Math136 - December 04, 2015
Introduction to Linear Algebra

Administrative Stuff

Instructor:	Gabriel Gauthier-Shalom
Office	MC 5113
Email:	g3gauthiershalom@uwaterloo.ca
Dropboxes:	Box 2 infront of MC4067
Assignments Due:	3PM

What is Linear Algebra

The central objects of study are linear equations. That is, equations of degree one.

For example, $x + y = 0$, $2x + 3z - y = 5$

So geometrically, we look at lines, planes, and higher dimensional analogues.

Why?

Because you will use it in many future courses in math, physics, etc. Also, proofs are really frickin important.

Vectors

A Vector $\vec{x} \in \mathbb{R}^n$ in n-dimensional real space is a tuple of n real numbers, which we write in a column.

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$$

$\mathbb{R}^n$ is the set of all such vectors.

Definitions

Two vectors $\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$ in $\mathbb{R}^n$ are **equal** if $x_1 = y_1, x_2 = y_2, \dots, x_n = y_n$

We can **sum** vectors $\vec{x}$ and $\vec{y} \in \mathbb{R}^n$ as follows: $\vec{x} + \vec{y} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{bmatrix}$

Summation Example: $\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

For $c \in \mathbb{R}$, **scalar multiplication** is defined as $c\vec{x} = \begin{bmatrix} cx_1 \\ \dots \\ cx_n \end{bmatrix}$

Scalar Multiplication Example: $4 \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ 0 \end{bmatrix}$

A **linear combination** of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k \in \mathbb{R}^n$ is the sum $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k$ with $c_1, c_2, \ldots, c_k$ being scalars.

E.g. if $\vec{v}_1 = \begin{bmatrix}1\\0\end{bmatrix}, \vec{v}_2 = \begin{bmatrix}2\\2\end{bmatrix}$ then $3\begin{bmatrix}1\\0\end{bmatrix} - 1\begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}1\\-2\end{bmatrix}$ is a linear combination of those vectors.

Vector Properties

If $\vec{x}, \vec{y}, \vec{z} \in \mathbb{R}^n$ and $c, d \in \mathbb{R}$ then:

$$\begin{aligned}
\text{PROPERTY 1}: &\quad \vec{x} + \vec{y} \in \mathbb{R}^n \\
\text{PROPERTY 2}: &\quad (\vec{x} + \vec{y}) + \vec{z} = (\vec{z} + \vec{x}) + \vec{y} \\
\text{PROPERTY 3}: &\quad (\vec{x} + \vec{y}) = (\vec{y} + \vec{x}) \\
\text{PROPERTY 4}: &\quad \exists\, \vec{0} \in \mathbb{R}^n \text{ such that } \vec{x} + \vec{0} = \vec{x} \text{ for any } x \in \mathbb{R}^n \\
\text{PROPERTY 5}: &\quad \text{For any } \vec{x} \in \mathbb{R}^n, \text{ there is a vector } -\vec{x} \in \mathbb{R}^n \text{ such that } \vec{x} + -\vec{x} = 0 \\
\text{PROPERTY 6}: &\quad c\vec{x} \in \mathbb{R}^n \\
\text{PROPERTY 7}: &\quad (c + d)\vec{x} = c\vec{x} + d\vec{x} \\
\text{PROPERTY 8}: &\quad c(d\vec{x}) = (cd)\vec{x} \\
\text{PROPERTY 9}: &\quad c(\vec{x} + \vec{y}) = c\vec{x} + c\vec{y} \\
\text{PROPERTY 10}: &\quad 1\,\vec{x} = \vec{x}
\end{aligned}$$

This list of properties will be very useful for lots of work we will do later on in the course. As an exercise left to the reader, try proving all of these properties.