Problem 1

Consider a set of well logs, $v_P(z)$, $v_S(z)$, $\rho(z)$, recorded in a borehole. We are interested to characterize the dependence between these three parameters using a probability density function, $f(v_P, v_S, \rho)$, constructed using a histogram of the observed parameters. Your assignment is to construct the PDF $f(v_P, v_S, \rho)$ and to motivate your discretization of the three variables. Explain why your function is a PDF and plot the function in 3D.

For reference, Figures 1(a)-1(f) show scatter plots and PDFs constructed for pairs of parameters. What is the relation between these 2D PDFs and your 3D PDF?

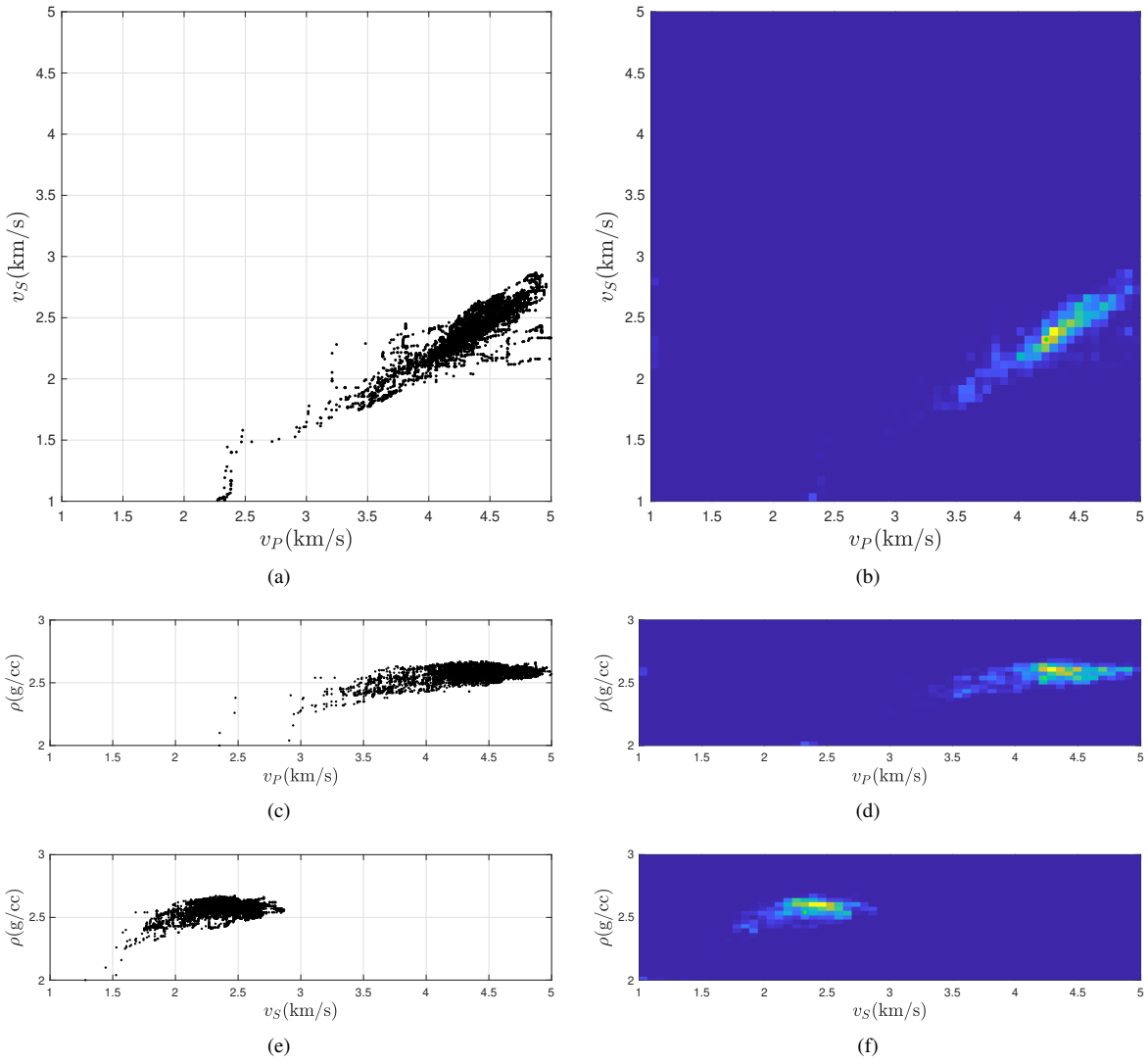


Figure 1: (a)-(b) $v_P - v_S$ scatter plot and histogram; (c)-(d) $v_P - \rho$ scatter plot and histogram; (e)-(f) $v_S - \rho$ scatter plot and histogram.

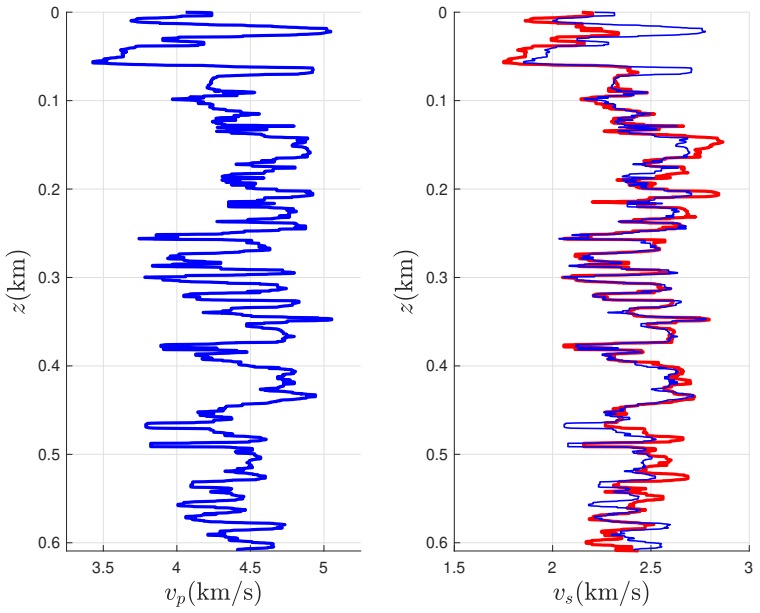
Problem 2

Assume that you are recording P-wave velocity (v_P) and S-wave velocity (v_S) well logs in boreholes A and B. You are interested predict v_S from v_P in borehole B using the relationship $v_S(v_P)$ derived from borehole A.[1]

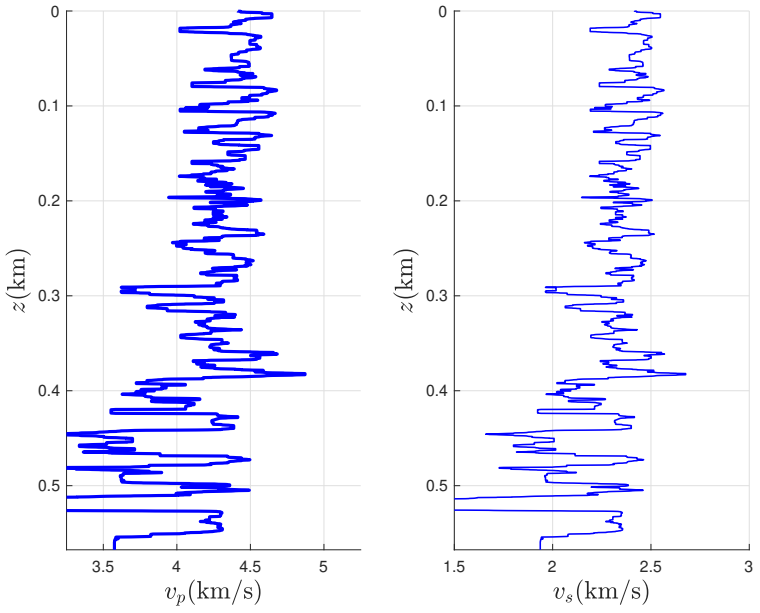
[1]The borehole B datafile contains v_S values – pretend you do not have these when you make your prediction.

1. Formulate an INVERSE PROBLEM which allows you to make this prediction. Explain what are the model and data parameters. Motivate your choice of parameters. What relation links the model and data parameters? What operator did you use to solve the INVERSE PROBLEM?

2. Plot the observed and predicted v_S in borehole A to demonstrate that your relation is reliable, Figure 2(a). Plot the predicted v_S in borehole B, Figure 2(b). Compare your predictions in borehole B with the corresponding observed data.



(a)



(b)

Figure 2: (a) v_P and (b) v_S well logs in borehole A (top) and in borehole B (bottom). The v_S panel shows the observed (thick line) and predicted (thin line) logs.

Extra credit: Assume that you are recording P-wave velocity (v_P), S-wave velocity (v_S) and density ρ well logs in boreholes A and B. You are interested to predict v_S from v_P and ρ in borehole B using the relationship $v_S\,(v_P, \rho)$ derived from borehole A.

1. Formulate an INVERSE PROBLEM which allows you to make this prediction. Explain what are the model and data parameters. Motivate your choice of parameters. What relation links the model and data parameters? What operator did you use to solve the INVERSE PROBLEM?

2. Plot the observed and predicted v_S in borehole A to demonstrate that your relation is reliable. Compare your predictions in borehole B with the corresponding observed data.

N.B. This is an individual assignment – your work is subject to the Mines Academic Integrity policy.

INSTRUCTIONS

FORMAT

- Submit the assignment to Canvas as a standalone **Jupyter notebook**.
- Make sure to run **Kernel/Restart & Run All** in Jupyter before submission.

CLARITY

- Include text documenting your reasoning and how you approached the solution.
- Show all intermediate mathematical derivation steps, if applicable.
- Include figures demonstrating the solution and explain their meaning.

PROGRAMMING

- Include detailed comments documenting the functionality of your codes.
- Organize your programs in clear functional blocks.
- Isolate repeated code in functions. Provide unit tests for all defined functions.
- Define and initialize all variables; indicate in comments their physical units.

POLICIES

- Incomplete or incorrect answers receive partial credit at the discretion of the grader.
- Submissions lose 25%/day if late for two days and are not graded afterward.
- Multiple submissions to Canvas are allowed, but only the last one is graded.

GRADING RUBRIC

Problem 1 (30 pts)

1. Build and plot the 3D probability density function. (20 pts)
2. Explain the relation between the 3D and the 2D probability density functions. (10 pts)

Problem 2 (50 pts)

1. Formulate the INVERSE PROBLEM, i.e., define the model and data parameters, as well as the operator linking model and data parameters. (20 pts)
2. Predict the v_S in borehole B. (20 pts)
3. Plot the predicted v_S in borehole B. (10 pts)

Code - 20 pts

Include all codes used with comments to explain their functionality.

Extra credit - 30 pts

Solve Problem 2 by developing a relationship between v_S and both v_P and ρ.