01 LOGIC

01_02 Truth Tables

If one proposition consists of n **independent** propositions,
Then there are 2^n cases which are represented by 2^n rows
in a truth table.

Construct the truth table for $\sim p \vee p$

Notice that $\sim p \vee p$ consists of one independent proposition.
So there are 2^1 cases or 2^1 rows in the truth table.

p	~p	p ∨ ~p
T	F	T
F	T	T

Notice also that the truth value of $\sim p \vee p$ is always T.

Construct the truth table for $\sim p \wedge p$

Notice that $\sim p \wedge p$ consists of one independent proposition.
So there are 2^1 cases or 2^1 rows in the truth table.

p	~p	p ∧ ~p
T	F	F
F	T	F

Notice also that the truth value of $\sim p \wedge p$ is always F.

For an implication $p \rightarrow q$, the implication $q \rightarrow p$ is called the converse of $p \rightarrow q$; the implication $\sim p \rightarrow \sim q$ is called the inverse of $p \rightarrow q$; the implication $\sim q \rightarrow \sim p$ is called the contrapositive of $p \rightarrow q$.

Construct the truth table for $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$.

Notice that $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ consists of two independent propositions. So there are 2^2 cases or 2^2 rows in the truth table.

p	q	~p	~q	p → q	~q → ~p	(p → q) ↔ (~q → ~p)
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Notice that the truth value of $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is always T.

Construct the truth table for $\sim q \oplus (q \leftrightarrow p)$.

Notice that $\sim q \oplus (q \leftrightarrow p)$ consists of two independent propositions. So there are 2^2 cases or 2^2 rows in the truth table.

p	q	~q	q ↔ p	~q ⊕ (q ↔ p)
T	T	F	T	T
T	F	T	F	T
F	T	F	F	F
F	F	T	T	F

Construct the truth table for ~p ∧ (q → r).

Notice that ~p ∧ (q → r) consists of three independent propositions. So there are 2^3 cases or 2^3 rows in the truth table.

p	q	r	~p	q → r	~p ∧ (q → r)
T	T	T	F	T	F
T	T	F	F	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Construct the truth table for p ↔ (q ⊕ r).

Notice that p ↔ (q ⊕ r) consists of three independent propositions. So there are 2^3 cases or 2^3 rows in the truth table.

p	q	r	$q \oplus r$	$p \leftrightarrow (q \oplus r)$
T	T	T	F	F
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	F	T
F	T	F	T	F
F	F	T	T	F
F	F	F	F	T

Precedence of Logical Operators

Operator	Precedence
$\sim$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5