07 Matrices

07_01 Definitions and Terminology on Matrices

Definition. A matrix is a rectangular array of numbers. A matrix is often denoted by a capital letter such as M.

The row number of a matrix M, denoted r(M), is the number of the rows in the rectangular array.

The column number of a matrix M, denoted c(M), is the number of columns in the rectangular array.

For the following matrix M, we have r(M) = 3 and c(M) = 2.

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$$

If M is a matrix with r(M) = r and c(M) = c, then M is called a $r \times c$ matrix. Read $r \times c$ as r by c. Often, M is written as M_{rxc}.

For the matrix M above, we can write it as M_{3x2} to indicate that M has 3 rows and 2 columns.

For the following matrix A, we have r(A) = 3, c(A) = 3. and A is a 3×3 matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

If M is a matrix such that r(M) = c(M), then M is called a square matrix of order r(M). Thus A is a square matrix of order 3.

For the following matrix B, we have r(B) = 1, c(B) = 7.

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{bmatrix}.$$

If M is a matrix such that r(M) = 1, then M is called a row matrix.

For the following matrix D, we have r(D) = 3, c(D) = 1.

$$D = \begin{bmatrix} 7 \\ 17 \\ 37 \end{bmatrix}.$$

If M is a matrix such that c(M) = 1, then M is called a column matrix.

Let A be the following matrix.

$$A = A_{3x4} = \begin{bmatrix} 1 & 2 & 3 & 10 \\ 4 & 5 & 6 & 11 \\ 7 & 8 & 9 & 12 \end{bmatrix}.$$

The (i, j)-element, denoted a_{ij}, is the element located at the intersection between i-th row and j-th column, where $1 \le i \le r(A)$ and $1 \le j \le c(A)$.

$$a_{11} = 1, a_{12} = 2, a_{13} = 3, a_{14} = 10,$$

$$a_{21} = 4, a_{22} = 5, a_{23} = 6, a_{24} = 11,$$

$$a_{31} = 7, a_{32} = 8, a_{33} = 9, a_{34} = 12.$$

In this case, we can write $A = A_{3x4} = (a_{ij})_{3x4}$.

the 1^{st} row of A is $[a_{11}, a_{12}, a_{13}, a_{14}]$,

the 2^{nd} row of A is $[a_{21}, a_{22}, a_{23}, a_{24}]$,

the 3^{rd} row of A is $[a_{31}, a_{32}, a_{33}, a_{34}]$.

In general, the i-th row in $A = A_{3x4} = (a_{ij})_{3x4}$ is

$$[a_{i1}, a_{i2}, a_{i3}, a_{i4}].$$

The 1^{st} column of A is

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$$

The 2^{nd} column of A is

$$\begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}$$

The 3rd column of A is

$$\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$$

The 4th column of A is

$$\begin{bmatrix} a_{14} \\ a_{24} \\ a_{34} \end{bmatrix}$$

In general, the jth column of A is

$$\begin{bmatrix} a_{1j} \\ a_{2j} \\ a_{3j} \end{bmatrix}$$

Often, a matrix $M_{m\times n}$ is represented as $(m_{ij})_{m\times n}$, where m_{ij}, $1 \leq i \leq m$ and $1 \leq j \leq n$, denotes the (i, j)-element of M.

A <u>unit matrix</u> (or an <u>identity matrix</u>) of order n, denoted U_n (or I_n), is a square matrix of order n with all elements in the diagonal equal to 1 and all off-diagonal elements equal to 0. For example,

$$U_3 \text{ or } I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A <u>diagonal matrix</u> of order n is a square matrix of n such that all the off-diagonal elements are equal to 0. For example, the following matrix is a diagonal matrix of order 3.

$$\begin{bmatrix} 10 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -9 \end{bmatrix}$$

A square matrix $M_{n\times n} = (m_{ij})_{n\times n}$ is called a <u>symmetric matrix</u> if $m_{ij} = m_{ji,}$ where $1 \leq i \neq j \leq n$. For example, the following matrix is a symmetric matrix.

$$\begin{bmatrix} 7 & -3 & 0 \\ -3 & 1 & 10 \\ 0 & 10 & -8 \end{bmatrix}$$

Note that every unit (or identity) matrix is a diagonal matrix and every diagonal matrix is a symmetric matrix.

A matrix is called a <u>zero-one matrix</u> if all the entries in the matrix are equal to either 0 or 1. The zero-one matrices are used in graph theory and other fields.