01 LOGIC

01_01 Propositions and Operations on Propositions

Proposition. A proposition (or statement) is a declarative sentence that is either true or false, but not both simultaneously.

Examples of propositions.

London is the capital of the United Kingdom.

Charleston is the capital of South Carolina.

A square has four sides.

2 + 3 = 5.

7 + 8 = 78.

Examples that are not propositions.

Turn off the computer.

What time is it?

$x + 8 = 16$.

$x + 2y = 3z$.

$x^2 > 7$.

There are two possible values for a position. One is called true, denoted by T, another one is called false, denoted by F.

We often use letters p, q, r, … to denote propositions. In this case, the letters p, q, r, … are called the propositional variables.

The following logical operations or connectives can be defined on the propositions.

Negation.

Conjunction.

Disjunction.

Exclusive OR.

Implication.

Biconditional Statement.

<u>The Negation of a Proposition.</u> Let p be a proposition. The negation of p, denoted ~p, is the sentence of

"it is not the case that p".

The truth table for the negation of p is as follows.

p	~p
T	F
F	T

<u>An Example for the negation of a proposition.</u>

The negation for the proposition of "Today is Sunday" is "It is not the case that Today is Sunday", which is the same as "Today is not Sunday".

<u>An Example for the negation of a proposition.</u>

The negation for the proposition p of "2 + 3 = 5" is "It is not the case that 2 + 3 = 5", which is the same as "2 + 3 != 5". Notice that the truth value of p is T and the truth value of ~p is F.

<u>An Example for the negation of a proposition.</u>

The negation for the proposition q of "7 + 8 = 78" is "It is not the case that 7 + 8 = 78", which is the same as "7 + 8 != 78". Notice that the truth value of q is F and the truth value of ~q is T.

The Conjunction of Two Propositions.

Let p and q be two propositions. The conjunction of p and q, denoted $p \wedge q$, is the proposition "p and q". The conjunction $p \wedge q$ is T only when both p and q are T.

The truth table for $p \wedge q$ is as follows.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

An example for the conjunction of two propositions.

Let p be a proposition of

London is the capital of the United Kingdom.

Let q be a proposition of

$2 + 3 = 5$.

Then the conjunction $p \wedge q$ is

London is the capital of the United Kingdom and $2 + 3 = 5$.

Since p is T and q is T, the truth value of $p \wedge q$ is T.

An example for the conjunction of two propositions.

Let p be a proposition of

A square has four sides.

Let q be a proposition of

$7 + 8 = 78$.

Then the conjunction $p \wedge q$ is

A square has four sides and $7 + 8 = 78$.

Since p is T and q is F, the truth value of $p \wedge q$ is F.

The Disjunction of Two Propositions.

Let p and q be two propositions. The disjunction of p and q, denoted $p \vee q$, is the proposition "p or q". The disjunction $p \vee q$ is F only when both p and q are F.

The truth table for $p \vee q$ is as follows.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

An example for the disjunction of two propositions.

Let p be a proposition of

Charleston is the capital of South Carolina.

Let q be a proposition of

$2 + 3 = 5$.

Then the disjunction $p \vee q$ is

Charleston is the capital of South Carolina or $2 + 3 = 5$.

Since p is F and q is T, the truth value of $p \vee q$ is T.

An example for the disjunction of two propositions.

Let p be a proposition of

A square has five sides.

Let q be a proposition of

$7 + 8 = 78.$

Then the disjunction $p \vee q$ is

A square has five sides or $7 + 8 = 78$.

Since p is F and q is F, the truth value of $p \vee q$ is F.

The Exclusive OR of Two Propositions.

Let p and q be two propositions. The exclusive OR of p and q, denoted $p \oplus q$, is the proposition which is T only when exactly one of p and q is T.

The truth table for $p \oplus q$ is as follows.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

An example for the exclusive OR of two propositions.

Let p be a proposition of

Charleston is the capital of South Carolina.

Let q be a proposition of

$2 + 3 = 5.$

Then $p \oplus q$ is T since p is F and q is T.

An example for the exclusive OR of two propositions.

Let p be a proposition of

A square has four sides.

Let q be a proposition of

$7 + 8 = 15.$

Then $p \oplus q$ is F since p is T and q is T.

The Implication of Two Propositions.

Let p and q be two propositions. The implication of p and q, denoted $p \to q$, is the proposition "if p, then q" or "p implies q". The implication $p \to q$ is F only when p is T and q is F.

The truth table for $p \to q$ is as follows.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

An example for the implication of two propositions.

Let p be a proposition of

Charleston is the capital of South Carolina.

Let q be a proposition of

$2 + 3 = 5.$

The implication $p \to q$ is

if Charleston is the capital of South Carolina, then $2 + 3 = 5$.

Then $p \to q$ is T since p is F and q is T.

An example for the implication of two propositions.

Let p be a proposition of

Charleston is the capital of South Carolina.

Let q be a proposition of

$2 + 3 = 6.$

The implication $p \to q$ is

if Charleston is the capital of South Carolina, then $2 + 3 = 6$.

Then $p \to q$ is T since p is F and q is F.

An example for the implication of two propositions.

Let p be a proposition of

Columbia is the capital of South Carolina.

Let q be a proposition of

$6 + 6 = 66.$

The implication $p \to q$ is

if Columbia is the capital of South Carolina, then $6 + 6 = 66$.

Then $p \to q$ is F since p is T and q is F.

The Biconditional Statement of Two Propositions.

Let p and q be two propositions. The biconditional statement of p and q, denoted $p \leftrightarrow q$, is the proposition "p if and only if q". $p \leftrightarrow q$ is T only when p and q have the same truth value. The truth table for $p \leftrightarrow q$ is as follows.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

An example for the biconditional statement of two propositions.

Let p be a proposition of

Charleston is the capital of South Carolina.

Let q be a proposition of

$2 + 3 = 5.$

Then $p \leftrightarrow q$ is F since p is F and q is T.

An example for the biconditional statement of two propositions.

Let p be a proposition of

A square has ten sides.

Let q be a proposition of

$7 + 8 = 56.$

Then $p \leftrightarrow q$ is T since p is F and q is F.