02 Set Theory

02_01_Definition

A set is an unordered collection of well-defined objects.
"Well-defined" should be understood as each object is either in the set or not in the set.

Examples of sets.

The tables in this classroom.
The students at this university.
The classes you are taking this semester.

The objects in a set are also called the members or elements of the set. We use $x \in S$ to denote the element x belongs to or is in the set S. If x does not belong to or is not in S, we write $x \notin S$.

The order of the elements in a set does not matter. For instance,

$$S = \{1, 2, 3\} = \{1, 3, 2\} = \{2, 1, 3\}$$
$$= \{2, 3, 1\} = \{3, 1, 2\} = \{3, 2, 1\}$$

Two ways of representing sets.

[1] The roster method.

The set of even positive integers less than or equal to 8.

$$S = \{2, 4, 6, 8\}.$$

The set of positive integers less than or equal to 1024.

$$T = \{1, 2, 3, \ldots, 1024\}.$$

The set of letters in English alphabet.

$U = \{a, b, c, d, \ldots, x, y, z\}$.

[2] The set builder method.

A = {x : x is a positive integers less than or equal to 32}

$= \{1, 2, 3, \ldots\ldots, 30, 31, 32\}$.

B = {x : x is a square of integers from 1 to 5}

$= \{1, 4, 9, 16, 25\}$.

C = {x : is a nonnegative power of 2}

$= \{2^0, 2^1, 2^2, 2^3, 2^4, \ldots\ldots\} = \{1, 2, 4, 8, 16, \ldots\ldots\}$

Common sets involving numbers.

N = the set of natural numbers = $\{1, 2, 3, \ldots\ldots\}$.

W = the set of whole numbers = $\{0, 1, 2, 3, \ldots\ldots\}$.

Z = the set of integers = $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\}$.

$\mathbf{Z}^+$ = the set of positive integers = $\{1, 2, 3, \ldots\ldots\}$ = **N.**

Q = the set of rational numbers

= {x : x = p/q, where p and q are integers and $q \neq 0$}.

I = the set of irrational numbers.

R = the set of real numbers.

$\mathbf{R}^+$ = the set of positive real numbers.

C = the set of complex numbers

= {a + bi: a and b are real numbers}.

The universal set, denoted U, is the set containing everything currently under consideration.

The empty set, denoted $\emptyset$ or { }, is a set which does not contain any element.