Chapter 3 Assignment

2. $\forall n \in S,\ P(n):\ \frac{n^2-5n+6}{2}$

$\frac{0^2-5(0)+6}{2} = 3$ is odd ✓

$1-5+6$

$\frac{1^2-5(1)+6}{2} = 1$ is odd ✓

$16-20+6$

$\frac{4^2-5(4)+6}{2} = 1$ is odd ✓

$25-25+6$

$\frac{5^2-5(5)+6}{2} = 3$ is odd ✓

4. proof by contrapositive

if n is even, then $11n-9$ is odd

Assume n is even, so $n = 2k$

$11n-9 = 11(2k)-9$

$22k-9 = (22k-8)+1 = 2(11k-4)+1$

$11k-4$ is an integer so $11n-9$ is odd

12.

$a=1$ and $b=1$

$(1)+(1) > (1)(1)$ $\quad 2>1$ True

But

$a=0$ and $b=0$

$(0)+(0) > (0)(0)$ $\quad 0>0$ false

Therefore, $\exists x \in \mathbb{Z},\ a+b > ab$

or

$\exists x \in \mathbb{Z},\ P(a,b)$

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$P(n)$: $3n+5$ is odd

$Q(n)$: n is an even integer

a)

b) proof by contraposition

$\sim Q(n)$: n is an odd integer $\quad n = 2k+1$

$\sim P(n)$: $3n+5$ is even $\quad 3(2k+1)+5 = 6k+3+5$

$6k+8 = 2(3k+4)$

$3k+4$ is an integer, so $3n+5$ is even

c) proof by contradiction

$P(n)$: $3n+5$ is odd

$\sim Q(n)$: n is odd $\quad n = 2k+1$

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(a) For every value where n is an integer, n^2 is less than 4

b) For some $n \in \mathbb{Z}$, $Q(n)$: n^3 equals n

c) for some $n \in \mathbb{Z}$, $\sim P(n)$: $n^2 \geq 4$

d) For every value $n \in \mathbb{Z}$, $n^2 \neq n$

e) If $n^3 = n$, then $n^2 < 4$

f) If $n^3 \neq n$, then $n^2 \geq 4$

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Direct Proof

$P(x): x^3 = x$ $\quad$ $Q(x): x^2 \geq 2$

$Q(x): x^2 < 2$ $\quad$ $\sim P(x): x^3 \neq x$

$x^3 - x = \quad x(x^2 - 1) = 0$

$x(x-1)(x+1) = 0 \quad x = 0 \quad x = -1 \quad x = 1$

so

$(0)^2 < 2 \quad -1^2 < 2 \quad 1^2 < 2$