Lecture 1, MATH 239 - Introduction to Combinatorics
Graph Theory 1 - Graph, Vertex, Edge, Complete Graph, Path, Cycle, Complete Bipartite Graph

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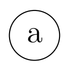
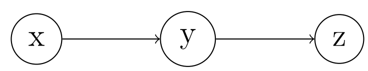
DEFINITION: A **graph** is a set of elements called **vertices** and a set of pairs of distinct vertices called **edges**. If G is a graph, we let $V(G)$ denote the set of vertices and $E(G)$ denote the set of edges.

We tend to draw graphs where vertices are points and edges are lines/curves connecting the pairs of vertices.

Example: $V = \{a\}, E = \emptyset$

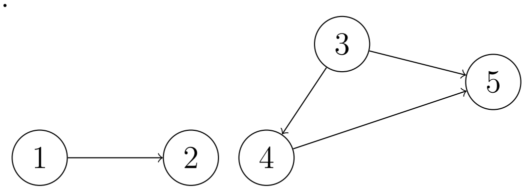


Example: $V = \{x, y, z\}, E = \{xy, yz\}$



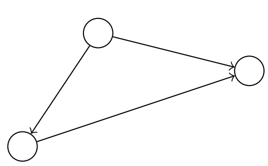
Example: $V = \emptyset, E = \emptyset$ known as the empty graph.

Example: $V = \{1, 2, 3, 4, 5\}, E = \{12, 34, 35, 45\}$

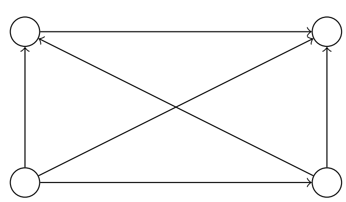


DEFINITION: K_n denotes the complete graph on n vertices where complete means all pairs of vertices are edges.

Example: K_3



K_4



DEFINITION: C_n denotes the cycle on n vertices. P_n denotes the path (or path graph) on n vertices.

DEFINITION: $K_{m,n}$ denotes a **complete bipartite graph** on m and n vertices. A complete bipartite graph is a special kind of bipartite graph where every vertex of the first set is connected to every vertex of the second set. $V = \{x_1, x_2, ..., x_m, y_1, y_2, ..., y_n\}$ $E = \{x_i y_j, \forall i | 1 \le i \le m, 1 \le j \le n\}$

Example: