02 Set Theory

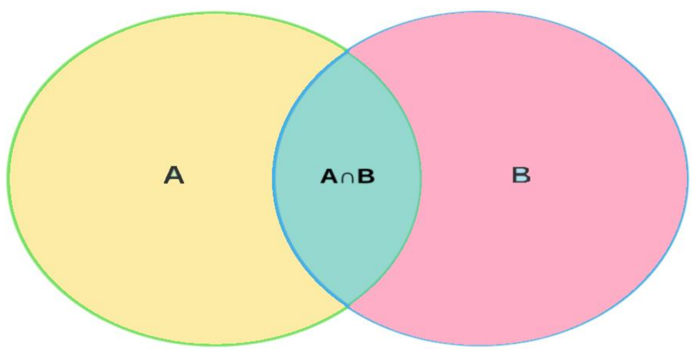
02_03_Set_Operations

The intersection of two sets A and B, denoted $A \cap B$, is a set

$$\{x : x \in A \text{ and } x \in B\}.$$

If $A \cap B = \emptyset$, then we say A and B are disjoint.

The Venn diagram for the intersection of A and B.



The intersection of two sets A and B.

Example. $\{1, 2, 3, 5, 7\} \cap \{10, 7, 8, 4, 6, 5\} = \{5, 7\}$.

Example. $\{x : x \text{ is even}\} \cap \{x : x \text{ is odd}\} = \emptyset$.

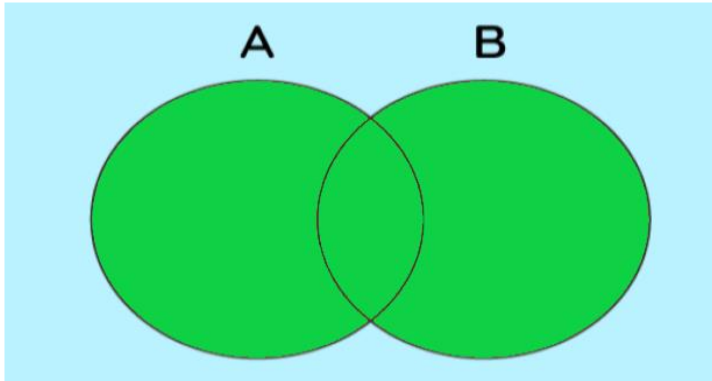
Note that $\emptyset \subseteq A \cap B \subseteq A \subseteq U$ and $\emptyset \subseteq A \cap B \subseteq B \subseteq U$.

Note also that $A \cap B = B \cap A$ and $A \cap B = A$ if $A \subseteq B$.

The union of two sets A and B, denoted $A \cup B$, is a set

$$\{x : x \in A \text{ or } x \in B\}.$$

The Venn diagram for the union of A and B.



The union of two sets A and B.

Example. $\{1, 2, 3, 5, 7\} \cup \{10, 7, 8, 4, 6, 5\}$

$= \{1, 2, 3, 5, 7, 10, 8, 4, 6\}$.

Example. $\{x : x \text{ is even}\} \cup \{x : x \text{ is odd}\}$

$= \{x : x \text{ is an integer}\}$.

Note that $\emptyset \subseteq A \cap B \subseteq A \subseteq A \cup B \subseteq U$ and

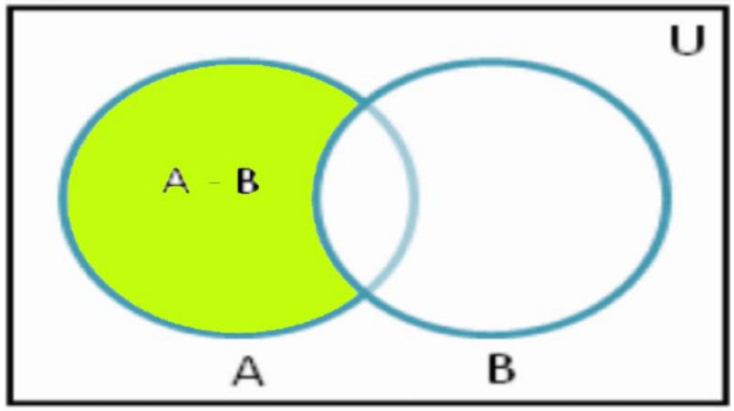
$\emptyset \subseteq A \cap B \subseteq B \subseteq A \cup B \subseteq U$.

Note also that $A \cup B = B \cup A$ and $A \cup B = B$ if $A \subseteq B$.

Let A and B be two sets. The difference of A and B, denoted $A - B$, is a set containing the elements of A that are not in B.

$A - B = \{x : x \in A \text{ and } x \notin B\}$.

The Venn diagram for the difference of A and B.



The difference of two sets A and B.

Example. {1, 2, 3, 5, 7} - {10, 7, 8, 4, 6, 5}

= {1, 2, 3}.

Example. {x : x is even} - {x : x is odd}

= {x : x is even}.

Note that $\emptyset \subseteq A - B \subseteq A \subseteq A \cup B \subseteq U$ and

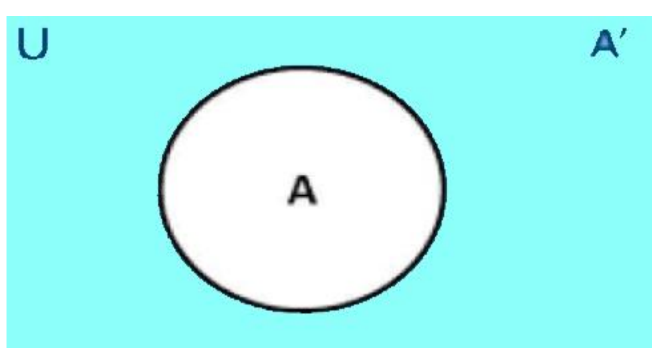
$\emptyset \subseteq B - A \subseteq B \subseteq A \cup B \subseteq U$.

Note also that $A - B = \{ \} = \emptyset$ if $A \subseteq B$.

If A is a set and U is the universal, then the complement of the A, denoted A' or A^c, is a set of U – A. Namely,

$A' = A^c = \{x : x \in U \text{ and } x \notin A\}$.

The Venn diagram for the complement of a set A^c.



The complement of a set A

Example. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A ={3, 5, 8}.

Then A' = A^c = {1, 2, 4, 6, 7, 9}.

Example. Let U = {x : x is an integer}, A = {x : x is even}.

A' = A^c = {x : x is odd}.

Note that U' = U^c = $\emptyset$ and $\emptyset' = \emptyset^c$ = U. Note also that

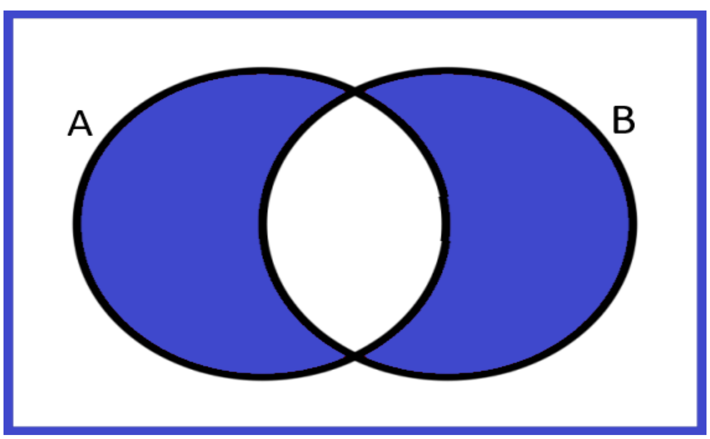
$A \cap A^c = \emptyset$, $A \cup A^c = U$ for any set A, and $A - B \neq$

$B - A$, in general.

The symmetric difference of A and B, denoted $A \oplus B$, is

A set of $(A - B) \cup (B - A)$.

The Venn diagram for the symmetric difference of A and B.



The symmetric difference of A and B

Example. Let A = {1, 2, 3, 5, 7} and B = {10, 7, 8, 4, 6, 5}

Then $A \oplus B = (A - B) \cup (B - A)$ = {1, 2, 3}$\cup$ {10, 8, 4, 6}

= {1, 2, 3, 4, 6, 8, 10}.

Note that $A \oplus B = B \oplus A$. Note also that

$(A - B) \cap (A \cap B) = \emptyset$, $(B - A) \cap (A \cap B) = \emptyset$,

$(A - B) \cap (B - A) = \emptyset$,

$(A \oplus B) \cap (A \cap B) = ((A - B) \cup (B - A)) \cap (A \cap B) = \emptyset$,

and

$A \cup B = (A \oplus B) \cup (A \cap B) = (A - B) \cup (B - A) \cup (A \cap B)$.