Answer **all** questions in the spaces provided.

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1 A block B of mass of 4 kg is placed on a rough plane which is inclined at an angle of 6° to the horizontal.

The coefficient of friction between B and the plane is μ

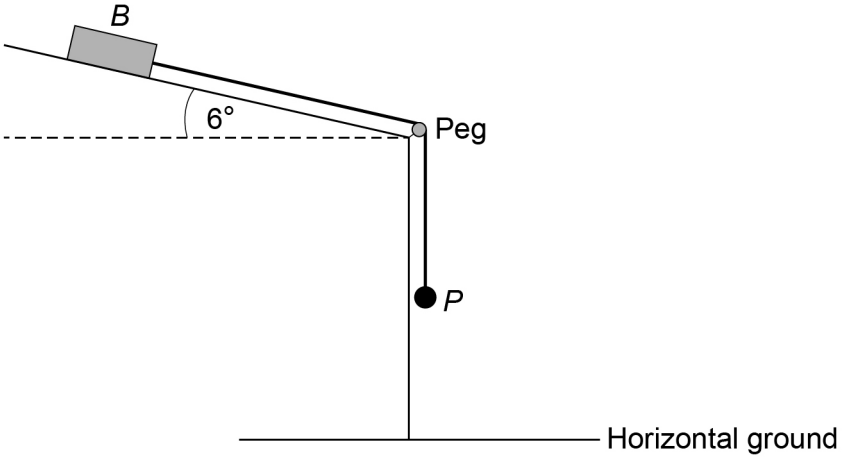
A light, inextensible string is attached to the block so that the string is parallel to the line of greatest slope of the plane.

The string passes over a smooth peg.

The other end of the string is attached to a particle P of mass 3 kg

When the block B is on the point of slipping down the plane, the particle P hangs in equilibrium vertically below the peg, as shown in **Figure 1**

Figure 1



1 (a) The peg is described as being smooth.

Explain what is meant by smooth.

[1 mark]

Do not write outside the box

1 (b) (i) Find the tension in the string.

Give your answer to three significant figures.

[1 mark]

Answer _____

1 (b) (ii) Find the value of μ

[4 marks]

$\mu =$ _____

1 (c) The string is cut so that B and P are no longer connected.

1 (c) (i) Write down the magnitude and direction of the acceleration of P

[1 mark]

Magnitude _____

Direction _____

1 (c) (ii) Find the magnitude of the friction force which acts on B

[2 marks]

Answer _____

9

2 A pebble of mass 25 grams is released from rest at a height h metres above horizontal ground.

A student models the pebble as experiencing a constant air resistance force of magnitude R newtons when it is moving through the air.

The pebble collides with the ground for the first time 0.55 seconds after it was released.

The pebble collides with the ground for the first time with a speed of 5.2 m s^{-1}

2 (a) Find the magnitude of the acceleration of the pebble as it moves through the air towards the ground according to the student's model.

Give your answer to three significant figures.

[1 mark]

Answer _____

2 (b) Find the value of R

[2 marks]

$R =$ _____

Do not write outside the box

2 (c) (i) Show that the kinetic energy of the pebble as it collides with the ground for the first time is 0.338 J

[1 mark]

2 (c) (ii) Hence use an energy method to find the value of h according to the student's model.

Give your answer to three significant figures.

[2 marks]

$h =$ _____

2 (d) After its first collision with the ground, the pebble rebounds vertically to a maximum height of 0.65 metres above the ground.

Find the magnitude of the impulse exerted on the pebble due to its first collision with the ground according to the student's model.

[4 marks]

Answer _____

10

Do not write outside the box

3 An electric scooter and rider of total mass 70 kg move along a straight horizontal track.

When the speed of the scooter and rider is v m s^{-1}, the total resistance force R newtons experienced by the scooter and rider is given by

$$R = 3v^k$$

where k is a positive constant.

The maximum power output of the scooter's electric motor is 250 W

3 (a) When the scooter and rider move with a speed of 4 m s^{-1} the maximum magnitude of their acceleration is 0.6 m s^{-2}

Show that $k = 1.386$ correct to four significant figures.

[4 marks]

Do not write outside the box

3 (b) Find the maximum possible speed of the scooter and rider on the straight horizontal track.

Give your answer to three significant figures.

[3 marks]

Answer _____

7

Turn over for the next question

4 A **non-uniform** rod AB of mass 219 kg and length 6.5 metres is held in equilibrium in a horizontal position by two light inextensible strings S_A and S_B

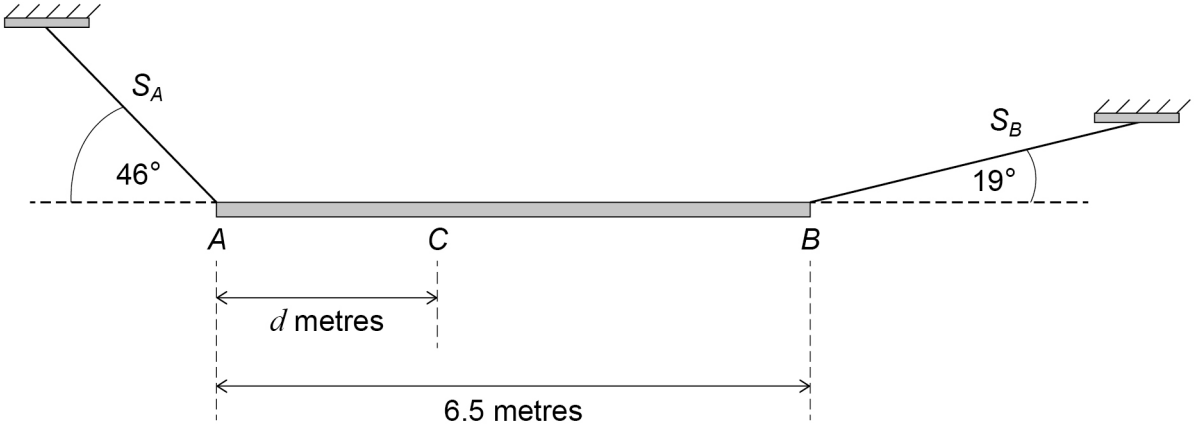
The centre of mass of the rod is located at the point C which is d metres from A

String S_A is attached to the rod at A, and the other end of this string is attached to a fixed point. This string makes an angle of 46° with the horizontal.

String S_B is attached to the rod at B, and the other end of this string is attached to a different fixed point. This string makes an angle of 19° with the horizontal.

The rod and strings are shown in **Figure 2**

Figure 2



4 (a) (i) The tension in string S_B is 1645 newtons.

By taking moments about A show that the value of d is 1.622 correct to four significant figures.

[3 marks]

Do not write outside the box

4 (a) (ii) The tension in string S_A is T_A newtons.

Find the value of T_A

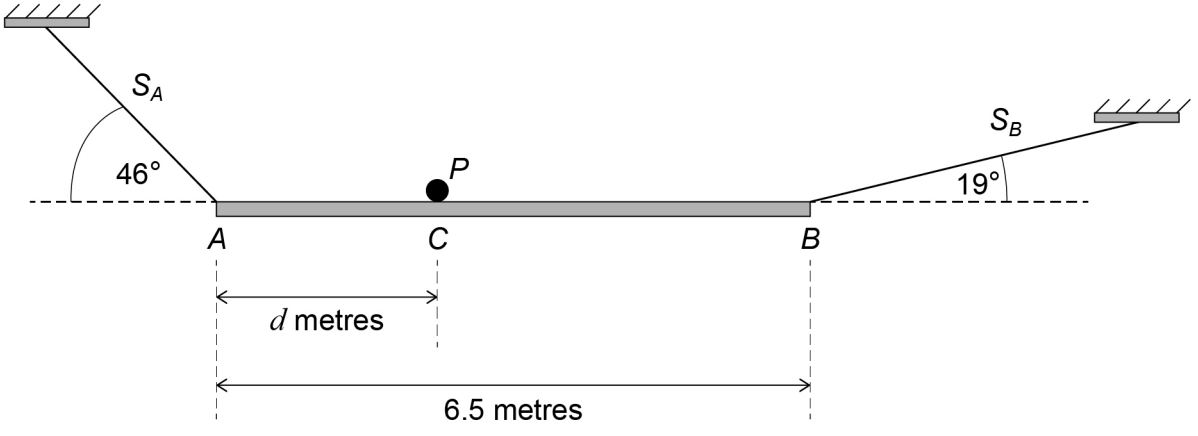
Give your answer to four significant figures.

[2 marks]

$T_A =$ _____

4 (b) A particle P of mass m kg is placed on the rod at C to form a system that is in equilibrium, as shown in **Figure 3**

Figure 3



If the tension in either string exceeds 3000 newtons, that string will break.

Find the range of values of m for which the system will remain in equilibrium.

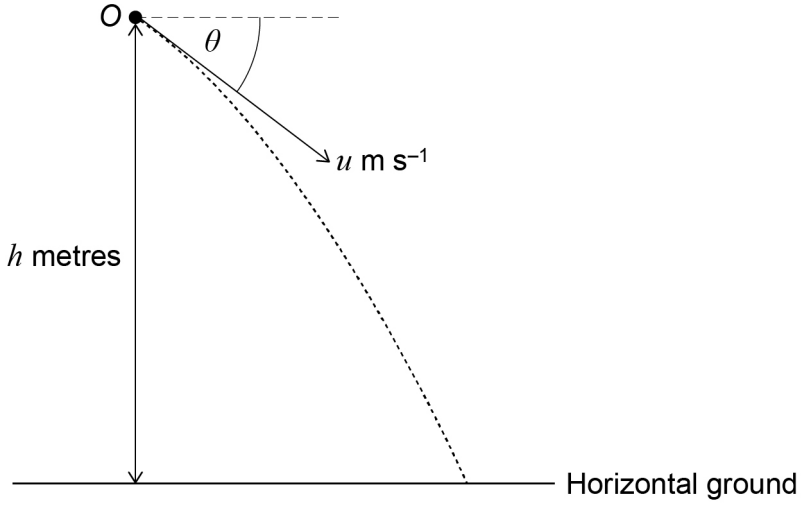
[4 marks]

Answer _____

9

5 A particle is projected with a speed of u m s^{-1} at an angle θ **below** the horizontal from a point O which is at a height of h metres above the horizontal ground, as shown in **Figure 4**

Figure 4



The horizontal displacement of the particle from O at time t seconds is x metres.

The vertical displacement of the particle above the horizontal ground at time t seconds is y metres.

Assume that the only force that the particle experiences whilst it is in the air is its weight.

5 (a) Show that

$$y = h - x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta$$

[6 marks]

Do not write outside the box

5 (b) A particle is projected with a speed of 20 m s^{-1} at an angle 25° below the horizontal from a point O which is at a height of 4 metres above the horizontal ground.

5 (b) (i) Use the result in **part (a)** to find the horizontal displacement of the particle from O when the particle collides with the horizontal ground for the first time.

[3 marks]

Answer _____

5 (b) (ii) The time taken for the particle to move from O to the horizontal ground for the first time is T seconds.

Find the value of T

[2 marks]

$T =$ _____

11

6 A body of mass 3.5 kg is acted upon by the force **F** newtons.

The position vector **r** metres of the body relative to a fixed origin at time t seconds is given by

$$\mathbf{r} = \begin{bmatrix} 6\cos 4t \\ 2 + 6\sin 4t \end{bmatrix}$$

6 (a) (i) The velocity of the body at time t seconds is **v** m s^{-1}

Find **v** in terms of t

[2 marks]

Answer _____

6 (a) (ii) Hence find the angle between the body's velocity and position vector when $t = \frac{\pi}{3}$

Give your answer to the nearest degree.

[6 marks]

Do not write outside the box

Answer

6 (b) It is given that $|\mathbf{F}| = k$ for all values of t

Find the value of k

[3 marks]

$k =$

11

Turn over for the next question

7 A light inextensible string of length $5d$ metres has one end attached to a fixed point O

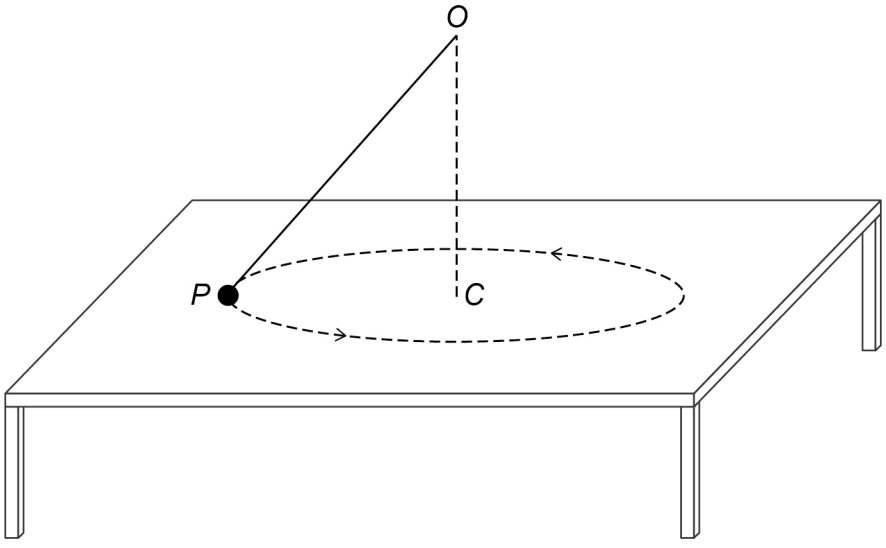
The other end of the string is attached to a particle P of mass m kg

The point C is on the surface of a smooth horizontal table.

The point O is vertically above the point C

The particle P is set into motion so that it moves with constant speed around the circumference of a horizontal circle of radius $3d$ metres and centre C on the table, as shown in **Figure 5**

Figure 5



The string is taut throughout the motion.

7 (a) It is given that the particle's constant speed is v m s^{-1}

Show that the tension in the string is given by $\dfrac{5mv^2}{9d}$

[3 marks]

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7 (b) In the case when $v = \sqrt{2gd}$, find an expression for the magnitude of the normal reaction force exerted on P due to its contact with the table.

Give your answer in terms of m and g

[4 marks]

Answer _____

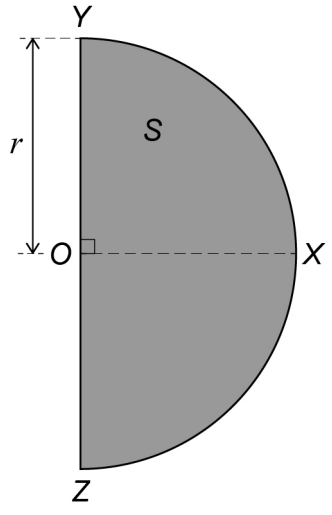
7

Turn over for the next question

8 **Figure 6** shows the semi-circular uniform lamina S and the points O, X, Y and Z, where

- the arc of the semi-circle is YXZ and the straight edge of the semi-circle is YZ
- the point O is the mid-point of YZ
- the radius OY has length r metres and the angle YOX is 90°

Figure 6

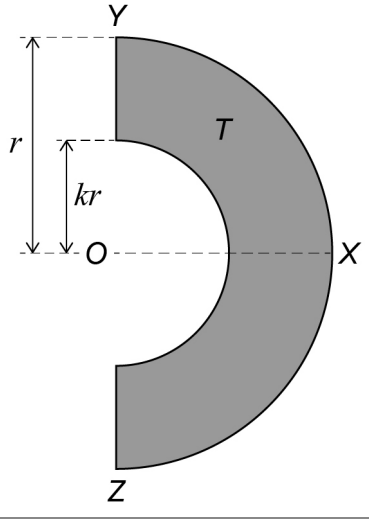


8 (a) Explain why the centre of mass of the lamina lies on the straight line OX

[1 mark]

8 (b) **Figure 7** shows the uniform lamina T which is made by removing a semi-circular region from S whose straight edge lies on YZ and O is the midpoint of its straight edge. The radius of the semi-circular region that has been removed is kr where $0 < k < 1$

Figure 7



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8 (b) (i) You are given that the centre of mass of a uniform semi-circular lamina with radius r is located at a distance of $\frac{4r}{3\pi}$ from the straight edge, along the radius perpendicular to the straight edge.

Find the distance of the centre of mass of T from O

Give your answer in the form $\frac{a\,r\left(1-k^{b}\right)}{c\,\pi\left(1-k^{d}\right)}$ where a, b, c and d are integers.

[5 marks]

Answer _____

8 (b) (ii) In the case when $k = 0.5$ the uniform lamina T is freely suspended from Y and hangs in equilibrium.

Find the angle between the straight line YZ and the vertical.

Give your answer to the nearest degree.

[3 marks]

Answer _____

9

9 A body of mass 5 kg is moving around the circumference of a circle of radius r metres and centre O

A force with magnitude F newtons acts on the body towards O where

$$F = \frac{5\pi^2}{r^3}$$

Assume that the body experiences no other forces.

It is given that the body moves around the circumference of the circle exactly 8 times every 25 seconds.

9 (a) Find the angular speed of the body.

Give your answer in an exact form.

[2 marks]

Answer _____

9 (b) Find the value of r

Give your answer as a fraction.

[3 marks]

Answer _____

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9 (c) Find the kinetic energy of the body.

[2 marks]

Answer _____

7

END OF QUESTIONS

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