04 Counting Methods

04_03_Examples

Example. How many car license plates can be formed if the plates consist of three distinct capital letters followed by four distinct digits.

[Solution]

P(26, 3)*P(10, 4) = 26*25*24*10*9*8*7 = 78,624,000.

Example. Suppose there are 8 mathematical faculty members and 6 computer science faculty members in a Department of Mathematics and Computer Science. How many ways are there to form a committee on developing a Data Science program such that the committee consisting of 6 members and

[1] the number of computer science faculty members is equal to the number of mathematical faculty members.

[2] the number of computer science faculty members is equal to or greater than the number of mathematical faculty members.

[Solution]

[1] In this case, the number of computer science faculty members in the committee = the number of mathematical faculty members = 3. Thus, the number of the committee is

$$C(8, 3)*C(6, 3) = 8!/((8 – 3)!\ 3!)*\ 6!/((6 – 3)!\ 3!)$$
$$= 56*20 = 1120.$$

[2] In this case, we have the following subcases.

Case 2.1

the number of computer science faculty members in the committee = 3,

the number of mathematical faculty members in the committee = 3.

Case 2.2

the number of computer science faculty members in the committee = 4,

the number of mathematical faculty members in the committee = 2.

Case 2.3

the number of computer science faculty members in the committee = 5,

the number of mathematical faculty members in the committee = 1.

Case 2.4

the number of computer science faculty members in the committee = 6,

the number of mathematical faculty members in the committee = 0.

Thus, the total number of committees can be formed is

C(8, 3)*C(6, 3) + C(8, 2)*C(6, 4) +
C(8, 1)*C(6, 5) + C(8, 0)*C(6, 6)
= 1589.

Example. Find the numbers of bit strings of length 8 having

[1] at least six 1s.

[2] at most three 1s.

[3] an equal number of 0s and 1s.

[Solution]

[1] C(8, 6) + C(8, 7) + C(8, 8) = 28 + 8 + 1 = 37.

[2] C(8, 3) + C(8, 2) + C(8, 1) + C (8, 0)

= 56 + 28 + 8 + 1 = 93.

[3] C(8, 4) = 70.

Example. Find the numbers of permutations of the integers 1, 2, 3, 4, 5, 6 such that

[1] each of the permutations has sequence 345.

[2] each of the permutations has sequences 21 and 56.

[Solution]

[1] Counting the sequence 345 as one unit, we have that the answer is 4! = 24.

[2] Counting the sequences 21 and 56 as two different units, we have that the answer is 4! = 24.

Example. There are 5 red balls and 5 blue balls in a bag. Find the number of ways of arranging the 10 balls such that the red balls and blue balls appear alternatively in one row.

[Solution]

The first pattern for arranging the 10 balls is

“R B R B R B R B R B”.

Under this pattern, the number of ways for arranging the 10 balls is 5!*5!.

The second pattern for arranging the 10 balls is

“B R B R B R B R B R”.

Under this pattern, the number of ways for arranging the 10 balls is 5!*5!.

Thus, the number of ways of arranging the 10 balls such that the red balls and blue balls appear alternatively in one row is 5!*5! + 5!*5! = 28,800.

The above problem can be generalized as follows.

<u>Example</u>. There are n red balls and n blue balls in a bag. Find the number of ways of arranging the 10 balls such that the red balls and blue balls appear alternatively in one row.

[Solution]

The answer is 2*n!*n!.

<u>Example.</u> Find the number of paths from (0, 0) to (4, 3) in which only horizontal moves to east and the vertical moves to north are allowed.

[Solution]

In each path, there are 7 moves. Each path is uniquely

decided by 4 horizontal moves or 3 vertical moves.
Thus, the number of the paths is
$C(7, 4) = C(4 + 3, 4) = C(4 + 3, 3) = C(7, 3) = 35.$

The above problem can be generalized as follows.

Example. Find the number of paths from $(0, 0)$ to (m, n) in which only horizontal moves to east and the vertical moves to north are allowed.

[Solution] The answer is

$$C(m + n, m) = C(m + n, n).$$