Computer Graph Exam 1*

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Anonymized for Submission

Abstract. Complete solutions with step-by-step explanations and diagrams for the questions you solved incorrectly during the exam to earn 0.25 points (out of 30) for each question (if your answers are typed in Latex or HTML and diagrams are drawn using either Latex or HTML Canvas, you will earn an additional 0.25 points for each question).

Keywords: Computer Graph · Exam · Solution

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Question 8: Rotation and Scaling Matrices

Question: Which of the following matrices are 2D rotation and scaling matrices at the same time?

Choices:

A. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

F. None of the other choices

Correct Answer: $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Explanation:
A 2D rotation matrix for an angle θ is defined as:

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

A 2D scaling matrix with factors s_x and s_y is defined as:

$$S(s_x, s_y) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

The matrix $[-1, 0], [0, -1]$ can be interpreted as both a scaling matrix (scaling by -1 on both axes, effectively mirroring the image) and a rotation matrix (rotating by 180 degrees, since $\cos 180° = -1$ and $\sin 180° = 0$).

Thus, $[-1, 0], [0, -1]$ satisfies the criteria for both a 2D rotation matrix and a scaling matrix.

Question 15: Cardinal Spline Derivative

Question: Which of the following can be the derivative (tangent vector) at $[0, 0]$ for a Cardinal spline (not necessarily Catmull-Rom) interpolating points $[300, 300]$, $[0, 0]$, $[0, 300]$, $[300, 0]$ (in this order)?

Choices:

A. $[-150, 0]$
B. $[100, 0]$
C. $[150, 0]$
D. $[-100, 0]$
E. $[-200, 0]$
F. None of the other choices

Correct Answer: $[-100, 0]$

Explanation:
For a Cardinal spline, the derivative at a point P_i can be given by the tension parameter c times the difference between the following and preceding points. Considering $P_1 = [0, 0]$, the derivative T_1 is calculated as:

$$T_1 = c \cdot (P_2 - P_0)$$

Given $P_0 = [300, 300]$ and $P_2 = [0, 300]$, we find:

$$T_1 = c \cdot \begin{bmatrix} 0 - 300 \\ 300 - 300 \end{bmatrix}$$

$$T_1 = c \cdot \begin{bmatrix} -300 \\ 0 \end{bmatrix}$$

The correct tangent vector at $[0, 0]$ must be a scalar multiple of this difference. Among the given options, $\begin{bmatrix} -100 \\ 0 \end{bmatrix}$ is the vector that is a scalar multiple of $\begin{bmatrix} -300 \\ 0 \end{bmatrix}$, implying that the tension parameter c is $\frac{1}{3}$.

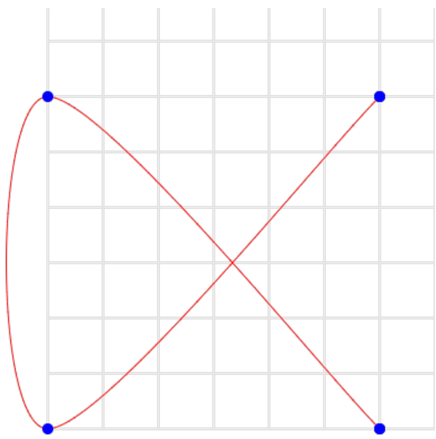

Fig. 1. Cardinal Spline visualization.

Question 26: Cubic and Quadratic Bezier Curves

Question: Which of the following cubic Bezier curves (given by the ordered list of control points) is the same as the quadratic Bezier curve with control points $[0, 0]$, $[0, 300]$, $[300, 300]$ (in this order)?

Choices:

A. None of the other choices
B. $[0, 0]$, $[0, 300]$, $[0, 300]$, $[300, 300]$
C. $[0, 0]$, $[0, 450]$, $[-150, 300]$, $[300, 300]$
D. $[0, 0]$, $[0, 100]$, $[200, 300]$, $[300, 300]$
E. It cannot be the same as any cubic curve
F. $[0, 0]$, $[0, 200]$, $[100, 300]$, $[300, 300]$

Correct Answer: $[0, 0]$, $[0, 200]$, $[100, 300]$, $[300, 300]$

Explanation:

To transform a quadratic Bezier curve with control points P_0, P_1, and P_2 to a cubic one, we derive new control points Q_0, Q_1, Q_2, and Q_3 such that the shapes of the two curves are identical. The cubic control points can be calculated using the relations:

$$Q_1 = \frac{2}{3}P_1 + \frac{1}{3}P_0$$
$$Q_2 = \frac{2}{3}P_1 + \frac{1}{3}P_2$$

Given the quadratic control points $P_0 = [0, 0]$, $P_1 = [0, 300]$, and $P_2 = [300, 300]$, we compute:

$$Q_1 = \frac{2}{3}[0, 300] + \frac{1}{3}[0, 0] = [0, 200]$$
$$Q_2 = \frac{2}{3}[0, 300] + \frac{1}{3}[300, 300] = [100, 300]$$

Thus, the cubic Bezier curve with control points $[0, 0]$, $[0, 200]$, $[100, 300]$, $[300, 300]$ is equivalent to the given quadratic Bezier curve.

Fig. 2. Quadratic Bezier Curve

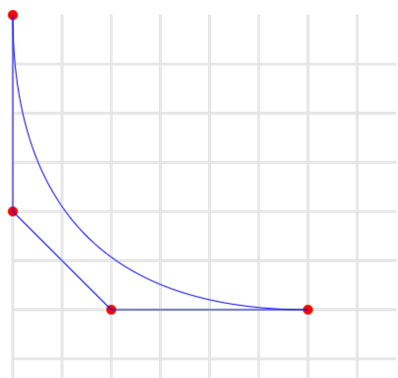

Fig. 3. Cubic Bezier Curve

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