You are tasked to evaluate the total amount of CO_2 emmited in the atmosphere by the US oil production from 1850 to 2100, assuming that the production follows historic trends, as depicted in Figure 1. In evaluating this graph, it appears that a Gaussian curve would be appropriate to describe the historic trend. You can use that curve to extrapolate oil production into the future.[1]

Find the optimal Gaussian that fits the observations, and predict the cummulative oil production until year 2100. Then, convert the total oil produced in the US up to that date into CO_2 emitted in the atmosphere assuming that all the oil undergoes this conversion (ignore other uses of oil products). Use the following conversion from crude oil to CO_2: 1 ton of CO_2 is equivalent with 3.15 barrels of oil.

What would the total amount of CO_2 emitted by the US be if nothing changes and historic trends continue?

Extra credit: As an alternative to the Gaussian curve, we can describe the cummulative oil production using the logistic curve defined by

$$Q = \frac{L}{1 + e^{-k(t-to)}} \ .$$

Formulate an INVERSE PROBLEM to solve for the logistic curve parameters k and L, and predict the cummulative oil production and total CO_2 emissiona until year 2100. Assume the reference time $to = 1975$, representing the approximate peak of the historic yearly oil production curve.

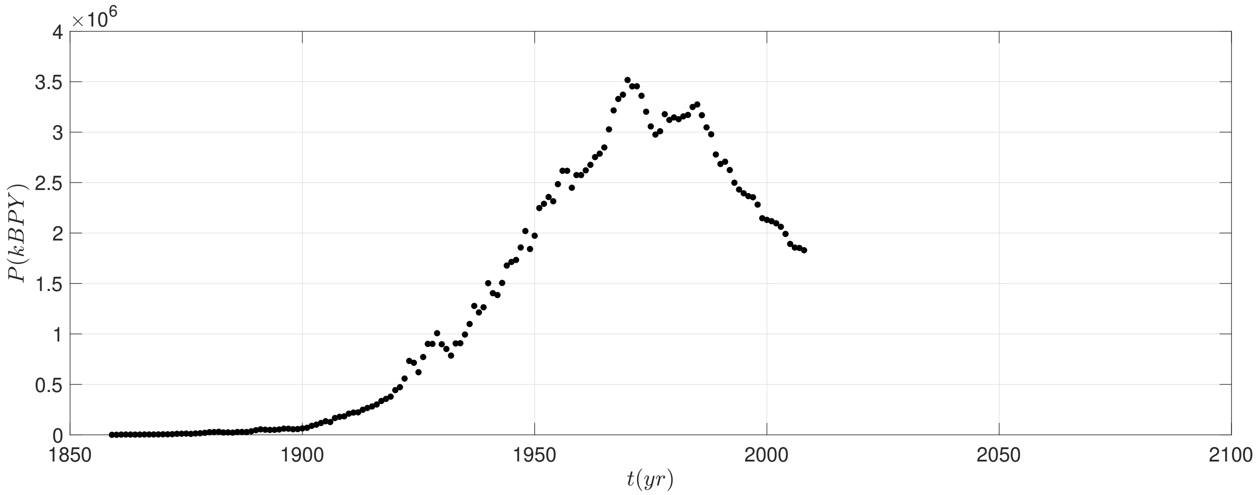

Figure 1: US yearly oil production (kBPY = tousands of barrels per year).

N.B. This is an individual assignment – your work is subject to the Mines Academic Integrity policy.

[1]This trend is approximate, as it did not account for recent developments, e.g., fracking, that actually increase significantly the US oil production and the corresponding CO_2 emissions.

INSTRUCTIONS

FORMAT

- Submit the assignment to Canvas as a standalone **Jupyter notebook**.
- Make sure to run **Kernel/Restart & Run All** in Jupyter before submission.

CLARITY

- Include text documenting your reasoning and how you approached the solution.
- Show all intermediate mathematical derivation steps, if applicable.
- Include figures demonstrating the solution and explain their meaning.

PROGRAMMING

- Include detailed comments documenting the functionality of your codes.
- Organize your programs in clear functional blocks.
- Isolate repeated code in functions. Provide unit tests for all defined functions.
- Define and initialize all variables; indicate in comments their physical units.

POLICIES

- Incomplete or incorrect answers receive partial credit at the discretion of the grader.
- Submissions lose 25%/day if late for two days and are not graded afterward.
- Multiple submissions to Canvas are allowed, but only the last one is graded.

GRADING RUBRIC

Part 1 - 30 pts

- Formulate the FORWARD PROBLEM, and justify all assumptions. (10 pts)
- Define model parameters. (5 pts)
- Define data parameters. (5 pts)
- Define the operator linking models and data. (10 pts)

Part 2 - 50 pts

- Formulate the INVERSE PROBLEM, and justify for all assumptions. (10 pts)
- Get the model parameters by inversion. (10 pts)
- Predict the yearly and cummulative production. (20 pts)
- Plot the observations and predictions on the same graph. (5 pts)
- Compute and report the total CO2 emissions. (5 pts)

Code - 20 pts

Include all codes used with comments to explain their functionality.

Extra credit - 40 pts

Solve the problem using the logistic curve formulation.