07 Matrices

07_01 Definitions and Terminology on Matrices

<u>Definition.</u> A <u>matrix</u> is a rectangular array of numbers. A matrix is often denoted by a capital letter such as M.

The <u>row number</u> of a matrix M, denoted r(M), is the number of the rows in the rectangular array.

The <u>column number</u> of a matrix M, denoted c(M), is the number of columns in the rectangular array.

For the following matrix M, we have r(M) = 3 and c(M) = 2.

$$\left[\begin{array}{cc}1&1\\0&2\\1&3\end{array}\right]$$

If M is a matrix with r(M) = r and c(M) = c, then M is called a $r \times c$ matrix. Read $r \times c$ as r by c. Often, M is written as M_{rxc} .

For the matrix M above, we can write it as M_{3x2} to indicate that M has 3 rows and 2 columns.

For the following matrix A, we have r(A) = 3, c(A) = 3. and A is a 3 × 3 matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

If M is a matrix such that r(M) = c(M), then M is called a square matrix of order r(M). Thus A is a square matrix of order 3.

For the following matrix B, we have r(B) = 1, c(B) = 7.

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$
.

If M is a matrix such that r(M) = 1, then M is called a row matrix.

For the following matrix D, we have r(D) = 3, c(D) = 1.

$$D = \begin{bmatrix} 7 \\ 17 \\ 37 \end{bmatrix}.$$

If M is a matrix such that c(M) = 1, then M is called a column matrix.

Let A be the following matrix.

$$A = A_{3x4} = \begin{bmatrix} 1 & 2 & 3 & 10 \\ 4 & 5 & 6 & 11 \\ 7 & 8 & 9 & 12 \end{bmatrix}.$$

The (i, j)-element, denoted a_{ij} , is the element located at the intersection between i-th row and j-th column, where $1 \le i \le r(A)$ and $1 \le j \le c(A)$.

$$a_{11} = 1, a_{12} = 2, a_{13} = 3, a_{14} = 10,$$

 $a_{21} = 4, a_{22} = 5, a_{23} = 6, a_{24} = 11,$
 $a_{31} = 7, a_{32} = 8, a_{33} = 9, a_{34} = 12.$

In this case, we can write $A = A_{3x4} = (a_{ij})_{3x4}$.

the 1st row of A is
$$[a_{11}, a_{12}, a_{13}, a_{14}]$$
,
the 2nd row of A is $[a_{21}, a_{22}, a_{23}, a_{24}]$,
the 3rd row of A is $[a_{31}, a_{32}, a_{33}, a_{34}]$.

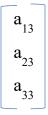
In general, the i-th row in
$$A = A_{3x4} = (a_{ij})_{3x4}$$
 is $[a_{i1}, a_{i2}, a_{i3}, a_{i4}].$

The 1st column of A is

The 2nd column of A is

a₁₂
a₂₂
a₃₂

The 3rd column of A is



The 4th column of A is

In general, the j^{th} column of A is

$$\begin{bmatrix} a_{1j} \\ a_{2j} \\ a_{3j} \end{bmatrix}$$

Often, a matrix $M_{m\times n}$ is represented as $(m_{ij})_{m\times n}$, where $m_{ij},\ 1\leq i\leq m$ and $1\leq j\leq n$, denotes the (i,j)-element of M.

A <u>unit matrix</u> (or an <u>identity matrix</u>) of order n, denoted U_n (or I_n), is a square matrix of order n with all elements in the diagonal equal to 1 and all off-diagonal elements

equal to 0. For example,

$$U_3 \text{ or } I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A <u>diagonal matrix</u> of order n is a square matrix of n such that all the off-diagonal elements are equal to 0. For example, the following matrix is a diagonal matrix of order 3.

A square matrix $M_{n\times n}=(m_{ij})_{n\times n}$ is called a <u>symmetric</u> <u>matrix</u> if $m_{ij}=m_{ji}$, where $1\leq i\neq j\leq n$. For example, the following matrix is a symmetric matrix.

Note that every unit (or identity) matrix is a diagonal matrix and every diagonal matrix is a symmetric matrix.

A matrix is called a <u>zero-one matrix</u> if all the entries in the matrix are equal to either 0 or 1. The zero-one matrices are used in graph theory and other fields.