02 Set Theory

02_02_Subsets and Power Sets

Two sets A and B are equal, denoted A = B, if A and B have the same elements. For example,

$$\{1, 2, 3, 4, 5\} = \{3, 2, 1, 5, 4\};$$

$$\{x : \text{ x is a positive integers less than 16}\}$$
$$= \{1, 2, 3, \ldots\ldots, 14, 15\}.$$

Note that A = B is the same as (or if and only if) that for each a in A (or $a \in A$), a is in B (or $a \in B$) and for each b in B (or $b \in B$), b is in A (or $b \in A$).

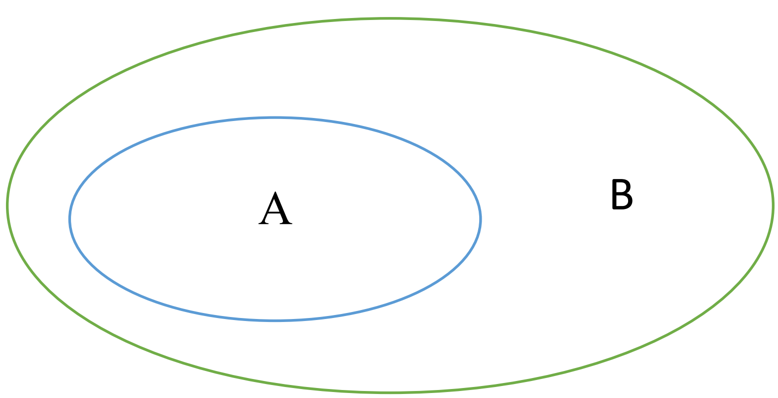
A set A is a subset of a set B, denoted $A \subseteq B$, if and only if each element of A is also an element of B. In this case, we can also say B is a superset of A. For example,

$$\{1, 2, 3\} \subseteq \{2, 1, 3\};$$

$$\{2, 3, 1\} \subseteq \{3, 1, 2, 7\};$$

$$\mathbf{N} = \text{the set of natural numbers} = \{1, 2, 3, \ldots\ldots\} \subseteq$$
$$\mathbf{Z} = \text{the set of integers} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\}.$$

The Venn diagram for A is a subset of B.



Note that for any set S, we have

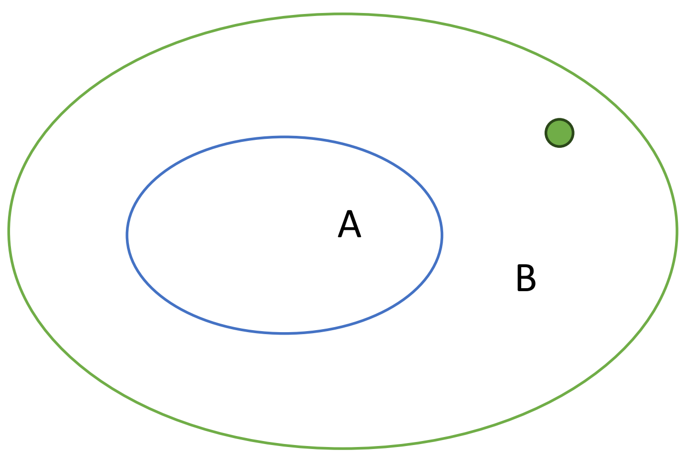
$$S \subseteq S \text{ and } \emptyset = \{\} \subseteq S \subseteq U.$$

Note also that A = B is the same as (or if and only if) that $A \subseteq B$ and $B \subseteq A$.

If $A \subseteq B$ and $A \neq B$, we say A is a proper subset of B. In this case, we can also say B is a proper superset of A. For instance, {2, 3, 1} is a proper subset of {3, 1, 2, 7} and **Z** is a proper superset of **N.**

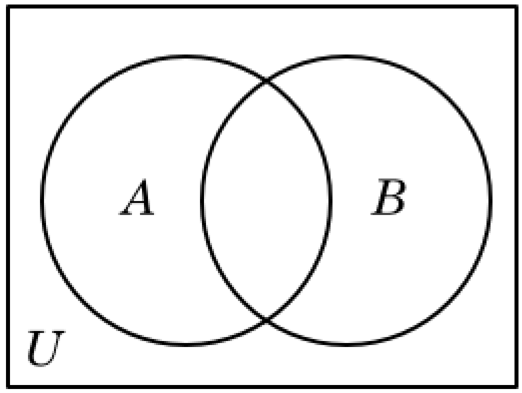
Note that A is a proper subset B (or B is a proper supset of A) if and only if for each element $a \in A$, $a \in B$, and there is one element in B which is not in A.

The Venn diagram for A is a proper subset of B.

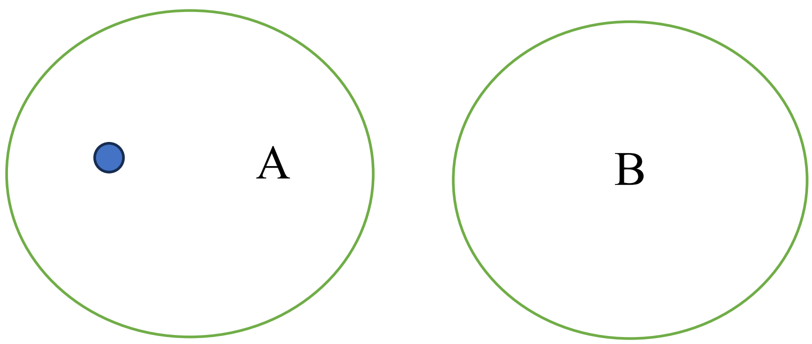


Note that A is not a subset of B means there is one element in A which is not in B.

The Venn diagrams for A is not a subset of B.



or



If a set S contains n elements, where n is a non-negative integer, then we say that <u>S is finite</u>. Otherwise, it is infinite. The <u>cardinality (or size)</u> of a finite set A, denoted |A|, is the number of elements in A. For instance, if S = {2, 4, 6, 8, 10}, then |S| = |{2, 4, 6, 8, 10}| = 5. Also,

$|\emptyset| = |\{\ \}| = 0,$

$|\{\emptyset\}| = 1,$

The set of integers is infinite.

<u>The power set of a set A, denoted P(A) (or 2^A), is a set</u>

consists of all the subsets of A. For instance,

$P(\emptyset) = P(\{\,\}) = \{\emptyset\}$,

$P(\{a\}) = \{\emptyset, \{a\}\}$,

$P(\{1, 2\}) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$,

$P(\{1, 2, 3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}\}$.

Note that $|P(A)| = 2^{|A|}$ if A is a finite set.