You are attending a football game and you are in possession of a sophisticated device that enables you to estimate the coordinates (position and height) of the football as a function of time, i.e., you have a smartphone. With just 3 s in the game, your team has just enough time to attempt a field goal from around the 40 yd line. Scoring would mean victory. You are video recording the free kick on your phone, which you cannot keep steady given all the commotion around you. There is also quite a bit of smoke nearby (don't ask!) and therefore you cannot observe the football at all times.

Your kicker's shot looks promising, i.e., the football seems to be on the right trajectory for a field goal. Unfortunately, while the football is still in the air, electrical power goes down and the whole stadium plunges in complete darkness. Nobody can see the football and when power returns 1 min later everybody is clueless if your kicker scored.

Everybody but you, that is. Since you have taken INVERSION, you decide to solve the mystery. You have the video of the football and you can estimate its x_i and z_i position as a function time t_i, for $i = 1 \dots N$. You know that your estimates are imperfect (you take your movie with a hand-held phone after all), and you estimate from the video the uncertainty of your measurement in both horizontal and vertical directions. If you find the launch coordinates x_0 and z_0, speed v and angle θ relative to the horizontal you can predict the trajectory of the football and determine whether your kicker actually scored.

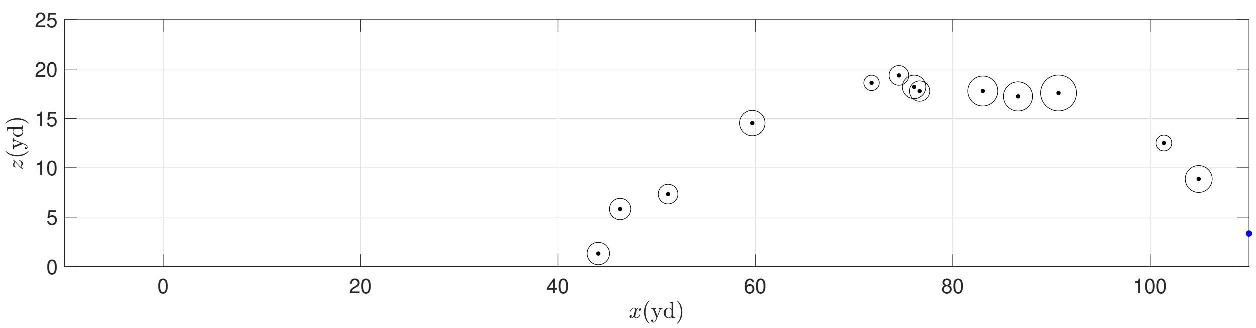

Figure 1: Football coordinates with uncertainties. The crossbar is indicated in blue.

You know that the forces acting on the ball are gravity (although the game is played somewhere in the Rockies and you do not know exactly the value of the gravitational acceleration g), and drag caused by air (although you do not know how strong this force is for a football), i.e., you do not know the air drag coefficient k. You vaguely recall that a football has a mass m of about 0.4 kg. You decide to assume that the gravitational acceleration is constant, since the football does not fly more than 25 yd above the ground and 70 yd toward the goal. Find all the parameters (x_0, z_0, v, θ, g and k) from your observations of the football by solving an INVERSE PROBLEM.

Extra credit: Formulate the problem in 3D, i.e., assume that the player kicks the ball at some distance away from the field center. Show how to find the 3D launch parameters, including the azimuth at which the football was kicked.

N.B. This is an individual assignment – your work is subject to the Mines Academic Integrity policy.

INSTRUCTIONS

FORMAT

- Submit the assignment to Canvas as a standalone **Jupyter notebook**.
- Make sure to run **Kernel/Restart & Run All** in Jupyter before submission.

CLARITY

- Include text documenting your reasoning and how you approached the solution.
- Show all intermediate mathematical derivation steps, if applicable.
- Include figures demonstrating the solution and explain their meaning.

PROGRAMMING

- Include detailed comments documenting the functionality of your codes.
- Organize your programs in clear functional blocks.
- Isolate repeated code in functions. Provide unit tests for all defined functions.
- Define and initialize all variables; indicate in comments their physical units.

POLICIES

- Incomplete or incorrect answers receive partial credit at the discretion of the grader.
- Submissions lose 25%/day if late for two days and are not graded afterward.
- Multiple submissions to Canvas are allowed, but only the last one is graded.

GRADING RUBRIC

Trajectory - 80 pts

- Formulate the INVERSE PROBLEMand justify all assumptions. (30 pts)
- Define model parameters. (5 pts)
- Define data parameters. (5 pts)
- Define the operator linking models and data. (10 pts)
- What are the values of x_0, z_0, v, θ, k and g? (10 pts)
- Plot of the data points with their uncertainty and the trajectory predicted from the model. (10 pts)
- Did your player score? Justify your answer. (10 pts)

Code - 20 pts

Include all codes used with comments to explain their functionality.

Extra credit - 25 pts

Formulate the 3D INVERSE PROBLEM and show how you could solve for all the launch parameters.