You are investigating the lithology in a carbonate formation using several well logs measured at 0.5 ft intervals, Figure 1. The well logs are:

- **Gamma ray log** (GR) – can be used to evaluate the amount of shale mixed with the carbonates;
- **Caliper log** (CALI) – can be used to characterize the condition of the borehole (the nominal value 6 in);
- **Porosity log** (ϕ) – an indirect measurement of the rock porosity;
- **Density log** (δ) – an indirect measurement of the rock density;

Your goal is to use all these data and Bayesian Inversion to construct the probability density function of P-wave velocity in the borehole, and then to find the most likely value of P-velocity as well as the uncertainty associated with this value at every depth. You know that velocity, porosity and density are not indepdendnt, as discussed next.

Wyllie

You know that the velocity is related to the porosity using the Wyllie equation

$$\frac{1}{v} = \frac{1-\phi}{v_M} + \frac{\phi}{v_F} \ , \tag{1}$$

where v_M and v_F are the matrix and fluid velocities, respectively, and that the density is linearly related to porosity by the expression

$$\delta = (1-\phi)\,\delta_M + \phi\delta_F \ , \tag{2}$$

where δ_M and δ_F are the matrix and fluid densities, respectively.

Gardner

The density is related to the velocity using the Gardner equation

$$\delta = 1.74 v^{0.25} \ , \tag{3}$$

where v and δ are given in km/s and g/cm^3, and that the density is also linearly related to porosity by the expression

$$\delta = (1-\phi)\,\delta_M + \phi\delta_F \ , \tag{4}$$

where δ_M and δ_F are the matrix and fluid densities, respectively.

Use the following constants, as needed:

$$\begin{aligned}
\delta_F &= 1.00 \ g/cm^3 \\
\delta_M &= 2.71 \ g/cm^3 \\
v_F &= 1.50 \ km/s \\
v_M &= 6.64 \ km/s
\end{aligned}$$

Use the Wyllie equation if your CWID is an even number, or the Gardner equation otherwise.

1. Construct the prior joint probability density functionbased on the observed values of ϕ and δ and for a P-wave velocity related to the GR log by the empirical relation

   $$v = 5.654 - 0.008\,GR \ , \tag{5}$$

   Specify what distributions you are using and justify your choice of parameters. Assume that a-priori all variables are independent.

2. Construct the theoretical joint probability density function assuming uncertainty relative to the theoretical prediction. Specify what distribution you are using and justify your choice of parameters.

3. Construct the posterior joint probability density function based on the prior and theoretical probability density functions.

4. Compare the model prior and posterior probability density functions and explain the observed differences.

Extra credit:

1. Repeat this exercise at all depth levels and plot the P-wave velocity distributions as a function of depth.

2. Redo the Bayesian inversion using other distributions for the various PDFs. Compare your results for the different assumptions.

3. Repeat the Bayesian inversion assuming that the logs are correlated from one depth level to another.

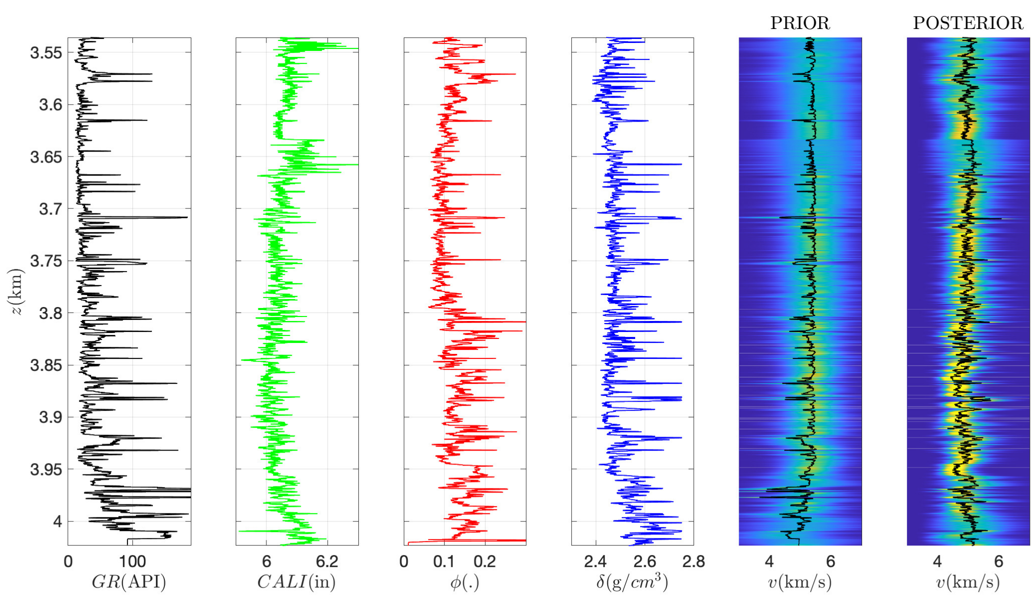

Figure 1: Well logs and the prior and posterior velocity PDFs.

N.B. This is an individual assignment – your work is subject to the Mines Academic Integrity policy.

INSTRUCTIONS

FORMAT

- Submit the assignment to Canvas as a standalone **Jupyter notebook**.
- Make sure to run **Kernel/Restart & Run All** in Jupyter before submission.

CLARITY

- Include text documenting your reasoning and how you approached the solution.
- Show all intermediate mathematical derivation steps, if applicable.
- Include figures demonstrating the solution and explain their meaning.

PROGRAMMING

- Include detailed comments documenting the functionality of your codes.
- Organize your programs in clear functional blocks.
- Isolate repeated code in functions. Provide unit tests for all defined functions.
- Define and initialize all variables; indicate in comments their physical units.

POLICIES

- Incomplete or incorrect answers receive partial credit at the discretion of the grader.
- Submissions lose 25%/day if late for two days and are not graded afterward.
- Multiple submissions to Canvas are allowed, but only the last one is graded.

GRADING RUBRIC

1. Construct the prior joint probability density function.
   - List all assumptions, equations used, distribution, procedure, and choice of parametrization.
   - Justify your choices. (15 pts)
   - Plot the prior joint probability density function. (15 pts)
2. Construct the theoretical joint probability density function.
   - List all assumptions, equations used, distribution, procedure, and choice of parametrization.
   - Justify your choices. (15 pts)
   - Plot the theoretical joint probability density function. (15 pts)
3. Construct the posterior joint probability density function.
   - List all assumptions, equations used, distribution, procedure, and choice of parametrization.
   - Justify your choices. (15 pts)
   - Plot the posterior joint probability density function. (15 pts)
4. Compare the model prior and posterior probability density functions and explain the differences. (10 pts)

Extra credit

1. Plot the P-wave velocity distributions as a function of depth. Explain your results. Use identical ranges for the prior and posterior tracks. (10 pts)
2. Choose other distributions for the various PDFs and redo the inversion at all depths. (10 pts)
   - List all assumptions, equations used, and distributions. Justify your choices.
   - Plot the prior, joint, and posterior probability density functions for a single depth.
   - Plot the P velocity distributions as a function of depth.
   - Compare the results obtained for the different assumptions. Explain the differences.
3. Repeat the inversion assuming that the logs have some correlation from one depth level to another. Use identical ranges for the prior and posterior tracks. (30 pts)