# 08 Relations.

## 08_04_The Representations of Relations

Let R be a relation from $A = \{a_1, a_2, \ldots, a_m\}$ to $B = \{b_1, b_2, \ldots, b_n\}$.

The matrix representation of R is an $m \times n$ matrix $M(R) = [m_{ij}]$ such that

$$m_{ij} = 1 \text{ if } (a_i, b_j) \in R,$$

$$m_{ij} = 0 \text{ if } (a_i, b_j) \notin R.$$

Example. Suppose $A = \{1, 2, 3\}$ and $B = \{w, x, y, z\}$. Let R be a relation from A to B and $R = \{(1, y), (2, x), (2, z), (3, w), (3, x)\}$. Then the matrix representation of R is

$$\begin{array}{c c} & \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix} \end{array}.$$

If A, B, and the matrix representation of a relation R from A to B are given, then we can find the elements in R.

Example. Suppose $A = \{1, 2\}$, $B = \{s, t, x, y, z\}$, and the representation of a relation R from A to B is as follows.

$$\begin{array}{c c} & \begin{array}{ccccc} s & t & x & y & z \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

Then R = {(1, s), (1, y), (2, s), (2, x), (2, y), (2, z)}.

Suppose R is a relation on a set A of size n and M(R) = $[m_{ij}]$ is the matrix representation of R. Then R is a reflexive relation if and only if $m_{ii} = 1$ for each i with $1 \leq i \leq n$.

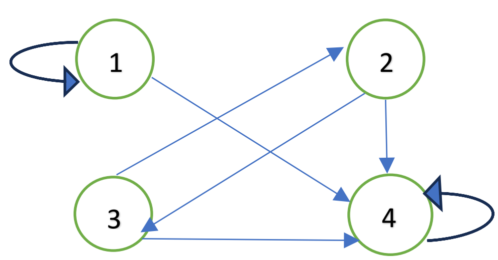
Suppose R is a relation on a set A of size n and M(R) = $[m_{ij}]$ is the matrix representation of R. Then R is a symmetric relation if and only if $m_{ij} = m_{ji}$ for each i with $1 \leq i \leq n$ and each j with $1 \leq j \leq n$.

Suppose R is a relation on a set A = $\{a_1, a_2, \ldots, a_n\}$. The <u>graphical representation of R</u> is a directed graph (or digraph) which can be formulated in the following steps.

[1] Use $a_1, a_2, \ldots, a_n$ in A to label n dots on a plane.

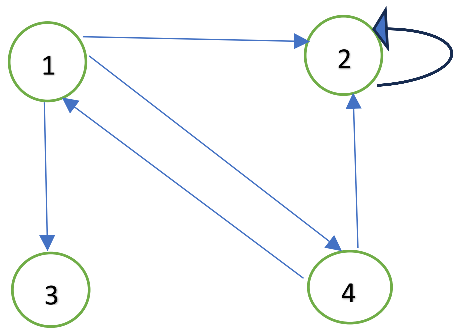
[2] Draw a directed edge (or arrow) between the dot labeled by $a_i$ and the dot labeled by $a_j$ if and only if $(a_i, a_j) \in R$. The directed edge from dot $a_i$ to dot $a_i$ is called a self-loop.

<u>Example.</u> Suppose A = {1, 2, 3, 4} and R = {(1, 1), (1, 4), (2, 3) (2, 4), (3, 2), (3, 4), (4, 4)}. Then the graphical representation of R is as follows.



If a set A and the graphical representation of a relation R on A are given, then we can find the elements in R.

<u>Example.</u> Suppose A = {1, 2, 3, 4} and the graphical representation of a relation R on A is as follows.




Then R = {(1, 2), (1, 3), (1, 4), (2, 2), (4, 1), (4, 2)}.

Suppose R is a relation on a set A of size n and D is the graphical representation of R. Then R is a reflexive relation if and only if for each dot in D there is a self-loop at it.

Suppose R is a relation on a set A of size n and D is the graphical representation of R. Then R is a symmetric relation if and only if the directed edge between the dot labelled as $a_i$ and the dot labelled as as $a_j$ appears in D if and only if the directed edge between the dot labelled as $a_j$ and the dot labelled as as $a_i$ appears in D, where $1 \leq i \leq n$ and $1 \leq j \leq n$.