# 03 Counting Methods

## 03_03_the Product Rule

The Product Rule. A task T can be decomposed into a sequence of two subtasks $A_1$ and $A_2$. If there are $a_1$ ways to complete subtask $A_1$ and $a_2$ ways to complete subtask $A_2$. Then there are $a_1 * a_2$ ways to complete task T.

Example. How many bitstrings of length two are there?
[Solution]. Task T is to form the bitstrings of length 2. It can be decomposed into two sequential subtasks $A_1$ and $A_2$, where $A_1$ is the task of filling in the first position and $A_2$ is the task of filling in the second position in the bitstrings. We can use 0 or 1 to fill in the first and second positions. Thus we have 2 ways to complete task $A_1$ and task $A_2$. Therefore we have $2*2 = 4$ ways to complete task T. Hence there are 4 bitstrings of length two.

The Product Rule above can be extended to the following form.
The Product Rule. A task T can be decomposed into a sequence of n subtasks $A_1, A_2, \ldots, A_n$. If there are $a_1$ ways to complete subtask $A_1$, $a_2$ ways to complete subtask $A_2, \ldots$, and $a_n$ ways to complete subtask $A_n$. Then there are $a_1 * a_2 * \ldots * a_n$ ways to complete task T.

Example. How many bitstrings of length ten are there?
[Solution]. Using the similar arguments as the ones on the

above example, we have there are $2^{10} = 1024$ bitstrings of length ten.

Example. How many different license plates are there if each plate consists of a sequence of four digits followed by four uppercase English letters?

[Solution] Task T is to form license plates. Subtask $A_1$ is to fill in the first position in license plates by using one of 10 digits. Subtask $A_2$ is to fill in the second position in license plates by using one of 10 digits. Subtask $A_3$ is to fill in the third position in license plates by using one of 10 digits. $A_4$ is to fill in the fourth position in license plates by using one of 10 digits. Subtask $A_5$ is to fill in the fifth position in license plates by using one of 26 uppercase English letters. Subtask $A_6$ is to fill in the sixth position in license plates by using one of 26 uppercase English letters. Subtask $A_7$ is to fill in the seventh position in license plates by using one of 26 uppercase English letters. Clearly, $a_1 = a_2 = a_3 = a_4 = 10$, $a_5 = a_6 = a_7 = 26$. Thus, by the product rule, there are 10 *10 *10 * 10 * 26 * 26 * 26 = 175,760,000 different license plates.

Example. There are three ways for one to travel from Aiken, SC to Augusta, GA, there are four ways for one to travel from Augusta, GA to Atlanta, GA, there are two ways for one to travel from Atlanta, GA to Los Angeles, CA. How many ways are there

for one to travel from Aiken, SC to Los Angeles, CA via Augusta, GA and Atlanta, GA?

[Solution]. Task T is for one to travel from Aiken, SC to Los Angeles, CA via Augusta, GA and Atlanta, GA. Subtask $A_1$ is for one to travel from Aiken, SC to Augusta, GA. Subtask $A_2$ is for one to travel from Augusta, GA to Atlanta, GA. Subtask $A_3$ is for one to travel from Atlanta, GA to Los Angeles, CA. Thus $a_1 = 3$, $a_2 = 4$, and $a_3 = 2$. Therefore, by the product rule, there are $3*4*2 = 24$ ways for one to travel from Aiken, SC to Los Angeles, CA via Augusta, GA and Atlanta, GA.