1 第二章

和答案对不上的有第 3、19、21、23 题

没做出来的有第 16 题

1.

$$\begin{aligned}\bar{A}_{45:\overline{20}|} &= \bar{A}^{\,1}_{45:\overline{20}|} + A_{45:\overset{\,1}{\overline{20}|}} \\ &= \int_0^{20} e^{-\delta t}\ {}_tp_{45}\ \mu_{45+t}\ dt + e^{-20t}\ {}_{20}p_{45} \\ &= 0.39814\end{aligned}$$

其中 ${}_tp_{45}\ \mu_{45+t} = -\frac{d}{dt}\frac{l_{45+t}}{l_{45}} = \frac{1}{58}$，${}_{20}p_{45} = \frac{l_{65}}{l_{45}} = 0.655172.$

2. $\bar{A}^{\,1}_{40:\overline{10}|} = \int_0^{10} e^{-\delta t}\ {}_tp_{40}\ \mu_{40+t}\ dt = 0.125330$

   其中 ${}_tp_{40}\ \mu_{40+t} = 1/60$

3. $E[Z] = \int_0^{50} v^t\ b_t\ {}_tp_{50}\ \mu_{50+t}\ dt = \frac{1}{50}\int_0^{50} 1.01^{-t}(1000-0.1t^2)\ dt = 730.18$

   其中 ${}_tp_{50}\ \mu_{50+t} = 1/50$

4. $E[Z] = \int_0^{75} e^{-\delta t}\ g(t)\ dt = (1-e^{-75\delta})/75\delta = 0.260395$

   同理 $E[Z^2] = (1-e^{-150\delta})/150\delta = 0.133260$，那么 $Var[Z] = E[Z^2] - (E[Z])^2 = 0.065454$

5.

k	v^{k+1}	b_{k+1}	${}_kp_x$	q_{x+k}	$v^{k+1}\ b_{k+1}\ {}_kp_x\ q_{x+k}$
0	0.943396	300000	1.0000	0.02	5660.3774
1	0.889996	350000	0.9800	0.04	12210.7512
2	0.839619	400000	0.9408	0.06	18957.9317

$E[Z] = \sum_{k=0}^{2} v^{k+1}\ b_{k+1}\ {}_kp_x\ q_{x+k} = 36829.06$

6.

k	v^{k+1}	b_{k+1}	${}_kp_x$	q_{x+k}	$v^{k+1}\ b_{k+1}\ {}_kp_x\ q_{x+k}$
0	0.943396	1	1.0000	0.1500	0.141509
1	0.881679	1	0.8500	0.2353	0.176336
2	0.816369	1	0.6500	0.3846	0.204092

$E[Z] = \sum_{k=0}^{2} v^{k+1}\ b_{k+1}\ {}_kp_x\ q_{x+k} = 0.521937$

7.

k	v^{k+1}	b_{k+1}	${}_kp_x$	q_{x+k}	$v^{k+1}\ b_{k+1})^2\ {}_kp_x\ q_{x+k}$	$(v^{k+1}\ b_{k+1}\ {}_kp_x\ q_{x+k}$
0	0.952381	1	1	0.021546	0.020520	0.019543
1	0.907029	1	0.978454	0.027202	0.024141	0.021897

$E[Z] = \sum_{k=0}^{1} v^{k+1}\ b_{k+1}\ {}_kp_x\ q_{x+k} = 0.044662$

$E[Z^2] = \sum_{k=0}^{1} (v^{k+1}\ b_{k+1})^2\ {}_kp_x\ q_{x+k} = 0.041440$

$Var[Z] = E[Z^2] - (E[Z])^2 = 0.039445$

8.（例 2-7）$A_x^{(4)} \approx \frac{i}{i^{(4)}} A_x = 0.251771$

其中 $1+i = (1+i^{(4)}/4)^4 \implies i^{(4)} = 0.058695$，$A_x = \sum_{k=0}^{\infty} v^{k+1}\ {}_kp_x\ q_{x+k} = 0.246295$

9. i. $z = 0$

$F_Z(z) = P\{Z = 0\} = P\{T \le 10\} = {}_{10}q_{35} = 1 - \exp(-\int_0^{10} \mu\ dt) = 0.451188$

ii. $0 < z \le v^{10}$

$F_Z(z) = P\{v^T \le z\} + P\{Z = 0\} = P\{T \ge \ln z/(-\delta)\} + 0.451188 = {}_tp_x + 0.451188 = e^{-0.06t} + 0.451188$，其中 $t = \ln z/(-\delta)$

iii. $v^{10} < z$

$F_Z(z) = 1$

综上

$$F_Z(z) = \begin{cases} 0.451188, & z = 0 \\ e^{-0.06t} + 0.451188, & 0 < z \le v^{10} \\ 1, & v^{10} < z \end{cases}$$

$F_Z^{-1}(0.9) = 0.263092\ (t = 13.352520)$

10.

$$(\bar{I}\bar{A})_x = \int_0^\infty t v^t\ {}_tp_x\ \mu_{x+t}\ dt = \int_0^\infty t e^{-\delta t}\ \mu e^{-\mu t}\ dt = \frac{\mu}{(\delta+\mu)^2}$$

$${}^2\bar{A}_x = \int_0^\infty (v^t)^2\ {}_tp_x\ \mu_{x+t}\ dt = \int_0^\infty e^{-2\delta t}\ \mu e^{-\mu t}\ dt = \frac{\mu}{2\delta+\mu}$$

因为 ${}^2\bar{A}_x = 0.25$，解得 $\mu = 0.04$，所以 $(\bar{I}\bar{A})_x = 4$.

11.

$$F_Z(z) = P\{Z \leq z\} = P\{v^T \leq z\} = P\{T \geq \ln z/(-\delta)\}$$
$$= {}_tp_x = (50-t)/50$$

其中 $t = \ln z/(-\delta)$. $F_Z^{-1}(0.6) = 0.449329$ $(t = 20)$

12. 从下图可以看到 $(IA)_x - A^1_{x:\overline{1}|}$ 和 $(IA)_{x+1} + A_{x+1}$ 对应的现金流相同，只不过前者折现到 x，后者折现到 $x+1$. 那么 $(IA)_x - A^1_{x:\overline{1}|} = A_{x:\overset{1}{\overline{1}|}}\,[(IA)_{x+1} + A_{x+1}]$，所以 $[(IA)_x - A^1_{x:\overline{1}|}]/[(IA)_{x+1} + A_{x+1}] = A_{x:\overset{1}{\overline{1}|}}$.

$(IA)_{x+1}$			1	2	3	4	5	6	7	8	$\cdots$
A_{x+1}			1	1	1	1	1	1	1	1	$\cdots$
$A^1_{x:\overline{1}|}$		1									
$(IA)_x$		1	2	3	4	5	6	7	8	9	$\cdots$
(x)	0	1	2	3	4	5	6	7	8	9	10

13. $(DA)^1_{x:\overline{5}|} = 6A^1_{x:\overline{5}|} - (IA)^1_{x:\overline{5}|} = 0.022081$

其中 $(IA)^1_{x:\overline{5}|} = (IA)_x - {}_5E_x\ [5A_{x+5} + (IA)_{x+5}] = 0.024809$，$A_{x:\overline{5}|} = A^1_{x:\overline{5}|} + A_{x:\overset{1}{\overline{5}|}} \implies A^1_{x:\overline{5}|} = 0.007815$

其中 ${}_5E_x = A_{x:\overset{1}{\overline{5}|}} = v^5\ {}_5p_x = 0.752185$

14. $(IA)^1_{40:\overline{10}|} = (IA)_{40} - [20\ {}_{20|}A_{40} + {}_{20|}(IA)_{40}]$

$(DA)^1_{x:\overline{5}|} = 21A^1_{40:\overline{20}|} - (IA)^1_{40:\overline{20}|} = 0.64531$

其中 $A^1_{40:\overline{20}|} = A_{40} - {}_{20|}A_{40} = 0.10441$.

又 $a \times (IA)^1_{40:\overline{20}|} = 100 \implies a = 64.63$，$b \times (DA)^1_{40:\overline{20}|} = 100 \implies b = 154.96$. 所以 $64.63x < 154.96(20-x) \implies x < 14.1136$，即 14 年.

15. 因为 $Z_3 = Z_1 + Z_2$，所以 $E[Z_3] = E[Z_1] + E[Z_2]$，$Var[Z_3] = Var[Z_1] + Var[Z_2] + 2Cov[Z_1, Z_2]$. 已知 $E[Z_2]$ 和 $Var[Z_2]$，需要求 $E[Z_1]$、$Var[Z_1]$

和 $Cov[Z_1, Z_2]$.

$$E[Z_1] = \int_0^{10} v^t \ {}_tp_x \ \mu_{x+t} \ dt = \int_0^{10} e^{-0.05t} \ 0.01e^{-0.01t} \ dt = 0.078694$$
$$Var[Z_1] = \int_0^{10} (v^t)^2 \ {}_tp_x \ \mu_{x+t} \ dt = \int_0^{10} e^{-0.1t} \ 0.01e^{-0.01t} \ dt = 0.078694$$

$Cov[Z_1, Z_2] = E[Z_1 Z_2] - E[Z_1]E[Z_2] = -0.027543$

其中 $E[Z_1 Z_2] = 0$. 那么 $E[Z_3]0.428694$，$Var[Z_3] = 0.234658$.

16. $F_Z(z) = P\{Z \le z\} = P\{(40-K)v^T \le z\} =$

17. $Z = Tv^T = Te^{-0.05T}$，$Z' = e^{-0.05T} - 0.05Te^{-0.05} = 0 \implies T = 20$，$Z$ 的最大值为 7.35758.

    $Z'' = -0.05e^{-0.05T} - 0.05(e^{-0.05T} - 0.05Te^{-0.05}) = -0.1e^{-0.05T} + 0.025Te^{-0.05} = 0 \implies T = 4$，证明 Z 确实在 $T = 20$ 时取最大值.

18. $20000A^1_{25:\overline{40}|} + PA_{20:\overset{1}{\overline{40}|}} + 10000_{40|}A_{65} = P \implies P = 2000.$

    其中 $A_{20:\overset{1}{\overline{40}|}} = v^{40} \ {}_{40}p_{25} = 0.16$

    $A^1_{25:\overline{40}|} = A_{25} - A_{20:\overset{1}{\overline{40}|}}A_{65} = 0.068$

    ${}_{40|}A_{65} = A_{20:\overset{1}{\overline{40}|}}A_{65} = 0.032$

19.
$$\begin{array}{ll} Z_1 = Tv^T & Z_2 = (K+1)v^{K+1} \\ v^{1/2}Z_1 = Tv^{T+1/2} & Z_2 \approx (T+1/2)v^{T+1/2} \end{array}$$

    那么 $v^T = 2/v^{1/2}(Z_2 - v^{1/2}Z_1)$，则 $E[v^T] = 2/v^{1/2}(E[Z_2] - v^{1/2}E[Z_1])$，即 $\bar{A}_x = 2/v^{1/2}[(IA)_x - v^{1/2}(\bar{I}\bar{A})_x] = 0.350730$

20.

$$A^1_{45:\overline{10}|} = \sum_{k=0}^{9} v^{k+1} \ {}_{k|}q_{45} = vq_{45} + \sum_{k=1}^{9} v^{k+1} \ {}_kp_{45} \ q_{45+k} = vq_{45} + \sum_{k=1}^{9} v^{k+1} \ {}_{45}p \ {}_{k-1}p_{46} \ q_{x+k}$$
$$A'^1_{45:\overline{10}|} = \sum_{k=0}^{9} v^{k+1} \ {}_{k|}q'_{45} = vq'_{45} + \sum_{k=1}^{9} v^{k+1} \ {}_kp'_{45} \ q_{45+k} = vq_{45} + \sum_{k=1}^{9} v^{k+1} \ p'_{45} \ {}_{k-1}p_{46} \ q_{x+k}$$

所以 $A'^1_{45:\overline{10}|} = 0.95 \times 0.02 + 0.98/0.99 \times 0.1405 = 0.158081$

21.

$$\begin{aligned} P\{Z>0.25\} &= P\{e^{-\delta T}>0.25\} = P\{T<34.66\} \\ &= \int_0^{34.66} {}_tp_x\ \mu_{x+t}\ dt = \int_0^{10} 0.01e^{-0.01t}\ dt + \int_{10}^{34.66} 0.02e^{-0.02t+0.1}\ dt = 0.447444 \end{aligned}$$

22. ${}_{2|2}A_{[60]} = {}_2E_{[60]}\,A_{[60]+2:\overline{2}|}^{\ 1} = 0.190258$

其中 ${}_2E_{[60]} = v^2\ {}_2p_{[60]} = 0.763408$

$A_{[60]+2:\overline{2}|}^{\ 1} = v\ q_{[60]+2} + v^2\ p_{[60]+2}\ q_{63} = 0.249222$

23.

$$\begin{aligned} E[Z_1] &= 1000(v\ q_{49} + v^2\ p_{49}\ q_{50}) = 27.055892 \\ E[Z_2] &= 1000(v\ q_{49}^{'} + v^2\ p_{49}^{'}\ q_{50}^{'}) = 45.033820 \\ E[Z_1^2] &= 1000^2(v^2\ q_{49} + v^4\ p_{49}\ q_{50}) = 24583.41893 \\ E[Z_2^2] &= 1000^2(v^2\ q_{49}^{'} + v^4\ p_{49}^{'}\ q_{50}^{'}) = 41087.4825 \\ Var[Z_1] &= E[Z_1^2] - (E[Z_1])^2 = 23851.39764 \\ Var[Z_2] &= E[Z_2^2] - (E[Z_2])^2 = 39059.43756 \\ E[Z] &= E(E[Z_i \mid I]) = 30.651478 \\ Var[Z] &= E(Var[Z_i \mid I]) + Var(E[Z_i \mid I]) = 26944.71857 \end{aligned}$$

其中 $I = \begin{pmatrix} 1 & 2 \\ 0.8 & 0.2 \end{pmatrix}$.

初始金额 $= 1.649\times\sqrt{100\times 26944.71857} + 100\times 30.651478 = 5771.955988$

24. $Var[Z] = {}^2A_{51} - (A_{51})^2 = 0.054849$

其中 $A_{51} - A_{50} = A_{51} - (v\ q_{50} + v\ p_{50}\ A_{51}) = 0.004 \implies A_{51} = 0.601999$

${}^2A_{51} - {}^2A_{50} = {}^2A_{51} - (v^2\ q_{50} + v^2\ p_{50}\ {}^2A_{51}) = 0.004 \implies {}^2A_{51} = 0.417252$