Weighted Finite State Transducers in Automatic Speech Recognition

ZRE lecture 10.04.2013

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Slides provided with permission, Daniel Povey
some slides from T. Schultz, M. Mohri and M. Riley

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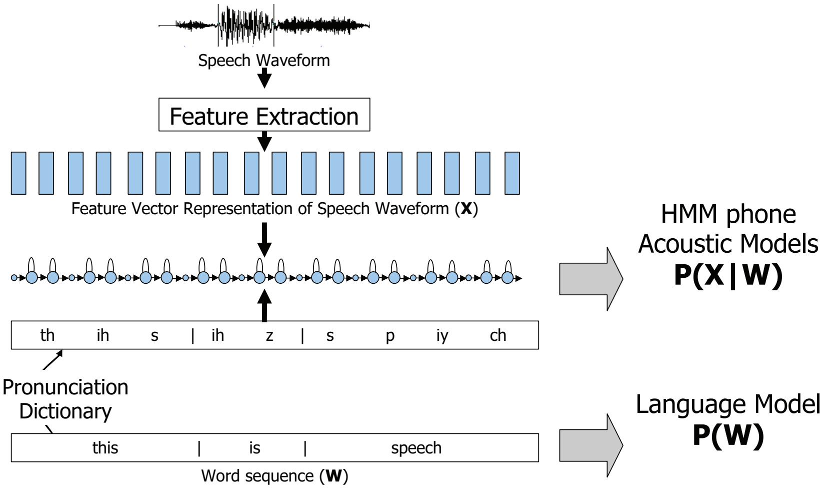
Automatic speech recognition



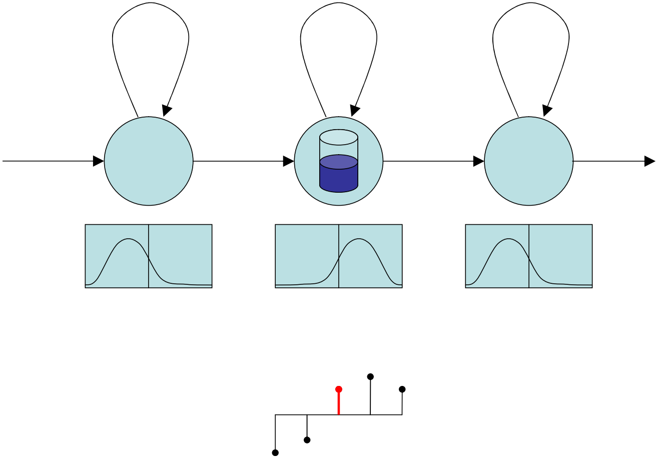
Automatic speech recognition

- Determine the most probable word sequence $\tilde{W}$ given the observed acoustic signal Y

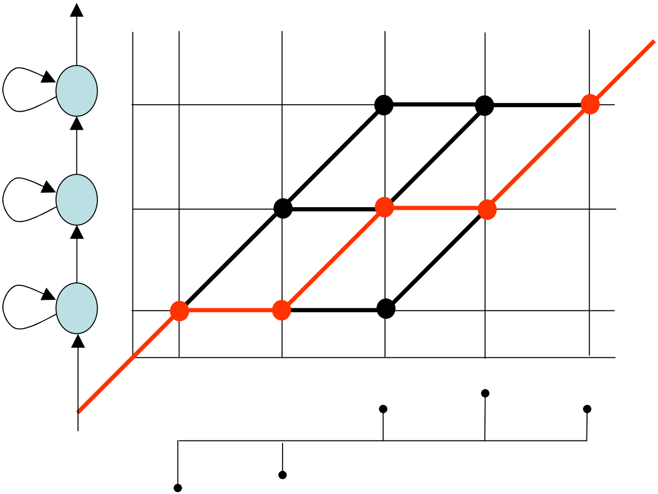
$$\tilde{W} = argmax\ P(W|Y) = \frac{argmax\ P(W)P(Y|W)}{P(Y)}$$

- Search for word sequence $\tilde{W}$ that maximizes P(W) and P(Y|W)
  - P(W) = language model (likelihood of word sequence)
  - P(Y|W) = acoustic model
    (likelihood of observed acoustic signal given word sequence)

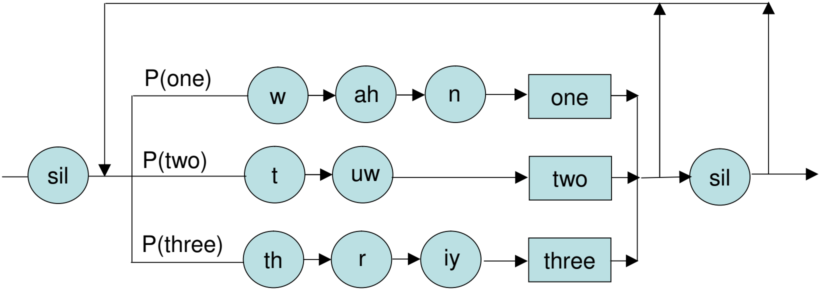
Hidden Markov Model, Token passing



Viterbi algorithm, trellis



Decoding graph/recognition network



Why Finite State Transducers?

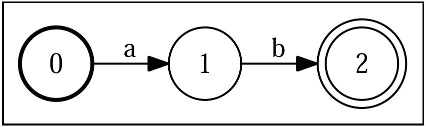
Motivation:

- most components (LM, lexicon, lattice) are finite-state
- unified framework for describing models
- integrate different models into a single model via composition operations
- improve search efficiency via optimization algorithms
- flexibility to extend (add new models)

→ *speed*: pre-compiled search space, near realtime performance on embedded systems

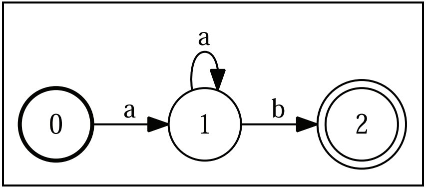
→ *flexibility*: same decoder used for hand-held devices and LVCSR

Finite State Acceptor (FSA)



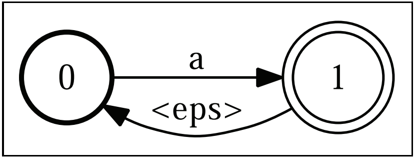
- An FSA "accepts" a set of strings
- (a string is a sequence of symbols).
- View FSA as a representation of a possibly infinite set of strings.
- This FSA accepts just the string ab, i.e. the set $\{ab\}$
- Numbers in circles are state labels (not really important).
- Labels are on arcs are the symbols.
- Start state(s) bold; final/accepting states have extra circle.
  - Note: it is sometimes assumed there is just one start state.

A less trivial FSA



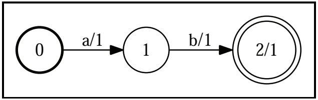
- The previous example doesn't show the power of FSAs because we could represent the set of strings finitely.
- This example represents the infinite set $\{ab, aab, aaab, \ldots\}$
- Note: a string is "accepted" (included in the set) if:
  - There is a path with that sequence of symbols on it.
  - That path is "successful' (starts at an initial state, ends at a final state).

The epsilon symbol



- The symbol ϵ has a special meaning in FSAs (and FSTs)
- It means "no symbol is there".
- This example represents the set of strings $\{a, aa, aaa, \ldots\}$
- If ϵ were treated as a normal symbol, this would be $\{a, a\epsilon a, a\epsilon a\epsilon a, \ldots\}$
- In text form, ϵ is sometimes written as <eps>
- Toolkits implementing FSAs/FSTs generally assume <eps> is the symbol numbered zero

Weighted finite state acceptors



- Like a normal FSA but with costs on the arcs and final-states
- Note: cost comes after "/". For final-state, "2/1" means final-cost 1 on state 2.
- View WFSA as a function from a string to a cost.
- In this view, unweighted FSA is f : string $\rightarrow \{0, \infty\}$.
- If multiple paths have the same string, take the one with the lowest cost.
- This example maps *ab* to $(3 = 1 + 1 + 1)$, all else to ∞.

Semirings

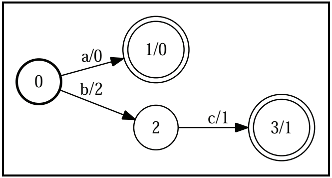
- The semiring concept makes WFSAs more general.
- A semiring is
  - A set of elements (e.g. $\mathbb{R}$)
  - Two special elements $\bar{1}$ and $\bar{0}$ (the identity element and zero)
  - Two operations, $\oplus$ (plus) and $\times$ (times) satsifying certain axioms.

Semiring examples. $\oplus_{\log}$ *is defined by:* $x \oplus_{\log} y = -\log(e^{-x} + e^{-y})$.

SEMIRING	SET	$\oplus$	$\otimes$	$\bar{0}$	$\bar{1}$
Boolean	$\{0,1\}$	$\vee$	$\wedge$	0	1
Probability	$\mathbb{R}_+$	$+$	$\times$	0	1
Log	$\mathbb{R} \cup \{-\infty, +\infty\}$	$\oplus_{\log}$	$+$	$+\infty$	0
Tropical	$\mathbb{R} \cup \{-\infty, +\infty\}$	min	$+$	$+\infty$	0

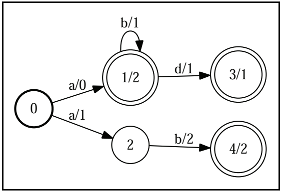
- In WFSAs, weights are multiplied along paths
- summed over paths with identical symbol-sequences

Weights vs. costs



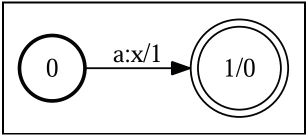
- Personally I use "cost" to refer to the numeric value, and "weight" when speaking abstractly, e.g.:
  - The acceptor above accepts *a* with unit weight.
  - It accepts *a* with zero cost.
  - It accepts *bc* with cost $4 = 2 + 1 + 1$
  - State 1 is final with unit weight.
  - The acceptor assigns zero weight to *xyz*.
  - It assigns infinite cost to *xyz*.

Weights and costs



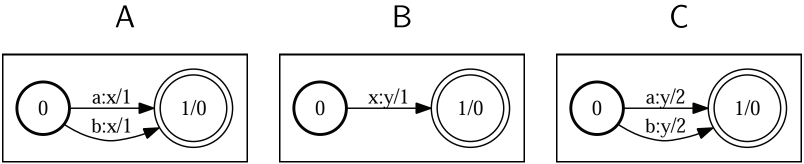
- Consider the WFSA above, with the tropical ("Viterbi-like") semiring. Take the string *ab*.
- We "multiply" ($\otimes$) the weights along paths; this means adding the costs.
- Two paths for *ab*:
  - One goes through states (0, 1, 1); cost is $(0 + 1 + 2) = 3$
  - One goes through states (0, 2, 3); cost is $(1 + 2 + 2) = 5$
- We add weights across different paths; tropical $\oplus$ is "take min cost" $\rightarrow$ this WFSA maps *ab* to 3

Weighted finite state transducers (WFST)



- Like a WFSA except with two labels on each arc.
- View it as a function from a (pair of strings) to a weight
- This one maps (a, x) to 1 and all else to ∞
- Note: view 1 and ∞ as costs. ∞ is $\bar{0}$ in semiring.
- Symbols on the left and right are termed “input” and “output” symbols.

Composition of WFSTs



- Notation: $C = A \circ B$ means, C is A composed with B.
- In special cases, composition is similar to function composition
- Composition algorithm "matches up" the "inner symbols"
  - i.e. those on the output (right) of A and input (left) of B

Composition algorithm

- Ignoring ϵ symbols, algorithm is quite simple.
- States in C correspond to tuples of (state in A, state in B).
  - But some of these may be inaccessible and pruned away.
- Maintain queue of pairs, initially the single pair $(0, 0)$ (start states).
- When processing a pair (s, t):
  - Consider each pair of (arc a from s), (arc b from t).
  - If these have matching symbols (output of a, input of b):
    - Create transition to state in C corresponding to (next-state of a, next-state of b)
    - If not seen before, add this pair to queue.
- With ϵ involved, need to be careful to avoid redundant paths...

Construction of decoding network

- WFST approach [Mohri et al.]
- exploit several knowledge sources (lexicon, grammar, phonetics) to find most likely spoken word sequence

$$HCLG = H \circ C \circ L \circ G \tag{1}$$

G probabilistic grammar or language model acceptor (word)

L lexicon (phones to words)

C context-dependent relabeling (ctx-dep-phone to phone)

H HMM structure (PDF labels to context-dependent phones)

Create H, C, L, G separately and compose them together

Language model

Estimate probability of a word sequence W:

$$P(W) = P(w_1, w_2, \ldots, w_N) \tag{2}$$

$$P(W) = P(w_1) \cdot P(w_2|w_1) \cdot P(w_3|w_1, w_2) \cdot \ldots \cdot P(w_N|w_1, \ldots, w_{N-1}) \tag{3}$$

Approximate by sharing histories h (e.g. bigram history):

$$P(W) \approx P(w_1) \cdot P(w_2|w_1) \cdot P(w_3|w_2) \cdot \ldots \cdot P(w_N|w_{N-1}) \tag{4}$$

What to do if a certain history was not observed in training texts?

- statistical smoothing of the distribution
- interpolation of different orders of history
- backing-off to shorter history

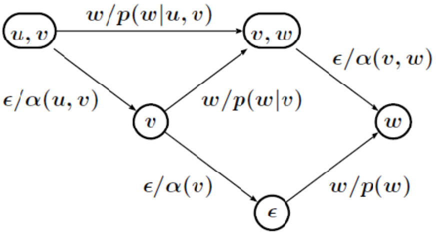
Language model acceptor G

- **G:** Grammar Transducer

Backing-off language model:

$$p(w|h) = \begin{cases} f(w|h) & : \text{if } N(w,h) > 0 \\ \alpha(h) \cdot f(w|\overline{h}) & : \text{if } N(w,h) = 0 \end{cases}$$

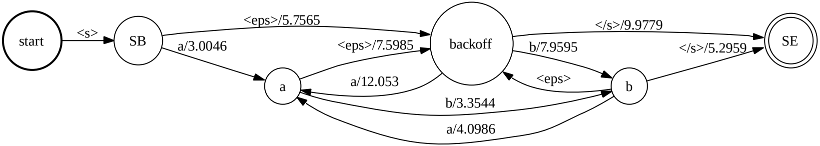
- **Input:** word
- **Weight:** history dependent word probability



Language models (ARPA back-off)

```
\1-grams:
-5.2347  a -3.3
-3.4568  b
 0.0000  <s> -2.5
-4.3333  </s>

\2-grams:
-1.4568  a b
-1.3049  <s> a
-1.78    b a
-2.30    b </s>
```



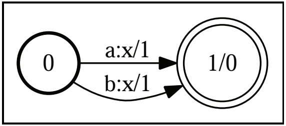
Pronunciation lexicon L

```
A          ax #1
ABERDEEN   ae b er d iy n
ABOARD     ax b r ao dd
ADD        ae dd #1
ABOVE      ax b ah v
```

Added disambiguation symbols:

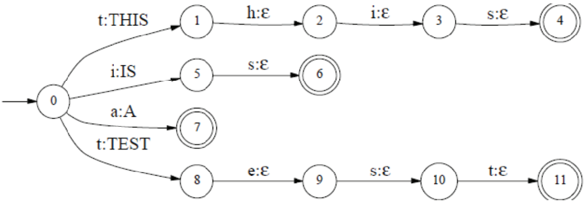
- if a phone sequences can output different words ("I scream for ice cream.")
- non-determinism: introduce disambiguation symbols, remove at last stage

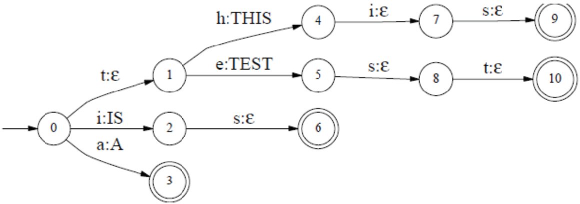
Deterministic WFSTs



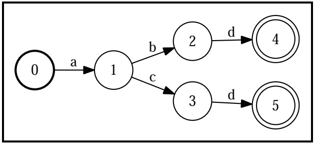
- Taken to mean "deterministic on the input symbol"
- I.e., no state can have > 1 arc out of it with the same input symbol.
- Some interpretations (e.g. Mohri/AT&T/OpenFst) allow ϵ input symbols (i.e. being ϵ-free is a separate issue).
- I prefer a definition that disallows epsilons, except as necessary to encode a string of output symbols on an arc.
- Regardless of definition, not all WFSTs can be determinized.

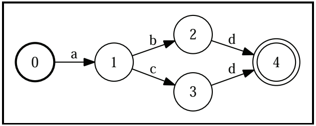
Determinization (like making tree-structured lexicon)





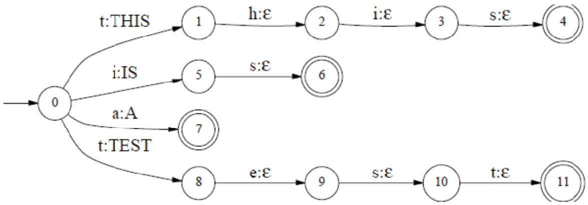
Minimal deterministic WFSTs

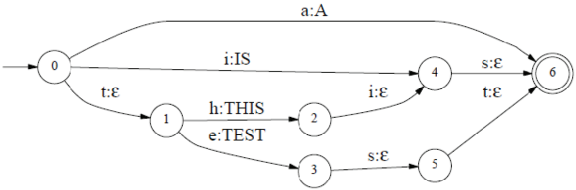




- Here, the left FSA is not minimal but the right one is.
- "Minimal" is normally only applied to *deterministic* FSAs.
- Think of it as suffix sharing, or combining redundant states.
- It's useful to save space (but not as crucial as determinization, for ASR).

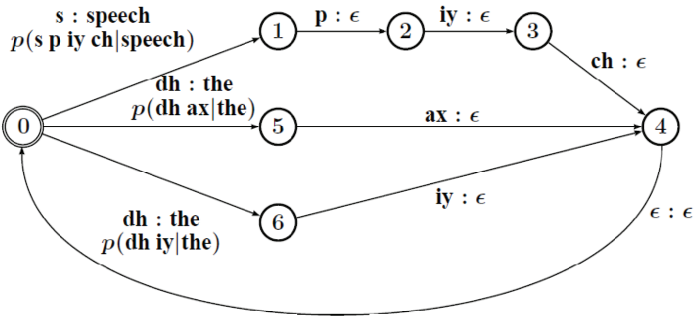
Minimization (like suffix sharing)





Pronunciation lexicon L

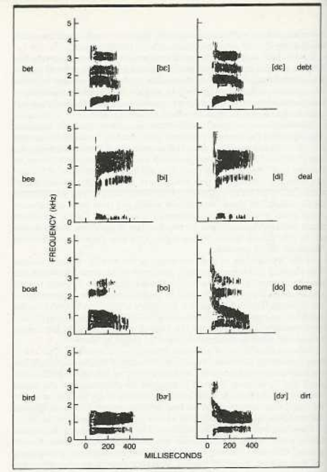
- **L:** Context-Dependency Transducer
  - **Input:** context-independent phone (phoneme)
  - **Output:** word
  - **Weight:** pronunciation probability



Context-dependency of phonemes

So far we use phonemes independent of context, but:

- co-articulation: pronunciation of phones changes with surrounding phones



Context-dependency transducer C

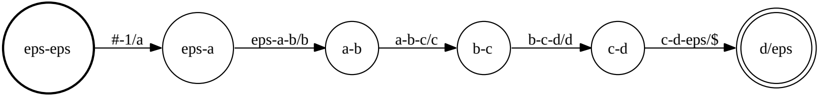
Introduce context-dependent phones:

- tri-phones: each model depends on predecessor and successor phoneme: a-b-c (b/a_c)
- implemented as context-dependency transducer C

Input: context-dependent phone (triphone)

Output: context-independent phone (phone)

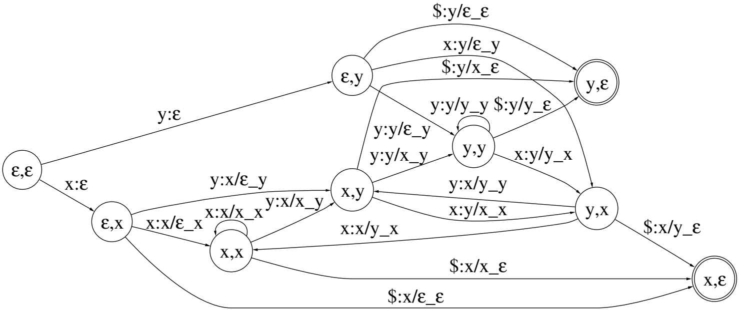
- shown is one path of it



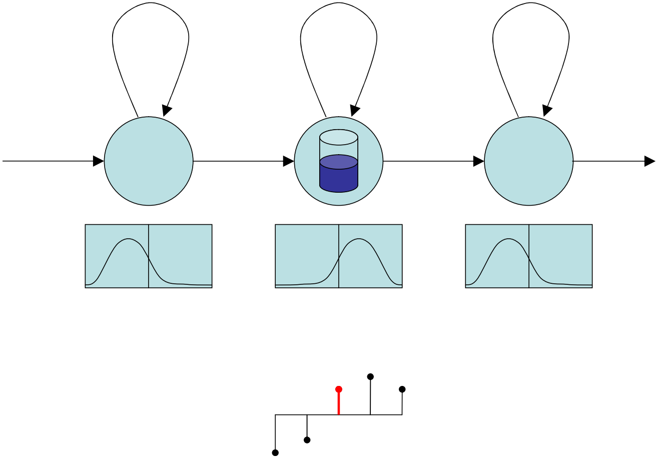
Context-dependency transducer C

(a) (b)

Figure 6: Context-dependent triphone transducer transition: (a) non-deterministic, (b) deterministic.

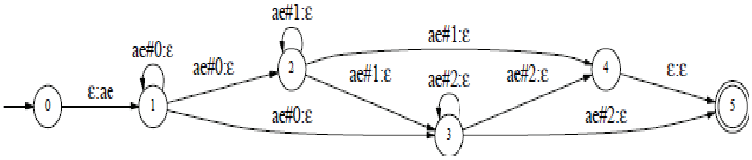


HMM as transducer



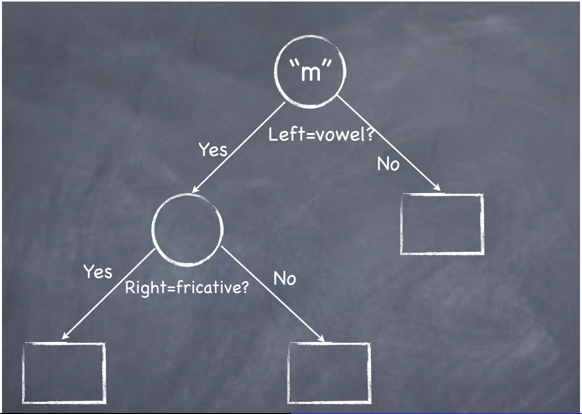
HMM as transducer (monophone)

- H: HMM Topology Transducer (maps states to phonemes)
  - **Input:** state
  - **Output:** context-dependent phone (triphone)
  - **Weight:** HMM transition probability

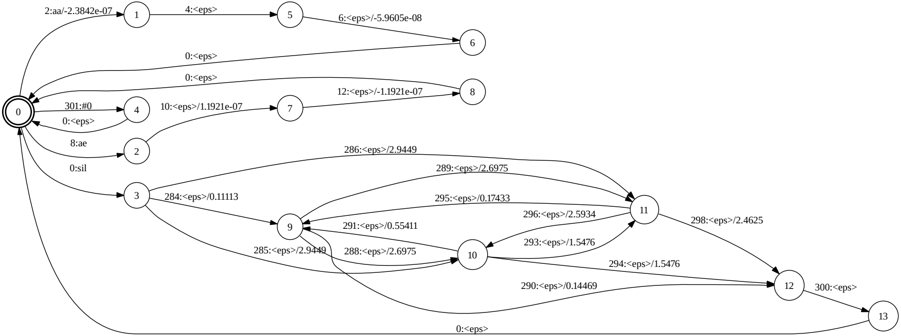


Phonetic decision tree

- too many context-dependent models (N^3) $\rightarrow$ clustering
- determine model-id (gaussian) based on phoneme context and state in HMM
- using questions about context (sets of phones)



HMM transducer H_a



(here shown for monophone case, without self-loops)

Construction of decoding network

WFST approach by [Mohri et al.]

$$HCLG = rds(min(det(H \circ det(C \circ det(L \circ G))))) \tag{5}$$

rds - remove disambiguation symbols
min - minimization, includes weight pushing

det - determinization

Kaldi toolkit [Povey et al.]

$$HCLG = asl(min(rds(det(H_a \circ min(det(C \circ min(det(L \circ G)))))))) \tag{6}$$

asl - add self loops

rds - remove disambiguation symbols

Decoding graph construction (complexities)

- Have to do things in a careful order or algorithms "blow up"
- Determinization for WFSTs can fail
  - need to insert "disambiguation symbols" into the lexicon.
  - need to "propagate these through" H and C.
- Need to guarantee that final HCLG is stochastic:
  - i.e. sums to one, like a properly normalized HMM
  - needed for optimal pruning (discard unlikely paths)
  - usually done by weight-pushing, but standard algorithm can fail, because FST representation of back-off LMs is non-stochastic
- We want to recover the phone sequence from the recognized path (words)
  - sometimes also the model-indices (PDF-ids) and the HCLG arcs that were used in best path

Decoding with WFSTs (finding best path)

- Solve

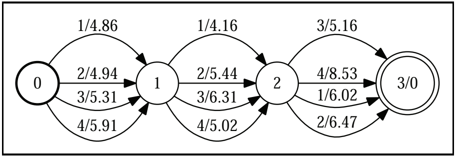
$$W' = argmax_W P(X|W)P(W)$$

- Compose recognizer as (H o C o L o G) which maps states to word sequences
- Decode by aligning the feature vectors X with HCLG "

i.e.,

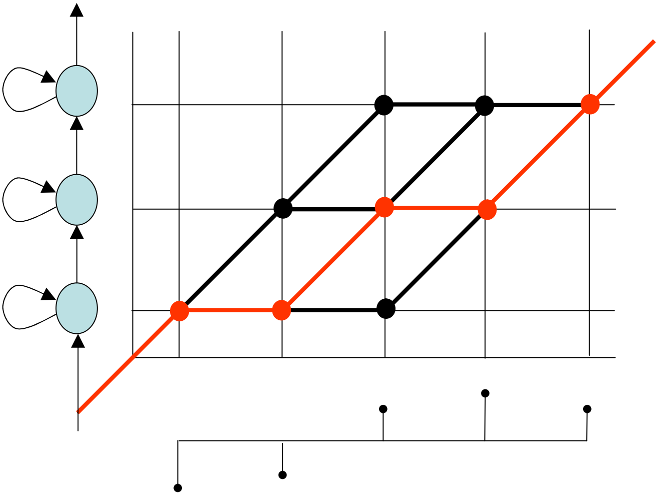
$$W' = argmax_W X \circ (H \circ C \circ L \circ G)$$

Decoding with WFSTs



- First– a "WFST definition" of the decoding problem.
- Let U be an FST that encodes the acoustic scores of an utterance (as above).
- Let $S = U \circ HCLG$ be called the search graph for an utterance.
- Note: if U has N frames (3, above), then
  - #states in S is $\leq (N + 1)$ times #states in $HCLG$.
  - Like $N + 1$ copies of $HCLG$.

Viterbi algorithm, trellis



Decoding with WFSTs

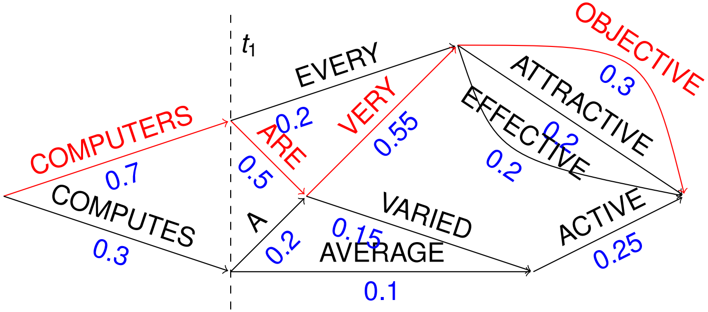
- With beam pruning, we search a subgraph of S.
- The set of "active states" on all frames, with arcs linking them as appropriate, is a subgraph of S.
- Let this be called the *beam-pruned subgraph* of S; call it B.
- A standard speech recognition decoder finds the best path through B.
- In our case, the output of the decoder is a linear WFST that consists of this best path.
- This contains the following useful information:
  - The word sequence, as output symbols.
  - The state alignment, as input symbols.
  - The cost of the path, as the total weight.

Decoding output

```
utt1  [ 2 6 6 6 6 10 ] [ 614 613 613 613 711 ] [ 122 123
utt1  SIL              th                      ax
utt1  <s>              THE
```

Word Lattice / Word Graph

Word Lattice: a compact representation of the search space



Lattices as WFSTs

The word "lattice" is used in the ASR literature as:

- Some kind of compact representation of the alternate word hypotheses for an utterance.
- Like an N-best list but with less redundancy.
- Usually has time information, sometimes state or word alignment information.
- Generally a directed acyclic graph with only one start node and only one end node.

Lattice Generation with WFSTs [Povey12]

Basic process (not memory efficient):

- Generate beam-pruned subgraph B of search graph S
- The states in B correspond to the active states on particular frames.
- Prune B with beam α to get pruned version P.
- Convert P to acceptor and lattice-determinize to get A (deterministic acceptor)

$\rightarrow$ No two paths in L have same word sequence (take best)

- Prune A with beam α to get final lattice L (in acceptor form).

Finite State Transducers for ASR

Pro's:

Fast: compact/minimal search space due to combined minimization of lexicon, phonemes, HMM's

Simple: easy construction of recognizer by composition from states, HMMs, phonemes, lexicon, grammar

Flexible: whatever new knowledge sources, the compose/optimize/search remains the same

Con's:

- composition of complex models generates a huge WFST
- search space increases, and huge memory is required
- esp. how to deal with huge language models

Ressources

OpenFST `http://www.openfst.org/` Library, developed at Google Research (M. Riley, J. Schalkwyk, W. Skut) and NYU's Courant Institute (C. Allauzen, M. Mohri)

Mohri08 M. Mohri et al., *"Speech Recognition with weighted finite state transducers."*

Povey11 D. Povey et al., *"The Kaldi Speech Recognition Toolkit."*

Povey12 D. Povey et al., *"Generating exact lattices in the WFST framework."*