(1).

为方便推导，这里做一些方便的记法,

在无摩擦的情况下任何债券组合都可以看做面值为1的零息债券的线性组合，计到期时间为T_i的这样的债券在t时刻的价格为$p(t,T_i)$,那么债券组合在s时刻的价值为

$$V_t=\sum_{i=1}^{d}\lambda_i p(t\Delta,T_i)$$

另外我们记$y(s,T_i)$为T_i到期的债券的收益率并作为风险因子， 则

$$p(s,T_i)=e^{-(T_i-s)(y(s,T_i))}$$
$$X_{t+1,i}=\Delta y(s,T_i)$$

因此有

$$V_t=\sum_{i=1}^{d}\lambda_i p(t\Delta,T_i)=\sum_{i=1}^{d}\lambda_i e^{-(T_i-t\Delta)(y(t\Delta,T_i))}$$

求导做一阶近似得

$$L_{t+1}^{\Delta}=\sum_{i=1}^{d}\lambda_i e^{-(T_i-t\Delta)(y(t\Delta,T_i))}(y(t\Delta,T_i)\Delta-(T_i-t\Delta)X_{t+1,i})$$

In [1]:

```
import pandas as pd
import numpy as np
import os
```

In [2]:

```
def get_data(comb_stock_codes):

    raw_data_list = []
    for r,d,fs in os.walk('../data/csmar_close'):
        for f in fs:
            if (not ('[DES]' in f)) and ('txt' in f):
                file_path = os.path.join(r,f)
                #print(file_path)
                raw_data_list.append(pd.read_csv(file_path,
                            sep='\t',
                            dtype={'Stkcd': str})[['Trddt','Stkcd','Adjprcwd']])
    working_data  =  pd.concat(raw_data_list,axis=0).set_index(['Trddt','Stkcd']
    workding_df = working_data.unstack('Stkcd')
    df = workding_df.T.dropna().T
    df.columns = df.columns.get_level_values(1)

    return df[comb_stock_codes]
```

In [ ]:

In [3]:

```
comb_stock_codes = ['000001','002330','000607','000021','600448',
                    '688233','000901','600722','900948','601208']
```

In [4]:

```
price_df = get_data(comb_stock_codes)
price_df.index = pd.to_datetime(price_df.index)
```

In [5]:

```
price_df
```

Out[5]:

Stkcd	000001	002330	000607	000021	600448	688233	00
Trddt							
2020-09-16	1646.285839	14.443544	15.971281	307.211266	9.074784	49.345117	73.49
2020-09-17	1660.147054	14.156722	15.933163	311.504821	9.052917	51.380277	72.84
2020-09-18	1713.459419	14.279646	16.047516	317.130859	9.031050	51.129642	74.14
2020-09-21	1691.068226	14.218184	16.199986	319.647771	9.140385	50.026846	75.90
2020-09-22	1660.147054	13.951849	15.818810	318.611396	9.009183	47.460338	73.36
...	...	...	...	...	...	...	
2025-09-09	1483.463745	10.015122	18.618745	310.673487	7.609705	32.888059	104.55
2025-09-10	1485.988790	10.200587	18.541808	307.098955	7.850242	32.735328	105.27
2025-09-11	1496.088969	10.283016	18.310997	322.795815	7.784641	34.354276	107.30
2025-09-12	1479.676179	10.200587	18.464871	338.803503	7.675306	35.494667	106.71
2025-09-15	1470.838523	11.230949	17.926312	340.046819	7.828375	35.179023	105.53

1212 rows × 10 columns

In [6]:

```
return_df = np.log(price_df.shift(-1)/price_df)
```

In [7]:

```
return_df
```

Out[7]:

Stkcd	000001	002330	000607	000021	600448	688233	000901
Trddt							
2020-09-16	0.008384	-0.020058	-0.002390	0.013879	-0.002413	0.040416	-0.008913
2020-09-17	0.031608	0.008646	0.007151	0.017900	-0.002418	-0.004890	0.017747
2020-09-18	-0.013154	-0.004313	0.009456	0.007905	0.012034	-0.021805	0.023469
2020-09-21	-0.018454	-0.018910	-0.023811	-0.003248	-0.014458	-0.052665	-0.034079
2020-09-22	0.003846	0.005857	0.000000	0.026142	0.019231	0.019455	0.003549
...	...	...	...	...	...	...	...
2025-09-09	0.001701	0.018349	-0.004141	-0.011572	0.031120	-0.004655	0.006869
2025-09-10	0.006774	0.008048	-0.012526	0.049850	-0.008392	0.048272	0.019107
2025-09-11	-0.011031	-0.008048	0.008368	0.048400	-0.014145	0.032656	-0.005510
2025-09-12	-0.005991	0.096228	-0.029600	0.003663	0.019747	-0.008932	-0.011111
2025-09-15	NaN	NaN	NaN	NaN	NaN	NaN	NaN

1212 rows × 10 columns

In [8]:

```
return_df.cov()
```

Out[8]:

Stkcd	000001	002330	000607	000021	600448	688233	000901	60
Stkcd								
000001	0.000339	0.000089	0.000074	0.000070	0.000052	0.000049	0.000065	0.00
002330	0.000089	0.000646	0.000283	0.000172	0.000239	0.000212	0.000186	0.00
000607	0.000074	0.000283	0.000870	0.000264	0.000291	0.000318	0.000251	0.00
000021	0.000070	0.000172	0.000264	0.000751	0.000154	0.000501	0.000287	0.00
600448	0.000052	0.000239	0.000291	0.000154	0.000643	0.000249	0.000167	0.00
688233	0.000049	0.000212	0.000318	0.000501	0.000249	0.001436	0.000345	0.00
000901	0.000065	0.000186	0.000251	0.000287	0.000167	0.000345	0.000639	0.00
600722	0.000105	0.000220	0.000252	0.000181	0.000255	0.000268	0.000250	0.00
900948	0.000088	0.000082	0.000084	0.000075	0.000080	0.000072	0.000106	0.00
601208	0.000068	0.000156	0.000184	0.000313	0.000160	0.000441	0.000269	0.00

In [9]:

```
return_df.mean()
```

Out[9]:

```
Stkcd
000001   -0.000093
002330   -0.000208
000607    0.000095
000021    0.000084
600448   -0.000122
688233   -0.000279
000901    0.000299
600722    0.000450
900948    0.001279
601208    0.000992
dtype: float64
```

(a)若假设X_t服从正态，记上述方差协方差矩阵为Σ,均值μ,联合分布为$N(\mu,\Sigma)$

(b) 给出组合 $b=(1/10,\ \ 1/10,\ \ 1/10,\ \ \ldots,1/10)'$ 由 则 $W_t \sim N(-V_t b'\mu, {V_t}^2 b'\Sigma b)$

(c)随机模拟结果如下

In [10]:

```
b = np.array([1/10 for i in range(10)])
loss_df = ((price_df.shift(-1)-price_df)*b).sum(axis=1)
sim_rounds =  1000
sim_list = []
for i in range(sim_rounds):


    idx = np.random.randint(len(loss_df))

    sim_list.append(loss_df.iloc[idx])
```

In [11]:

```
sim_df = pd.Series(sim_list)
```

In [12]:

```
print(f'估计均值为{sim_df.mean():.5f}')
print(f'估计方差为{sim_df.var():.5f}')
```

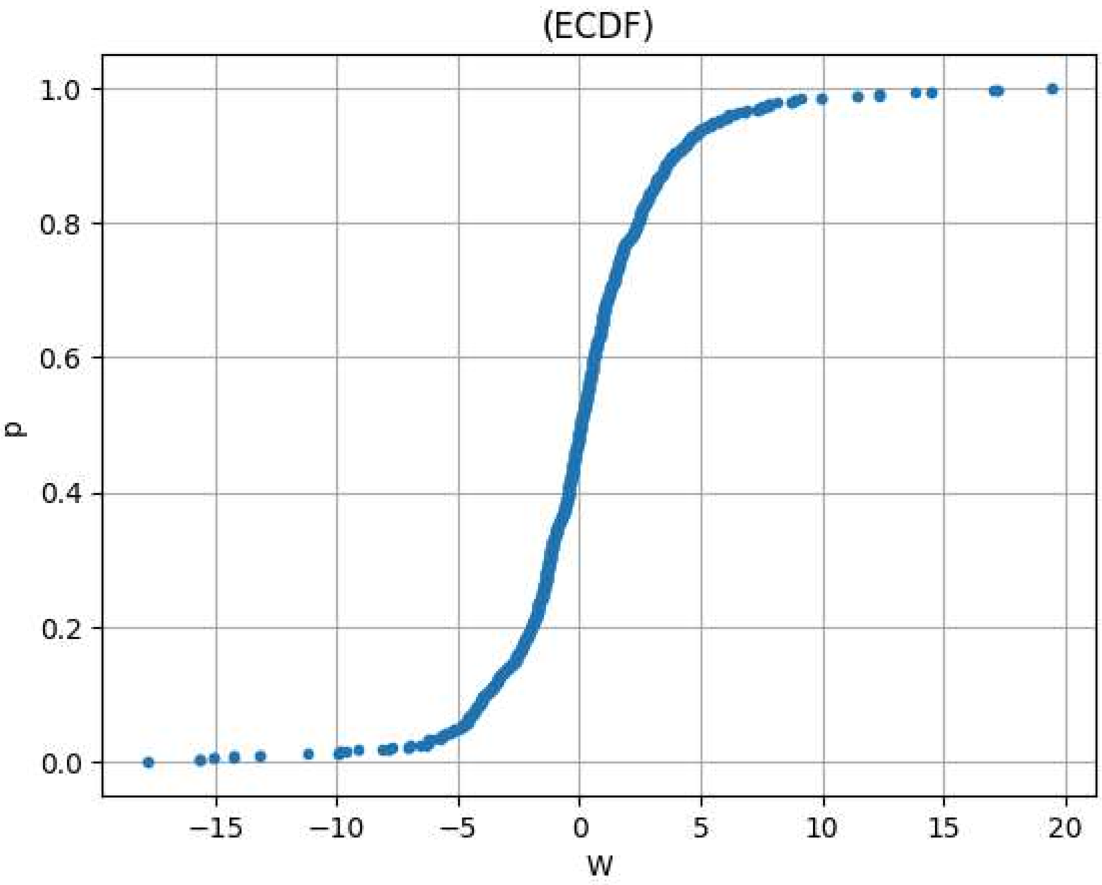
```
估计均值为0.12623
估计方差为14.11890
```

In [13]:

```
import matplotlib.pyplot as plt

w_values = sim_df.sort_values()
# 计算累积概率
cumulative_prob = np.arange(1, len(w_values) + 1) / len(w_values)

# 绘制经验分布图
plt.plot(w_values, cumulative_prob, marker='.', linestyle='none')
plt.title('(ECDF)')
plt.xlabel('W')
plt.ylabel('p')
plt.grid(True)
plt.show()
```



In [ ]:

若改为月度的，重采样后得到结果

In [14]:

```
month_price_df = price_df.resample('M').last()
```

```
/var/folders/qb/_zmdxt6n29n5h6hxbt7v0v0c0000gn/T/ipykernel_9360/2731593369.py:1:
FutureWarning: 'M' is deprecated and will be removed in a future version, please
use 'ME' instead.
  month_price_df = price_df.resample('M').last()
```

In [15]:
```
month_price_df
```

Out[15]:

Stkcd	000001	002330	000607	000021	600448	688233	00
Trddt							
2020-09-30	1617.497162	13.869900	15.285163	305.582676	8.549977	45.916424	70.55
2020-10-31	1892.588966	13.624052	15.018341	313.725626	8.703046	45.034186	63.97
2020-11-30	2104.772177	14.423057	16.276224	304.546303	8.593710	44.803602	68.14
2020-12-31	2062.122283	13.746977	15.018345	281.449936	8.025169	43.831137	65.53
2021-01-31	2461.965023	12.087507	14.217875	306.174891	6.997423	43.921366	60.12
...	...	...	...	...	...	...	
2025-05-31	1415.602597	9.355690	15.079645	265.645520	7.872105	26.654952	77.43
2025-06-30	1523.864458	9.438120	17.580096	291.246683	7.631571	32.348407	81.43
2025-07-31	1544.064815	9.685407	18.003250	288.760050	7.237966	32.765872	86.01
2025-08-31	1521.339415	10.015123	17.003070	339.425161	7.303567	37.225616	130.17
2025-09-30	1470.838523	11.230949	17.926312	340.046819	7.828375	35.179023	105.53

61 rows × 10 columns

In [16]:
```
month_return_df = np.log(month_price_df.shift(-1)/month_price_df)
```

In [17]:
```
#协方差矩阵为
month_return_df.cov()
```

Out[17]:

Stkcd	000001	002330	000607	000021	600448	688233	000901
Stkcd							
000001	0.007001	0.001637	0.001573	0.000386	0.001033	-0.000957	0.000277
002330	0.001637	0.009440	0.004554	-0.000041	0.003944	0.002521	0.001600
000607	0.001573	0.004554	0.008369	0.003209	0.003935	0.005032	0.002684
000021	0.000386	-0.000041	0.003209	0.013858	0.000975	0.009923	0.006311
600448	0.001033	0.003944	0.003935	0.000975	0.007956	0.003732	0.003701
688233	-0.000957	0.002521	0.005032	0.009923	0.003732	0.025177	0.009116
000901	0.000277	0.001600	0.002684	0.006311	0.003701	0.009116	0.011745
600722	-0.000380	0.003310	0.005499	0.002870	0.005046	0.005516	0.004285
900948	0.000404	-0.001604	-0.001139	-0.002047	0.000311	-0.002894	-0.000258
601208	0.001978	0.001680	0.003306	0.005333	0.002721	0.009977	0.006277

In [18]:

```
#均值向量为
month_return_df.mean()
```

Out[18]:

```
Stkcd
000001   -0.001584
002330   -0.003517
000607    0.002656
000021    0.001781
600448   -0.001470
688233   -0.004440
000901    0.006711
600722    0.010571
900948    0.024885
601208    0.019777
dtype: float64
```

In [19]:

```
#随机模拟结果如下
b = np.array([1/10 for i in range(10)])
month_loss_df = ((month_price_df.shift(-1)-month_price_df)*b).sum(axis=1)
sim_rounds =  5000
sim_list = []
for i in range(sim_rounds):


    idx = np.random.randint(len(month_loss_df))

    sim_list.append(month_loss_df.iloc[idx])

```

In [20]:

```
month_sim_df = pd.Series(sim_list)
```

In [21]:

```
print(f'月度估计均值为{month_sim_df.mean():.5f}')
print(f'月度估计方差为{month_sim_df.var():.5f}')
```

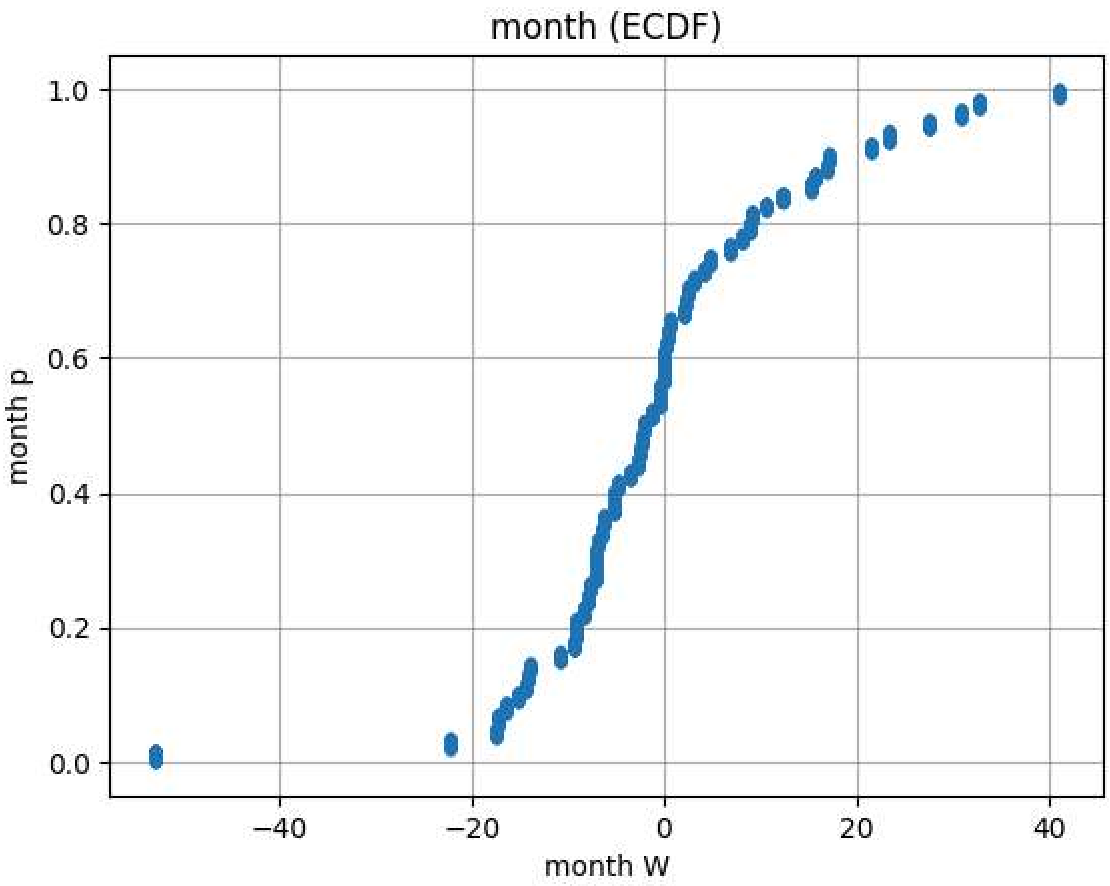
```
月度估计均值为-0.47998
月度估计方差为213.97543
```

In [25]:

```
import matplotlib.pyplot as plt

w_values = month_sim_df.sort_values()
# 计算累积概率
cumulative_prob = np.arange(1, len(w_values) + 1) / len(w_values)

# 绘制经验分布图
plt.plot(w_values, cumulative_prob, marker='.', linestyle='none')
plt.title('month (ECDF)')
plt.xlabel('month W')
plt.ylabel('month p')
plt.grid(True)
plt.show()
```



In [ ]: