12 Probability

12_02_Conditional Probability and Independence of Events

Suppose A and B are two events in an experiment. The probability that event A occurs given event B occurs is called a **conditional probability**, denoted **P(A | B),** which is read as “the probability of A given B”.

Example. In the experiment of tossing three coins, we define A as the event of getting at least two heads and B as the event of getting one or two tails. Find the probabilities of P(A | B).

[Solution] The sample space Ω = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}, the event A = {HHH, HHT, HTH, THH}, and the event B = {HHT, HTH, HTT, THH, THT, TTH}. Thus $P(A \mid B) = 3/6 = 1/2$. Note that $P(A \mid B) = (3/8)/(6/8) = P(A \cap B)/P(B)$.

Suppose A and B are two events in an experiment. The probability that event A occurs given event B occurs is called a **conditional probability**, denoted **$P(A \mid B)$,** which is read as "the probability of A given B". **If $P(B) > 0$, then $P(A \mid B) = P(A \cap B)/P(B)$,** further, **$P(A \cap B) = P(B)\, P(A \mid B)$.** Also, **if $P(A) > 0$, then $P(B \mid A) = P(B \cap A)/P(A)$.** Thus **$P(A \cap B) = P(B \cap A) = P(A)\, P(B \mid A) = P(A)\, P(B \mid A)$.**

Example. The defective rate of a collection of 1000 cellphones is 5%. Draw sequentially two cellphones from that collection without replacement. Find the probability of the second pick is the first time of drawing a non-defective cellphone.

[Solution] Define A as the event of drawing a defective cellphone in the first pick. Define B as the event of drawing a non-defective cellphone in the second pick. We need to find $P(A \cap B)$.

$P(A \cap B) = P(A)\, P(B \mid A) = 0.05 * (1000 - 1000*0.05)/999 \approx 0.0475$.

Suppose A and B are two events in an experiment. If $P(A) = 0$ or $P(B) = 0$, then A and B are **independent**. If $P(A) \neq 0$ and $P(B) \neq 0$, then that A and B are **independent** is defined by any one of three equivalent conditions below.

[1] $P(B \mid A) = P(B)$,

[2] $P(A \mid B) = P(A)$,

[3] $P(A \cap B) = P(A)\, P(B)$.

Example. Roll a balanced regular tetrahedron with faces labelled as 1, 2, 3, and 4. Find the probability of the event of getting 1 or 2 on faces. Find the probability of the event of getting 1 or 3 on faces.

[Solution] Define A as the event of getting face 1 or face 2. Define B as the event of getting face 1 or face 3. Then $A \cap B$ is the event of getting face 1. Note that the sample space is {face 1, face 2, face 3, face 4}. Thus

$$P(A) = 2/4 = 1/2,\ P(B) = 2/4 = 1/2, \text{ and } P(A \cap B) = 1/4 = P(A)\,P(B).$$

Hence event A and event B are independent.