09 Principle of Mathematical Induction.

09_04_Additional Examples

Example. Let P(n) be the statement that

$$\sim(p_1 \vee p_2 \vee \ \ldots \vee p_n) \Leftrightarrow \sim p_1 \wedge \sim p_2 \wedge \ \ldots \wedge \sim p_n,$$

where p_1, p_2, … , p_n are propositions and n is an integer with $n \geq 1$. Prove that P(n) is true.

[Proof]

Step 1. P(1) is true since $\sim p_1 \Leftrightarrow \sim p_1$.

Step 2. Assume that P(n) is true for $n = k \geq 1$, i.e.,

$$\sim(p_1 \vee p_2 \vee \ \ldots \vee p_k) \Leftrightarrow \sim p_1 \wedge \sim p_2 \wedge \ \ldots \wedge \sim p_k.$$

Now

$$\sim(p_1 \vee p_2 \vee \ \ldots \vee p_k \vee p_{k+1})$$

$$\Leftrightarrow$$

$$\sim((p_1 \vee p_2 \vee \ \ldots \vee p_k) \vee p_{k+1})$$

//by De Morgan's laws with

//two propositions

$$\Leftrightarrow$$

$$\sim(p_1 \vee p_2 \vee \ \ldots \vee p_k) \wedge \sim p_{k+1}$$

$$\Leftrightarrow$$

$$(\sim p_1 \wedge \sim p_2 \wedge \ \ldots \wedge \sim p_k) \wedge \sim p_{k+1}.$$

$$\Leftrightarrow$$

$$\sim p_1 \wedge \sim p_2 \wedge \ \ldots \wedge \sim p_k \wedge \sim p_{k+1}.$$

Hence P(k + 1) is true.

By the principle of mathematical induction, we have that P(n) is true.

<u>Example.</u> Let P(n) be the statement that

$$A \cap (B_1 \cup B_2 \cup \ldots \cup B_n) =$$
$$(A \cap B_1) \cup (A \cap B_2) \cup \ldots \cup (A \cap B_n),$$

where $A, B_1, B_2, \ldots, B_n$ are sets and n is an integer with $n \geq 1$. Prove that P(n) is true.

[Proof]

Step 1. P(1) is true since both LHS of P(1) and RHS of P(1) are equal to $A \cap B_1$.

Step 2. Assume that P(n) is true for $n = k \geq 1$, i.e.,

$$A \cap (B_1 \cup B_2 \cup \ldots \cup B_k) =$$
$$(A \cap B_1) \cup (A \cap B_2) \cup \ldots \cup (A \cap B_k).$$

Now

$$A \cap (B_1 \cup B_2 \cup \ldots \cup B_k \cup B_{k+1})$$
$$= A \cap ((B_1 \cup B_2 \cup \ldots \cup B_k) \cup B_{k+1})$$

// by the distributive laws

$$= (A \cap (B_1 \cup B_2 \cup \ldots \cup B_k)) \cup (A \cap B_{k+1})$$
$$= (A \cap B_1) \cup (A \cap B_2) \cup \ldots \cup (A \cap B_k) \cup (A \cap B_{k+1})$$

Hence P(k + 1) is true.

By the principle of mathematical induction, we have that P(n) is true.

<u>Example.</u> Let P(n) be the statement that

$$A \cup (B_1 \cap B_2 \cap \ldots \cap B_n) =$$

$$(A \cup B_1) \cap (A \cup B_2) \cap \ldots \cap (A \cup B_n),$$

where $A, B_1, B_2, \ldots, B_n$ are sets and n is an integer with $n \geq 1$. Prove that P(n) is true.

[Proof]

Step 1. P(1) is true since both LHS of P(1) and RHS of P(1) are equal to $A \cup B_1$.

Step 2. Assume that P(n) is true for $n = k \geq 1$, i.e.,

$$A \cup (B_1 \cap B_2 \cap \ldots \cap B_k) =$$

$$(A \cup B_1) \cap (A \cup B_2) \cap \ldots \cap (A \cup B_k).$$

Now

$$A \cup (B_1 \cap B_2 \cap \ldots \cap B_k \cap B_{k+1})$$

$$= A \cup ((B_1 \cap B_2 \cap \ldots \cap B_k) \cap B_{k+1})$$

// by the distributive laws

$$= (A \cup (B_1 \cap B_2 \cap \ldots \cap B_k)) \cap (A \cup B_{k+1})$$

$$= (A \cup B_1) \cap (A \cup B_2) \cap \ldots \cap (A \cup B_k) \cap (A \cup B_{k+1})$$

Hence P(k + 1) is true.

By the principle of mathematical induction, we have that P(n) is true.