Consider the function

$$f(x) = \sqcap\left(\frac{x - x_a}{a}\right) + \sqcap\left(\frac{x - x_b}{b}\right) \tag{1}$$

where $\sqcap(x)$ is the boxcar function defined by

$$\sqcap(x) = \begin{cases} 0 & |x| > \frac{1}{2} \\ \frac{1}{2} & |x| = \frac{1}{2} \\ 1 & |x| < \frac{1}{2} \end{cases}, \tag{2}$$

and x_a, x_b, a and b are known positive real numbers.

1. Sketch the function $f(x)$.
2. Is $f(x)$ a probability density function? If yes, justify your answer. If not, scale it to become a PDF.
3. If $x_a \neq x_b$, find the mean of the probability density function. What condition must a and b satisfy such that the mean of the probability density function is halfway between x_a and x_b? Justify your answer.
4. If $x_a = x_b$ and $a \neq b$, what is the standard deviation of the probability density function?

Extra credit: Consider the function

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma_a} e^{-\frac{1}{2}\left(\frac{x - x_a}{\sigma_a}\right)^2} + \frac{1}{\sqrt{2\pi}\sigma_b} e^{-\frac{1}{2}\left(\frac{x - x_b}{\sigma_b}\right)^2}. \tag{3}$$

1. Sketch the function $g(x)$.
2. Is $g(x)$ a probability density function? If yes, justify your answer. If not, scale it to become a PDF.
3. Compute the mean of $g(x)$.
4. Compute the standard deviation of $g(x)$ when $x_a = x_o$ and $x_b = -x_o$ with $x_o \neq 0$ and $\sigma_a = \sigma_b = \sigma_o$ with $\sigma_o \neq 0$.

N.B. This is an individual assignment – your work is subject to the Mines Academic Integrity policy.

INSTRUCTIONS

FORMAT

- Submit the assignment to Canvas as a standalone **Jupyter notebook**.
- Make sure to run **Kernel/Restart & Run All** in Jupyter before submission.

CLARITY

- Include text documenting your reasoning and how you approached the solution.
- Show all intermediate mathematical derivation steps, if applicable.
- Include figures demonstrating the solution and explain their meaning.

PROGRAMMING

- Include detailed comments documenting the functionality of your codes.
- Organize your programs in clear functional blocks.
- Isolate repeated code in functions. Provide unit tests for all defined functions.
- Define and initialize all variables; indicate in comments their physical units.

POLICIES

- Incomplete or incorrect answers receive partial credit at the discretion of the grader.
- Submissions lose 25%/day if late for two days and are not graded afterward.
- Multiple submissions to Canvas are allowed, but only the last one is graded.

GRADING RUBRIC

Boxcar problem - 100 pts

- Sketch the function $f(x)$. (20 pts)
  Hint: Is a single sketch representative of all cases? If no, draw multiple sketches and properly label the axes.
- Is $f(x)$ a probability density function? If yes, justify your answer. If not, transform it into a PDF. (20 pts)
- If $x_a \neq x_b$, find the mean of the probability density function. (20 pts)
- What condition must a and b satisfy such that the mean of the probability density function is halfway between x_a and x_b? Justify your answer. (20 pts)
- If $x_a = x_b$ and $a \neq b$, what is the standard deviation of the probability density function? (20 pts)

Gaussian problem - 50 pts

- Sketch the function $g(x)$. (5 pts)
- Is $g(x)$ a probability density function? If yes, justify your answer. If not, transform it into a PDF. (5 pts)
- Find the mean of the probability density function $g(x)$. (20 pts)
- Find the standard deviation of the probability density function $g(x)$? (20 pts)