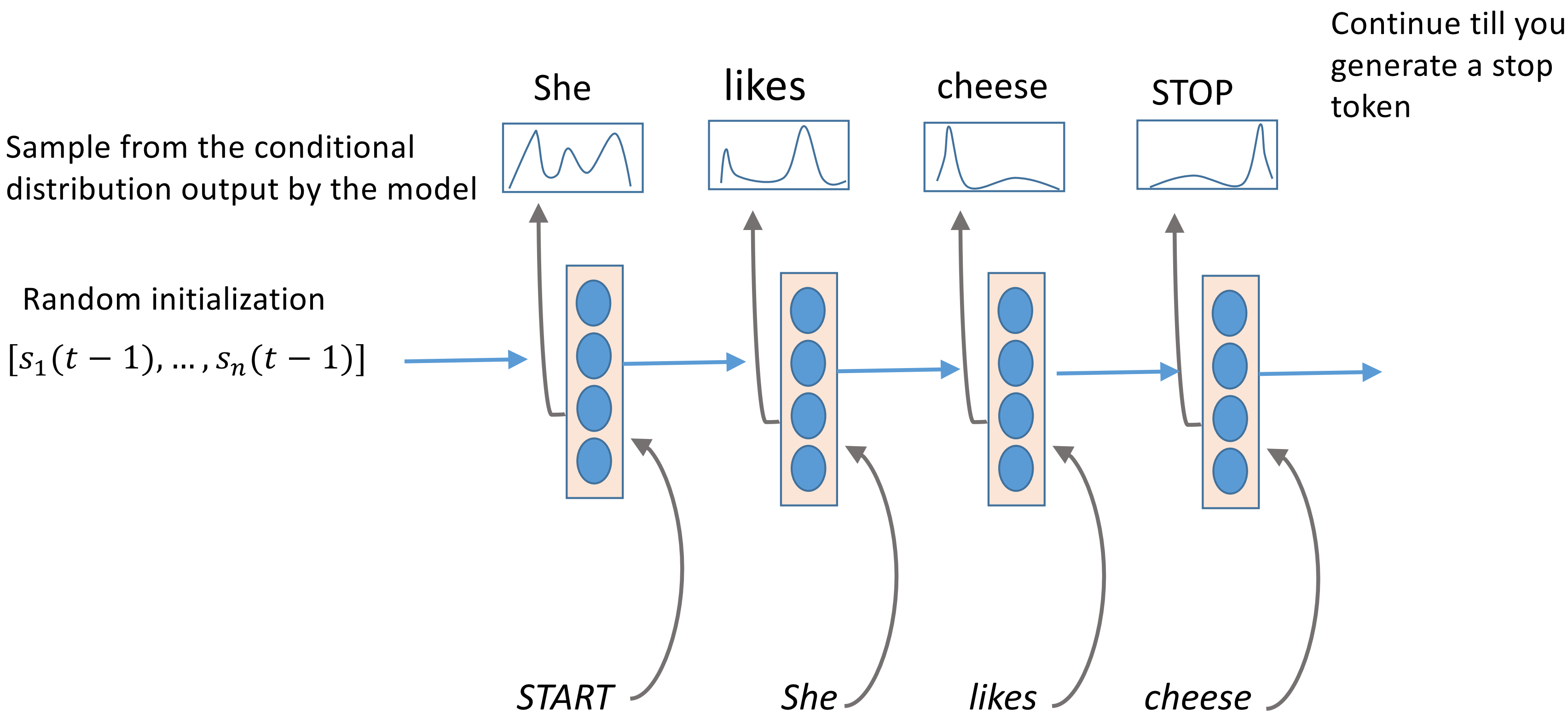
Generating a new sentence with the model



Generating a new sentence with the model



Generating a new sentence with the model



Perplexity

- A measure of how hard it is to guess the next word.
- The exponentiation of the cross-entropy

$$Perplexity = 2^{H(p)} = 2^{-\sum_x p(x)(log_2(q(x))}$$

- A commonly used measure of how well a language model is doing
- Measures how confused the model is (how many choices it has reduced the next word to)

Getting more context

- We predict/generate a new token, based on a prior sequence.
- Our generated output is contextually informed by the past
- But wait….if our training data is whole sentences, can’t we do the same thing from the ”future” (i.e. the next word or rest of sentence)?
- Sure we can. Just feed in the sequence backwards.

RNN: predicting the "past" based on the "future"

$$prediction = \boxed{\hat{w}(t-1)} = \max_k [y_1(t),\ldots,y_k(t),\ldots,y_m(t)]$$

Output layer

$$y_k(t) = g\left(\sum_k s_j(t) v_{jk}\right) \qquad g(z_k) = \frac{e^{z_k}}{\sum_m e^{z_m}}$$

Hidden layer

$$s_j(t) = \sigma\left(\sum_i u_{ij}\, x_i(t)\right) \qquad \sigma(z) = (1+e^{-z})^{-1}$$

Input +
next state

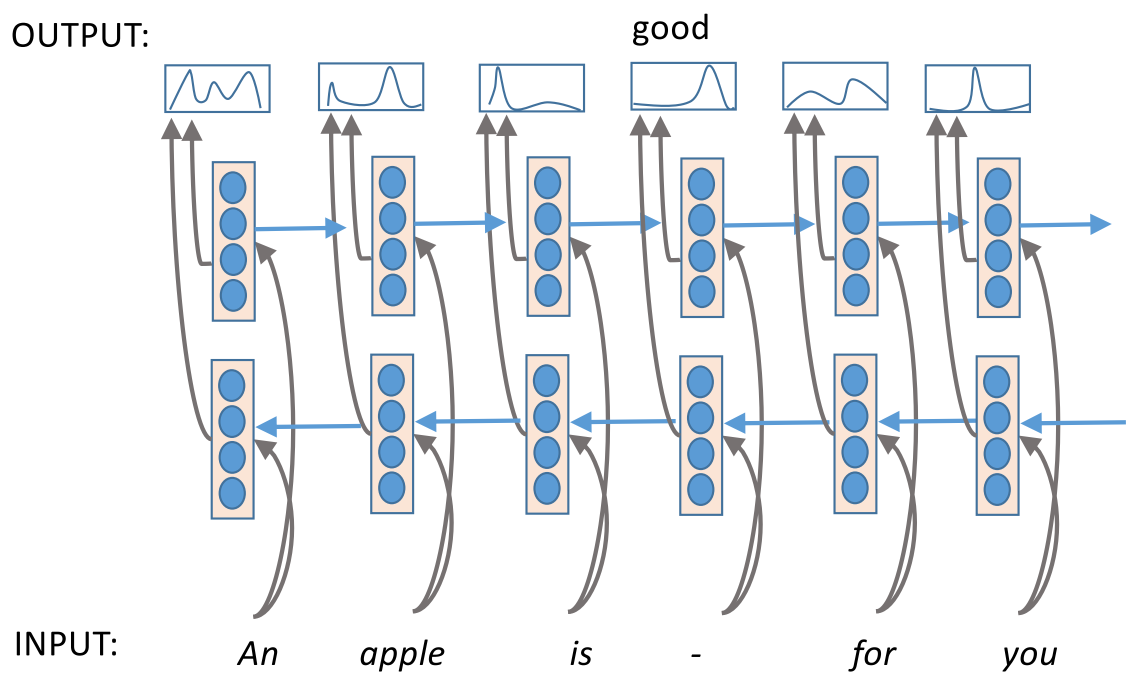
$$\mathrm{x}(t) = w(t) + s(t+1)$$

$$w(t) \qquad s(t+1) = [s_1(t+1), \ldots, s_n(t+1)]$$

INPUT word w(t): a 1003 element
one-hot vector encoding word *t*.

Bidirectional RNN

- Inform output layer's probability distribution using a forward layer and a backwards layer
- The generated token(s) are influenced by both previous and subsequent context



Multi-layer RNN

- You can have multiple hidden layers, where layer n feeds into layer n+1

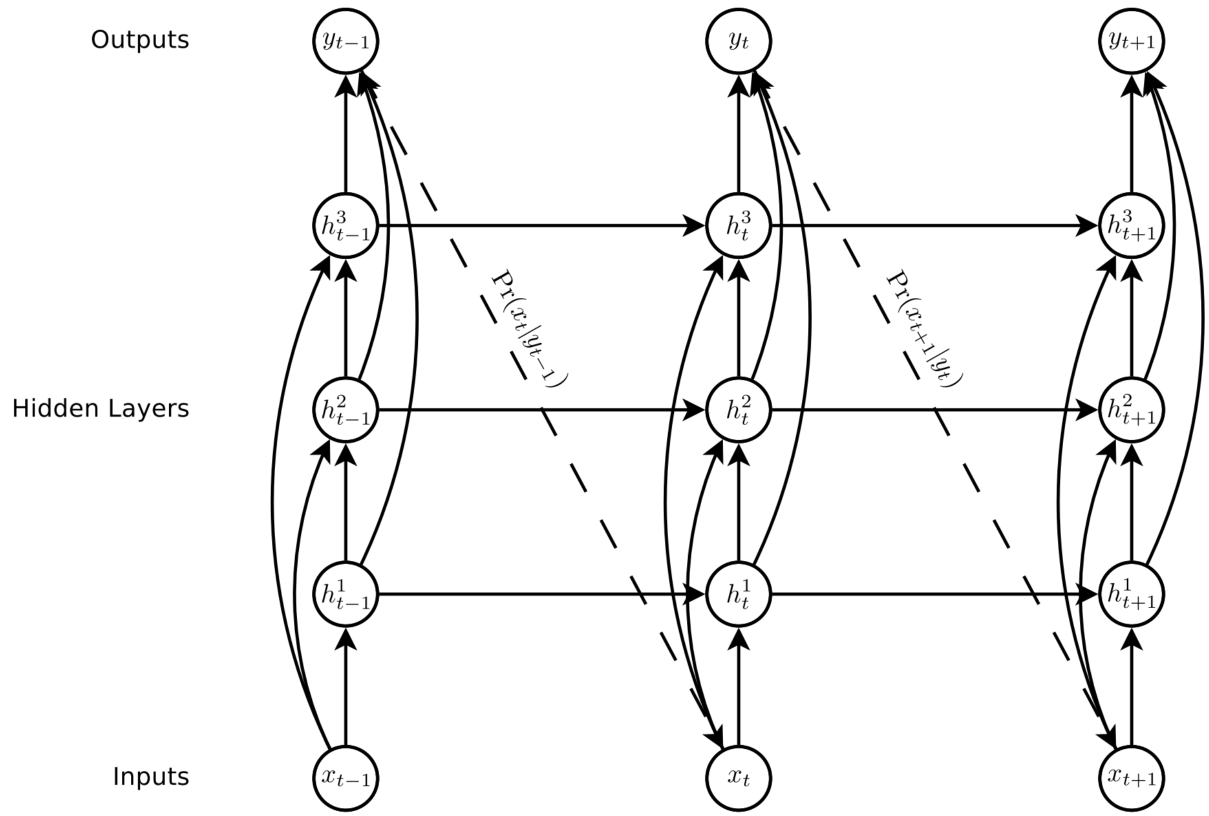


Image from Graves, Alex. "Generating sequences with recurrent neural networks." *arXiv preprint arXiv:1308.0850* (2013).

Long-Short Term Memories

Hochreiter, Sepp, and Jürgen Schmidhuber. "Long short-term memory." *Neural computation* 9.8 (1997): 1735-1780.

Here’s a problem. What can learn to do it?

- X is a finite-length sequence composed of tokens, where each token $x_n \in \mathbb{R} \cup \{a, b\}$.

- The length of X is unknown.

- Before beginning, the total = 0.

- Iterate through X and do the following
  - If $x_n = a$, add x_{n+1} to the total.
  - If $x_n = b$, return the total and reset the total to 0.

Let’s play

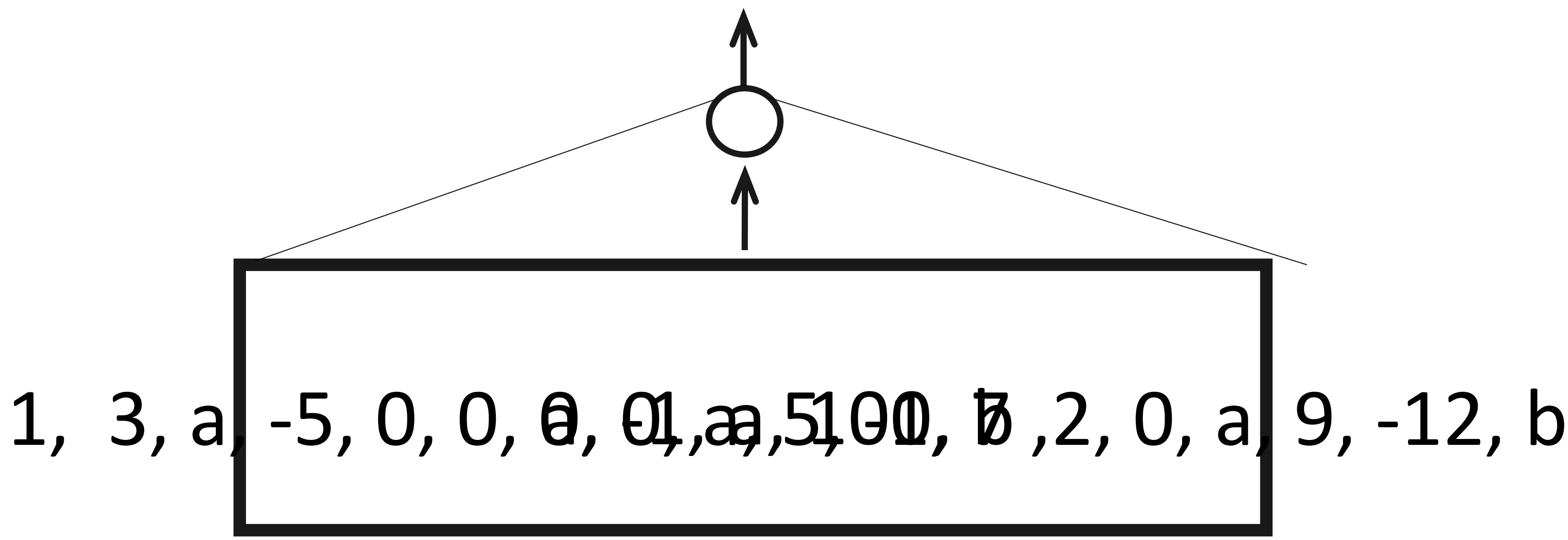
a, -1, a, 100, b = 99

1, 3, -5, a, 5, -1, 8 , 2, 0, a, 9, b = 14

1, 3, a, -5, 0, 0, 0, 0, a, 5, -1, 7 ,2, 0, a, 9, -12, b = 9

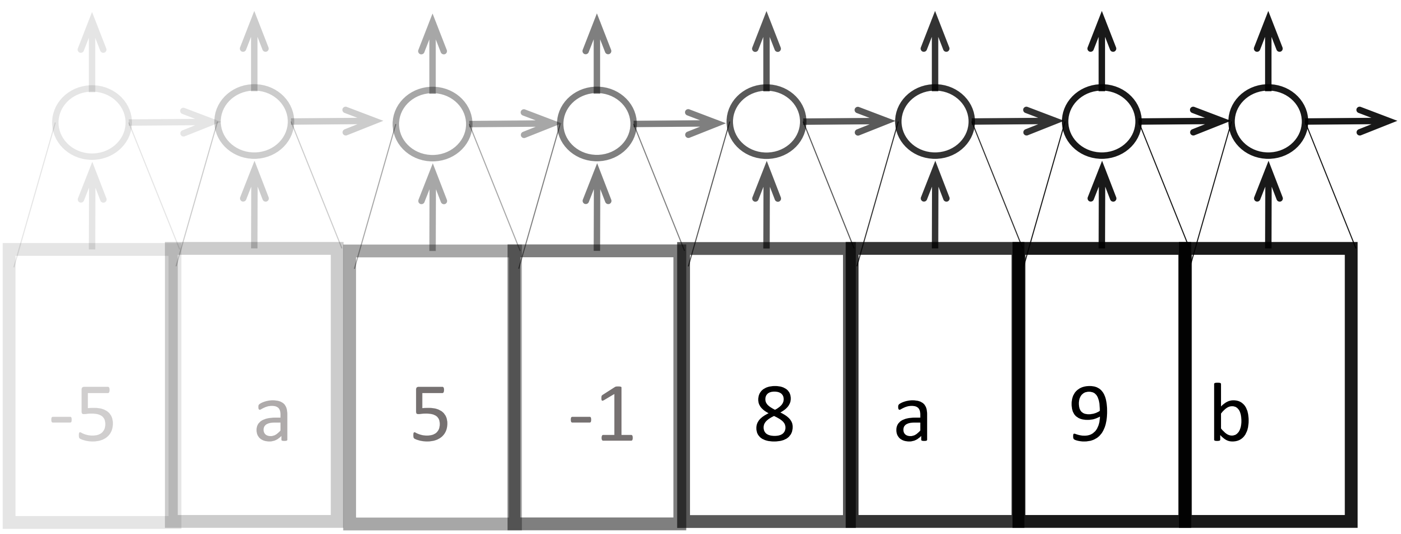
Feed-forward: Fixed-length time window

- If your network needs to connect information from outside the window, you lose.



RNN: exponentially decaying influence

- If your network needs to connect information from a distant timestep, the influence of the earlier one tends to get lost
- Why? Exponential decay.



Long Short Term Memory Units (LSTMs)

- Added a way of storing data over many time steps without decay
- Let networks to handle problems with long term dependencies

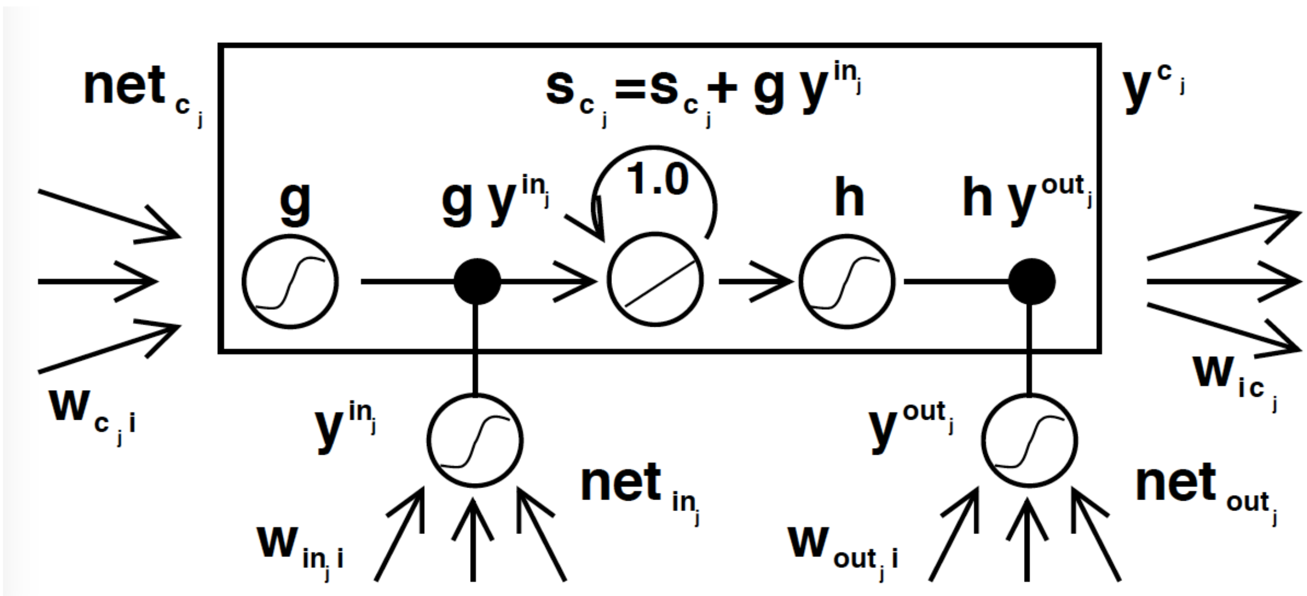


Image from: Hochreiter, Sepp, and Jürgen Schmidhuber. "Long short-term memory." *Neural computation* 9.8 (1997): 1735-1780.

LSTM training

- Error is propagated indefinitely through its memory cell, the constant error carousel (CEC)
- Error flow back through the unit is truncated at the incoming weights.

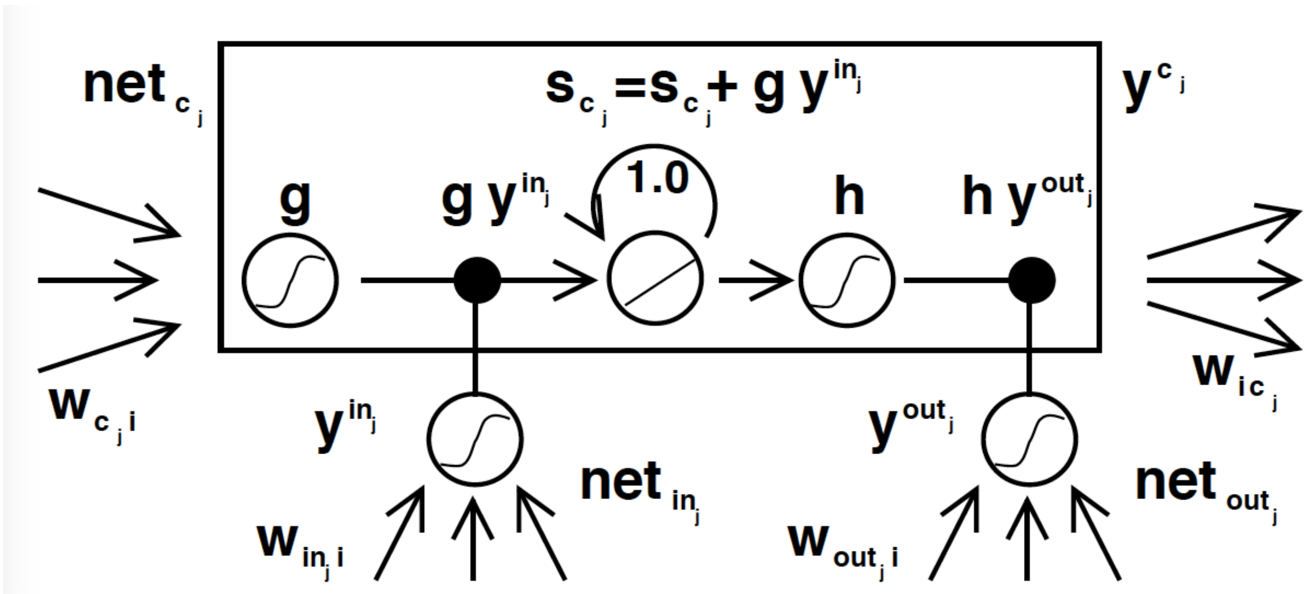
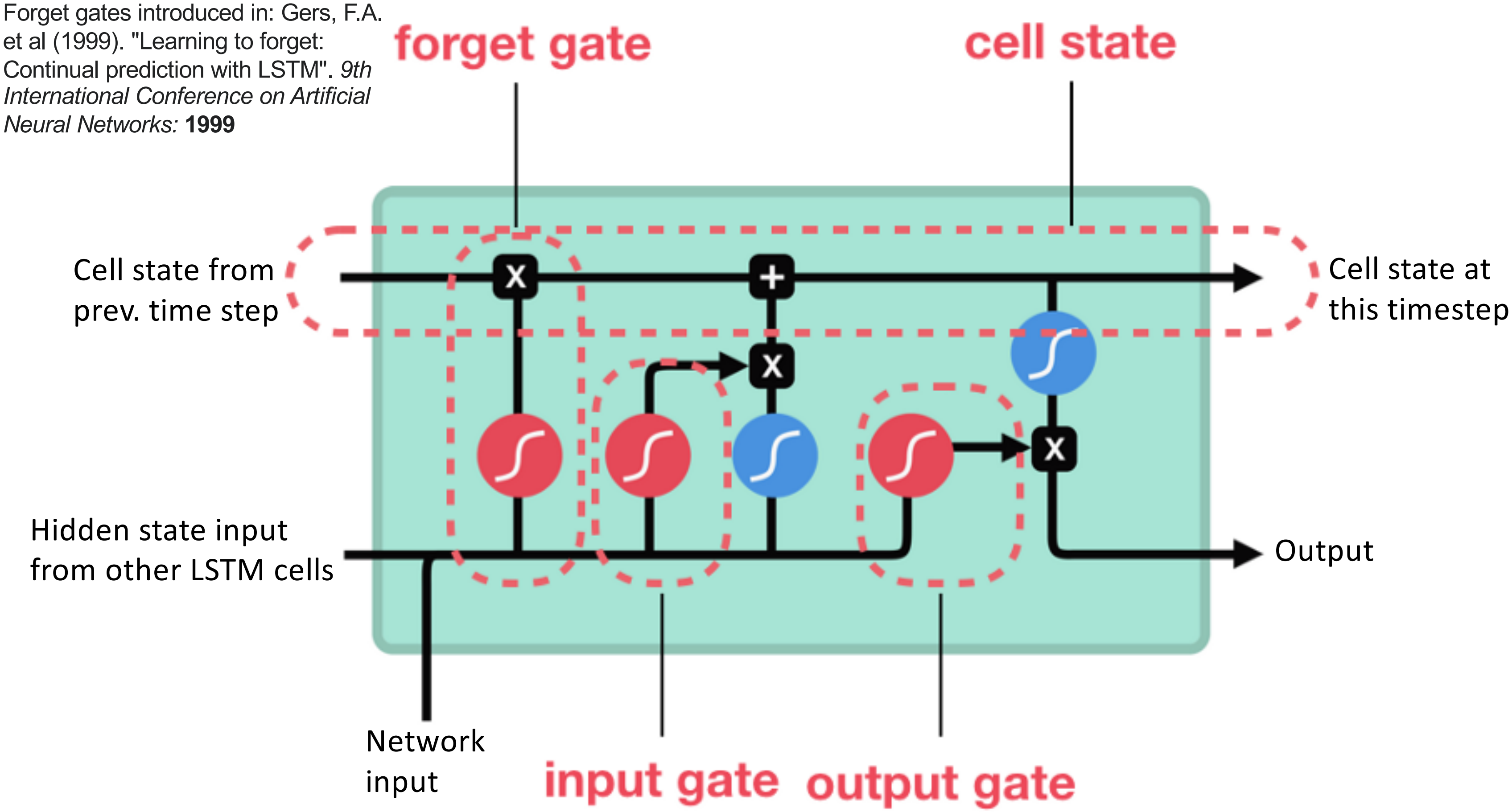


Image from: Hochreiter, Sepp, and Jürgen Schmidhuber. "Long short-term memory." *Neural computation* 9.8 (1997): 1735-1780.

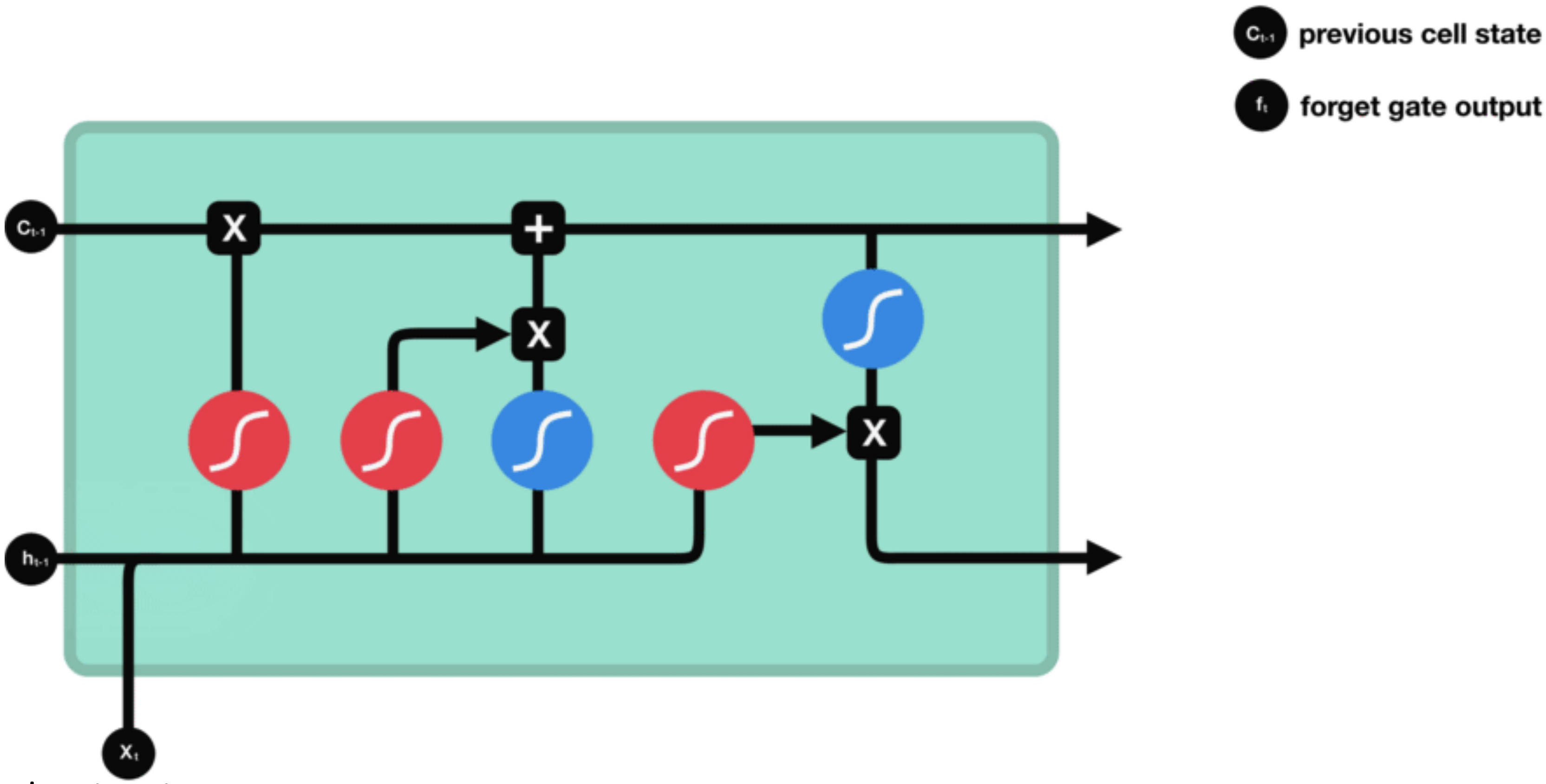
An easy-to-follow-visual of a modern LSTM

Forget gates introduced in: Gers, F.A. et al (1999). "Learning to forget: Continual prediction with LSTM". *9th International Conference on Artificial Neural Networks:* **1999**



Amazing gifs from Michael Phi's https://towardsdatascience.com/illustrated-guide-to-lstms-and-gru-s-a-step-by-step-explanation-44e9eb85bf21

Forget Gate

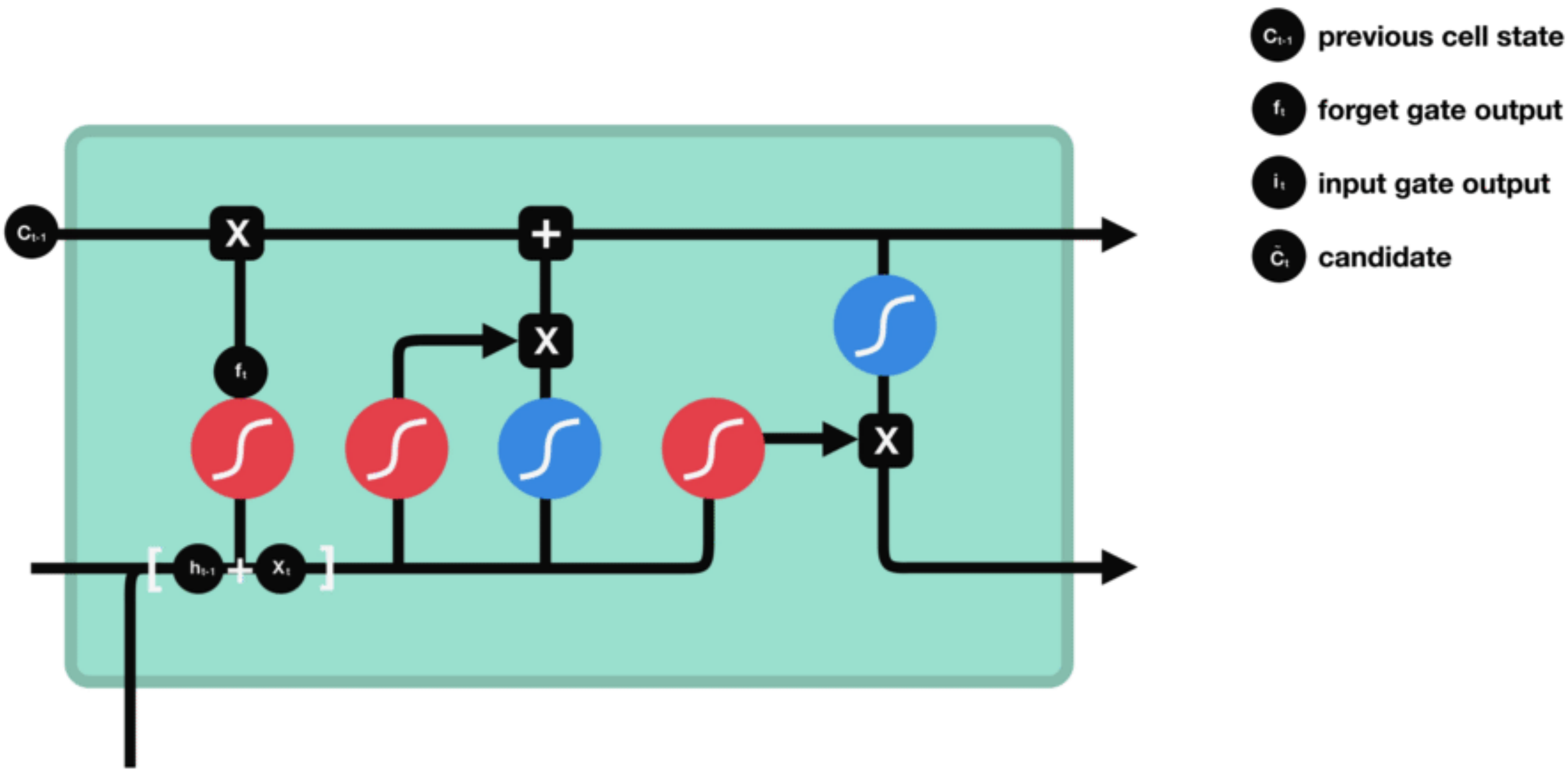


Hidden state passed in from other cells

New input vector

Amazing gifs from Michael Phi's https://towardsdatascience.com/illustrated-guide-to-lstms-and-gru-s-a-step-by-step-explanation-44e9eb85bf21

Input Gate



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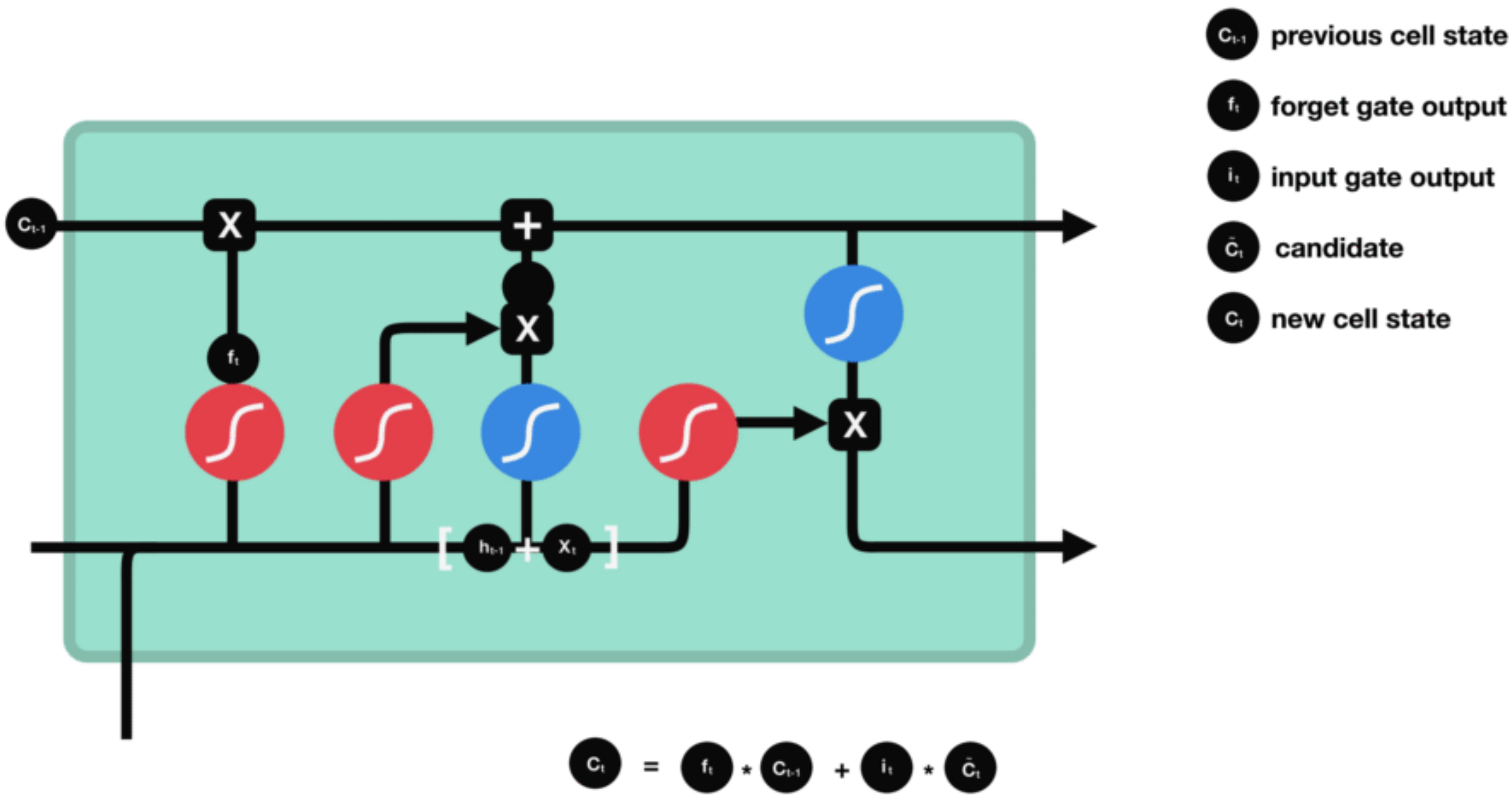
Cell State



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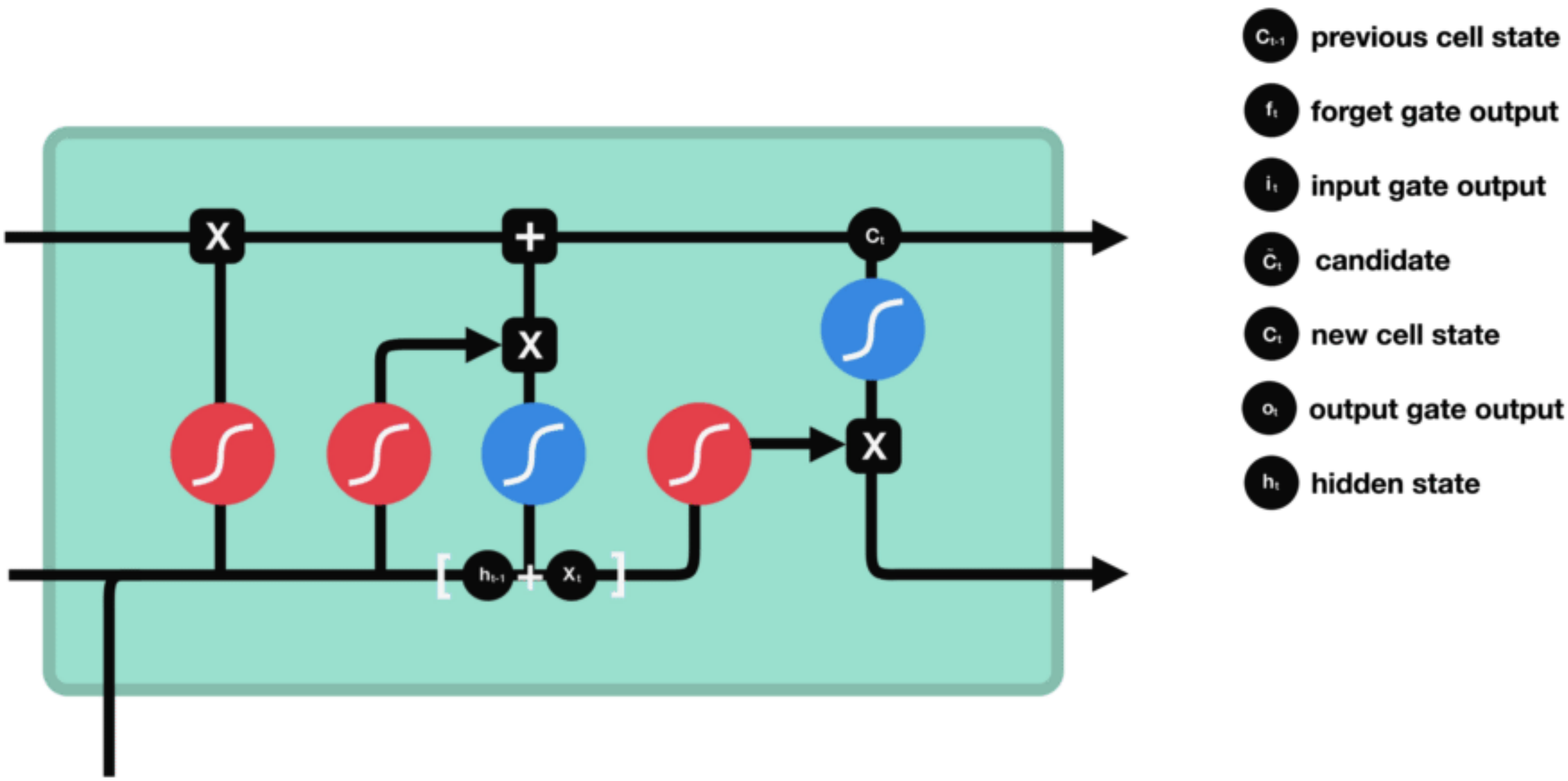
Output Gate



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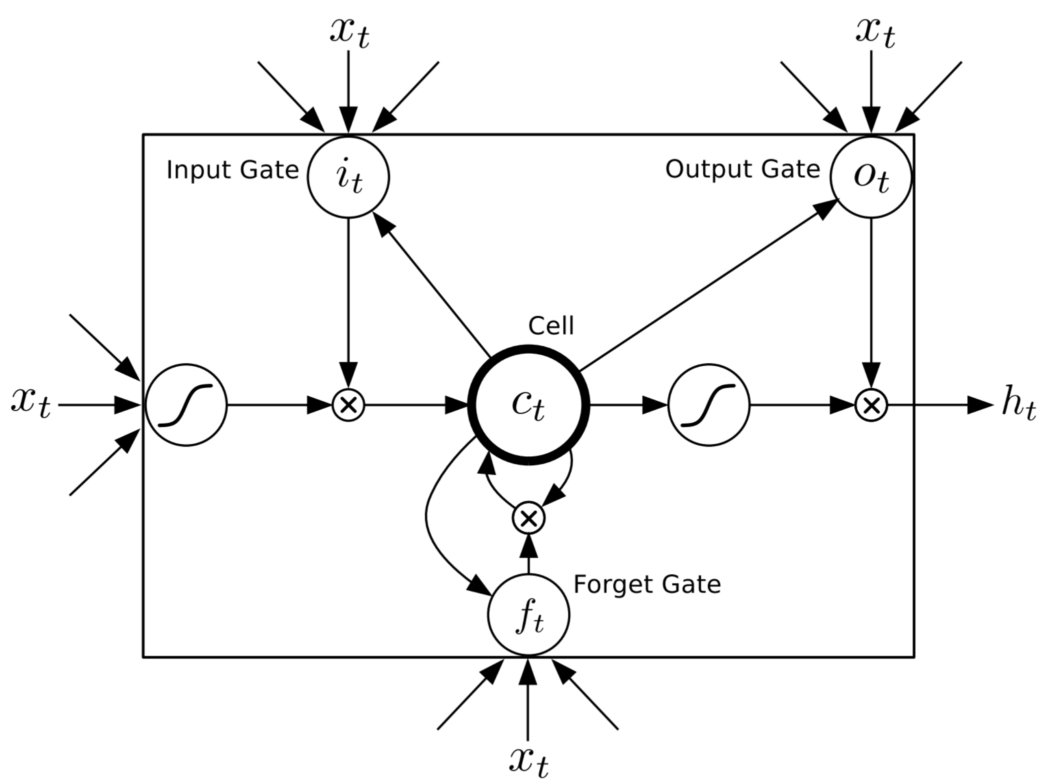
The math of the modern LSTM



$$i_t = \sigma\left(W_{xi}x_t + W_{hi}h_{t-1} + W_{ci}c_{t-1} + b_i\right)$$
$$f_t = \sigma\left(W_{xf}x_t + W_{hf}h_{t-1} + W_{cf}c_{t-1} + b_f\right)$$
$$c_t = f_t c_{t-1} + i_t \tanh\left(W_{xc}x_t + W_{hc}h_{t-1} + b_c\right)$$
$$o_t = \sigma\left(W_{xo}x_t + W_{ho}h_{t-1} + W_{co}c_t + b_o\right)$$
$$h_t = o_t \tanh(c_t)$$

Image adapted from Graves, Alex. "Generating sequences with recurrent neural networks." *arXiv preprint arXiv:1308.0850* (2013).

Forget gates introduced in: Gers, F.A. et al (1999). "Learning to forget: Continual prediction with LSTM". *9th International Conference on Artificial Neural Networks:* **1999**

How many weights for a single LSTM unit?

Input gate $$i_t = \sigma\left(\boxed{W_{xi}} x_t + \boxed{W_{hi}} h_{t-1} + \boxed{W_{ci}} c_{t-1} + \boxed{b_i}\right)$$

forget gate $$f_t = \sigma\left(\boxed{W_{xf}} x_t + \boxed{W_{hf}} h_{t-1} + \boxed{W_{cf}} c_{t-1} + \boxed{b_f}\right)$$

memory $$c_t = f_t c_{t-1} + i_t \tanh\left(\boxed{W_{xc}} x_t + \boxed{W_{hc}} h_{t-1} + \boxed{b_c}\right)$$

output gate $$o_t = \sigma\left(\boxed{W_{xo}} x_t + \boxed{W_{ho}} h_{t-1} + \boxed{W_{co}} c_t + \boxed{b_o}\right)$$

final output $$h_t = o_t \tanh(c_t)$$

$$number\ of\ weights = 4|x| + 4|h| + 3 + 4$$

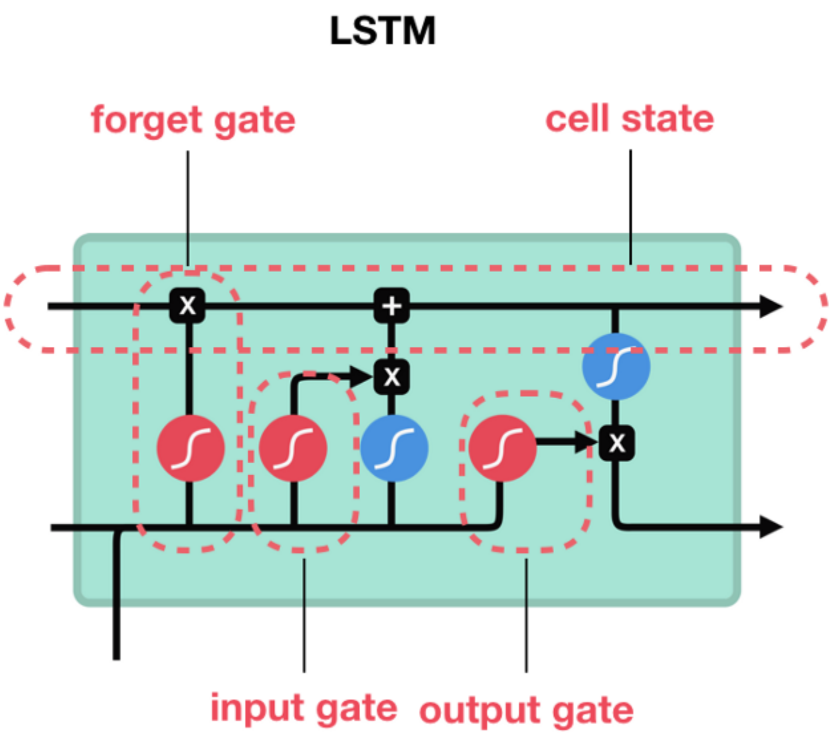
How many weights for this network?

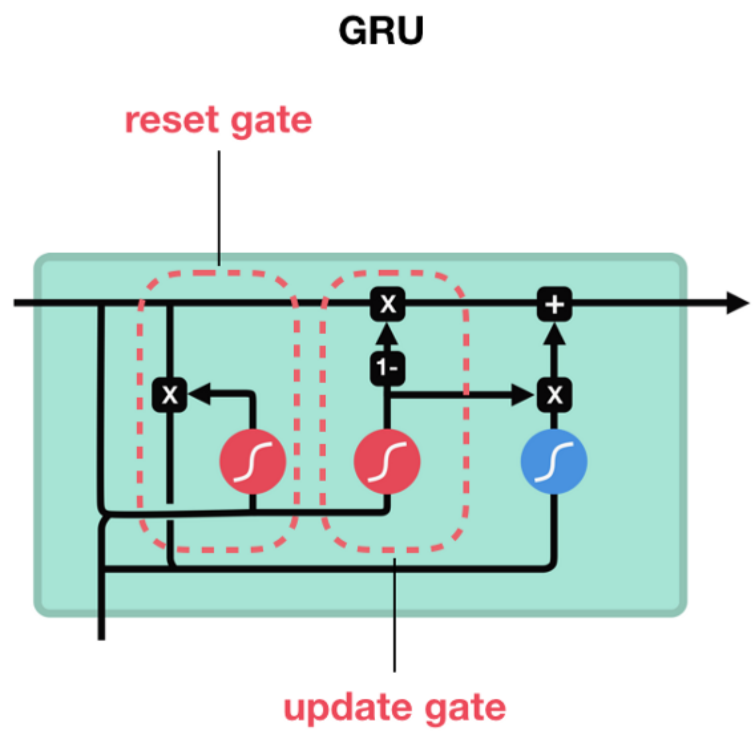
- Input: 50,000 word vocabulary, 4 LSTM layers of 100 cells per layer

- Compare that to a vanilla RNN with the same number of layers and vocabulary size….

- Can we shrink closer to a vanilla RNN but keep advantages of an LSTM?

GRU: A simplified LSTM





Images from Michael Phi's https://towardsdatascience.com/illustrated-guide-to-lstms-and-gru-s-a-step-by-step-explanation-44e9eb85bf21

GRU: The Math

Final output

A linear interpolation between previous output and candidate output

$$h_t = (1 - z_t)h_{t-1} + z_t\tilde{h}_t$$

Update gate

Determines how much to make the output be influenced by the previous hidden state vs the current input.

$$z_t = \sigma(W_z\mathbf{x}_t + U_z\mathbf{h}_{t-1})$$

Vector of all reset gates in the hidden layer

Candidate output

$$\tilde{h}_t = tanh(W_{\tilde{h}}\mathbf{x}_t + U_{\tilde{h}}\,(\mathbf{r}_t\odot\mathbf{h}_{t-1}))$$

Vector of all outputs in the hidden layer

Reset gate

Determines how hard to reset this unit's output

$$r_t = \sigma(W_r x_t + U_r h_{t-1})$$

Math based on: Chung, Junyoung, et al. "Empirical evaluation of gated recurrent neural networks on sequence modeling." *arXiv preprint arXiv:1412.3555* (2014)

LSTM/GRU Plusses and Minuses

- Lets networks handle problems with long term dependencies
- This lets LSTMs (or GRU) solve problems simple recurrent architectures cannot

- Still has trouble with XOR (time-delayed XOR where you XOR two inputs that are an unknown number of time steps apart)
- Lots of extra weights compared to regular cells
- Long and slow to train
- Not easy to inspect networks to understand them