Part 1

Consider the function

$$f\left(x,y\right) = yx^2 - xy^2 \ , \tag{1}$$

where $x \in [1,2]$ and $y \in [0,1]$.

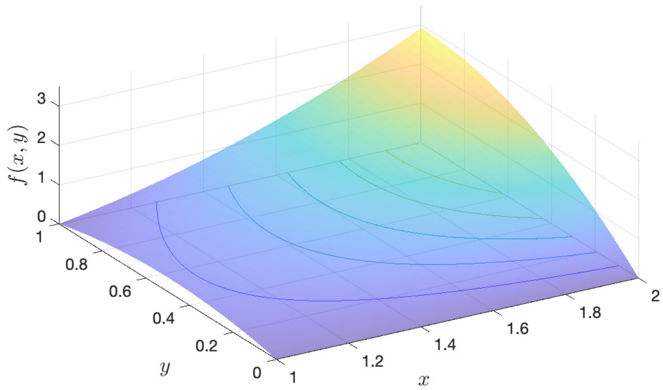

Figure 1: probability density function derived from the function $f\left(x,y\right)$, equation 1.

- Can this function be a probability density function? Motivate your answer.
- If not, modify the function so it can be used as a probability density function on the given interval.
- Derive the expression for the marginal probability density functions $f_X\left(x\right)$ and $f_Y\left(y\right)$. Plot the marginal PDFs.
- Derive the expression for the conditional probability density function $f\left(x|\overline{y}=\frac{1}{4}\right)$. Plot the conditional PDF.
- If $f\left(x,y\right)$ is a probability density function over the space $\{x,y\}$, what are the probabilities of event $\mathcal{A}$ corresponding to $x \in \left[1,\frac{3}{2}\right]$ and $y \in \left[0,\frac{1}{2}\right]$ and $\mathcal{B}$ corresponding to $x \in \left[\frac{3}{2},2\right]$ and $y \in \left[\frac{1}{2},1\right]$?

Part 2

Consider the following function:

$$f(x,y,z) = 1 + \sin(5\left(x^2 + y^2 + z^2\right)) \ , \tag{2}$$

where

$$\begin{aligned} x &\in [-\pi/2, +\pi/2] \ , \\ y &\in [-\pi/4, +\pi/4] \ , \\ z &\in [-\pi/3, +\pi/3] \ . \end{aligned}$$

Is this function usable as probability density functions? If not, scale it such that it can be used as a probability density function, and plot it as in Figure 2.

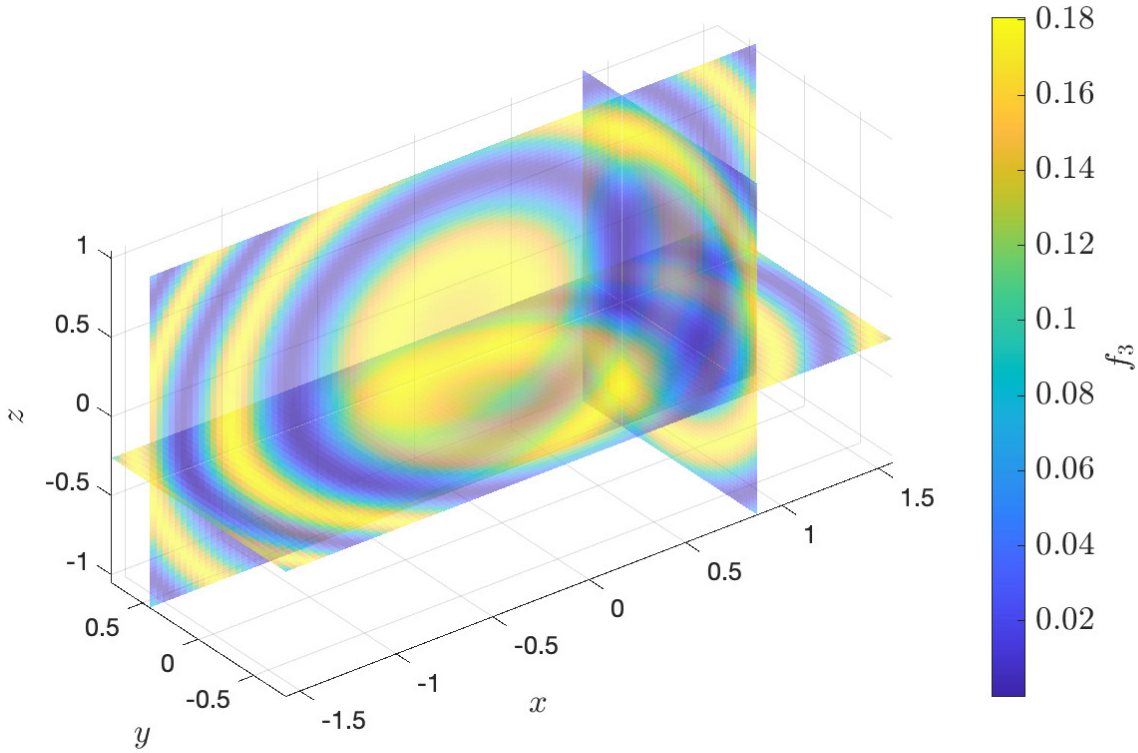

Figure 2: 3D PDF.

Using the 3D PDF defined earlier, compute and plot the marginal probability density functions

$$f_{XY}(x,y) \ , \tag{3}$$

and

$$f_Z(z) \ . \tag{4}$$

Extra credit: Consider the following function:

$$f(x,y,z,u,v) = 1 + \sin(5\left(x^2 + y^2 + z^2 + u^2 + v^2\right)) \ , \tag{5}$$

where

$$
\begin{aligned}
x &\in [-\pi/2, +\pi/2] \ , \\
y &\in [-\pi/4, +\pi/4] \ , \\
z &\in [-\pi/3, +\pi/3] \ , \\
u &\in [-\pi/4, +\pi/4] \ , \\
v &\in [-\pi/2, +\pi/2] \ .
\end{aligned}
$$

- Is this function usable as probability density functions? If not, scale it such that it can be used as a PDF.
- How many marginal PDFs could we define for this function?
- Plot the marginal PDFs $f_{XZV}(x, z, v)$ and $f_{YU}(y, u)$.

N.B. This is an individual assignment – your work is subject to the Mines Academic Integrity policy.

INSTRUCTIONS

Format

- Submit the assignment to Canvas as a standalone **Jupyter notebook**.
- Make sure to run **Kernel/Restart & Run All** in Jupyter before submission.

Clarity

- Include text documenting your reasoning and how you approached the solution.
- Show all intermediate mathematical derivation steps, if applicable.
- Include figures demonstrating the solution and explain their meaning.

Programming

- Include detailed comments documenting the functionality of your codes.
- Organize your programs in clear functional blocks.
- Isolate repeated code in functions. Provide unit tests for all defined functions.
- Define and initialize all variables; indicate in comments their physical units.

Policies

- Incomplete or incorrect answers receive partial credit at the discretion of the grader.
- Submissions lose 25%/day if late for two days and are not graded afterward.
- Multiple submissions to Canvas are allowed, but only the last one is graded.

GRADING RUBRIC

Part 1 - 40 pts

- Can $f\left(x,y\right)$ be used as a PDF? Explain. (5 pts)
- Modify $f\left(x,y\right)$ to become a PDF. (5 pts)
- Derive the marginal PDF s $f_X(x)$ and $f_Y(y)$ and plot them. (10 pts)
- Derive the expression for the conditional PDF $f\left(x|\overline{y}=\frac{1}{4}\right)$ and plot it. (10 pts)
- Compute the probabilities of events A and B. (10 pts)

Part 2 - 40 pts

- Can the function $f\left(x,y,z\right)$ be used as a PDF? (5 pts)
- If not, modify it to become a PDF. (5 pts)
- Plot the 3D joint PDF $f\left(x,y,z\right)$. (10 pts)
- Compute and plot the 2D marginal PDF $f_{XY}\left(x,y\right)$. (10 pts)
- Compute and plot the 1D marginal PDF $f_Z\left(z\right)$. (10 pts)

Code - 20 pts

Include all codes used with comments to explain their functionality.

Extra credit - 30 pts

- Analyze the 5D PDF and ensure it can be used as a PDF. (10 pts)
- Determine the number of possible marginal PDFs. Motivate your answer. (10 pts)
- Compute and plot the 3D marginal PDF $f_{XZV}\left(x,z,v\right)$. (5 pts)
- Compute and plot the 2D marginal PDF $f_{YU}\left(y,u\right)$. (5 pts)