Chapter 4: Continuous Random Variables & Probability Distributions

MAS291 - STATISTICS & PROBABILITY

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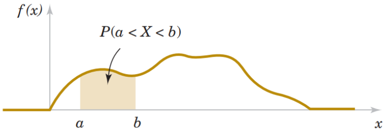
Table of Contents

Probability Density Function (pdf)

Definition 1.1

For a continuous random variable X, a probability density function is a function such that

1. $f(x) \geq 0$.
2. $\displaystyle\int_{-\infty}^{+\infty} f(x)dx = 1$.
3. $\forall\, a, b \in \mathbb{R}$, $P(a \leq X \leq b) = \displaystyle\int_a^b f(x)dx = P(a < X \leq b) = P(a \leq X < b) = P(a < X < b)$



Remark 1.1. $P(X = x_0) = 0$ (different from pmf)

pdf - Example

Example 1.1. Let X be a continuous random variable and suppose that $f(x) = cx^2$, for $-1 < x < 2$, and 0 otherwise is the pdf of X.

a) Find c.

b) Find $P(X \geqslant 0)$.

Example 1.2. Let X be a continuous random variable and suppose that $f(x) = cx^2$, for $-1 < x < 2$, and 0 otherwise is the pdf of X.

a) Find c.

b) Find $P(X \geqslant 0)$.

Solution.

a) $\int_{-\infty}^{\infty} f(x)dx = \int_{-1}^{2} cx^2 dx = c(8/3 + 1/3) = 3c = 1 \Longleftrightarrow c = 1/3$

b) $P(X \geqslant 0) = \int_0^2 (1/3)\mathrm{x}^2 dx = 8/9$

pdf - Example

Example 1.3. The lifetime, in years, of some electronic component is a continuous random variable with the probability density function

$$f(x) = \begin{cases} \dfrac{k}{x^3} & , x > 1 \\ 0 & , \text{otherwise} \end{cases}$$

Find k and compute the probability for the lifetime to exceed 5 years.

pdf - Example

Example 1.4. The lifetime, in years, of some electronic component is a continuous random variable with the probability density function

$$f(x) = \begin{cases} \dfrac{k}{x^3} & , x > 1 \\ 0 & , \text{otherwise} \end{cases}$$

Find k and compute the probability for the lifetime to exceed 5 years.

Solution.

$$\cdot \int_{-\infty}^{\infty} f(x)dx = \int_{1}^{\infty} \frac{k}{x^3}dx = -k(0-1)/2 = 1 \Longleftrightarrow k = 2$$

Table of Contents

Cumulative Distribution Functions (cdf)

Definition 2.1

The cumulative distribution function of a continuous random variable X is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)du, \ \forall x \in \mathbb{R}$$

pdf from the cdf: Given $F(x)$, $f(x) = F'(x)$ as long as the derivative exists.

Remark 2.1.

1. $P(a \leq X \leq b) = F(b) - F(a)$.
2. $F(x) \leq F(y)$, if $x \leq y$ (increasing).
3. $F(x) = P(X \leq x) = P(X < x)$.
4. $\lim\limits_{x \to -\infty} F(x) = 0$, $\lim\limits_{x \to +\infty} F(x) = 1$.

cdf - Example

Example 2.1. The time until a chemical reaction is complete (in milliseconds) is approximated by the cumulative distribution function

$$F(x) = \begin{cases} 0 & , x < 0 \\ 1 - e^{-0.01x} & , x \geq 0 \end{cases}$$

a) Determine the probability density function of X.

b) Compute the probability that a reaction completes within 200 milliseconds.

cdf - Example

Example 2.2. The time until a chemical reaction is complete (in milliseconds) is approximated by the cumulative distribution function

$$F(x) = \begin{cases} 0 & , x < 0 \\ 1 - e^{-0.01x} & , x \geq 0 \end{cases}$$

a) Determine the probability density function of X.

b) Compute the probability that a reaction completes within 200 milliseconds.

Solution.

a)

$$f(x) = \begin{cases} 0 & , x < 0 \\ 0.01e^{-0.01x} & , x \geq 0 \end{cases}$$

b) $P(X \leq 200) = F(200) = 1 - e^{-2} = 0.8647.$

Table of Contents

Definition 3.1

Suppose that X is a continuous random variable with probability density function $f(x)$. Then

1. The mean or expected value of X is $\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$
2. The variance of X is

$$\sigma^2 = V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx = \int_{-\infty}^{\infty} x^2 f(x) - \mu^2$$

3. The standard deviation of X is $\sigma = \sqrt{\sigma^2}$

Expected Value of a Function of a Continuous Random Variable

If X is a continuous random variable with probability density function $f(x)$,

$$E[h(X)] = \int_{-\infty}^{\infty} h(x) f(x) dx$$

Mean and Variance - Example

Example 3.1. Given the pdf of X, $f(x) = \begin{cases} \dfrac{3}{x^4}, & x > 1 \\ 0, & \text{otherwise} \end{cases}$. Find the mean and variance of X.

Example 3.2. Suppose that $f(x) = \dfrac{x}{8}$, $3 < x < 5$. Determine the mean and variance of X.

Example 3.3. The probability density function of the weight of packages delivered by a post office is $f(x) = \dfrac{70}{69x^2}$ for $1 < x < 70$ pounds.

a. Determine the mean and variance of weight.
b. If the shipping cost is \$2.5 per pound, what is the average shipping cost of a package?
c. Determine the probability that the weight of a package exceeds 50 pounds?

Table of Contents

Continuous Uniform Distribution

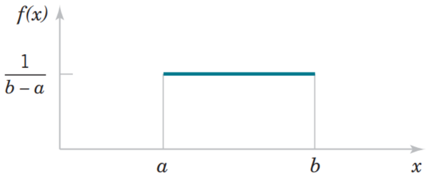
Definition 4.1

Continuous uniform distribution is a continuous random variable X with probability density function

$$f(x) = \frac{1}{b-a},\ a \leq x \leq b$$

Mean and Variance. If X is a continuous uniform random variable over $a \leq x \leq b$ then

$$\mu = E(X) = \frac{(a+b)}{2} \text{ and } \sigma^2 = V(X) = \frac{(b-a)^2}{12}$$



Example 4.1. Let the continuous random variable X denote the current measured in a thin copper wire in milliamperes. Assume the range of X is $[0, 20 \text{ mA}]$, and assume that the probability density function of X is

$$f(x) = 0.05,\ 0 \leq x \leq 20$$

a) Compute the probability that a measurement of current is between 5 and 10 milliamperes.
b) Compute mean and variance of X.

Continuous Uniform Distribution - Example

Example 4.2. Let the continuous random variable X denote the current measured in a thin copper wire in milliamperes. Assume the range of X is $[0, 20 \text{ mA}]$, and assume that the probability density function of X is

$$f(x) = 0.05,\ 0 \le x \le 20$$

a) Compute the probability that a measurement of current is between 5 and 10 milliamperes.
b) Compute mean and variance of X.

Solution.
a) $P(5 < X < 10) = (10 - 5)(0.05) = 0.25$
b) $E(X) = (a + b)/2 = (0 + 20)/2 = 10$
$V(X) = (b\breve{\ }a)^2/12 = (20\breve{\ }0)^2/12 = 33.33$

Table of Contents

5 Normal Distribution

Definition 5.1

A normal random variable X with parameter μ and $\sigma > 0$ if its probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\dfrac{-(x-\mu)^2}{2\sigma^2}}$$

Mean and Variance: $X \sim \mathcal{N}(\mu, \sigma^2) \Rightarrow E(X) = \mu,\ V(X) = \sigma^2$

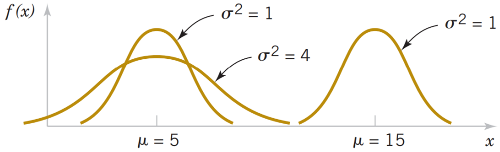


Figure: Normal pdfs for selected values of the parameters μ and σ^2.

Normal Distribution (Gaussian distribution)

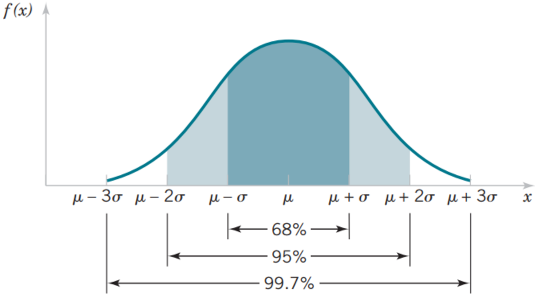


Figure: Probabilities associated with a normal distribution.

For any normal random variable,

$$P(\mu - \sigma < X < \mu + \sigma) = 0.6827$$

$$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9545$$

$$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$$

Standardize: If $X \sim \mathcal{N}(\mu, \sigma^2)$ **then** $Z = \dfrac{X-\mu}{\sigma} \sim \mathcal{N}(0,1)$

Definition 5.2

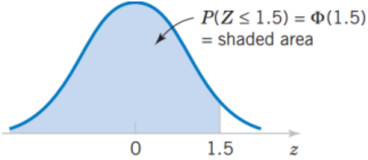
A normal random variable with $\mu = 0$, $\sigma^2 = 1$ is called a standard normal random variable and is denoted as Z. **Notation:** $Z \sim \mathcal{N}(0,1)$.

The probability density function of Z is $f(x) = \dfrac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$

The cumulative distribution function of a standard normal random variable is denoted as

$$\Phi(z) = P(Z \le z)$$

Example 5.1. Assume Z is a standard normal random variable. Appendix Table III provides probabilities of the form $\Phi(z) = P(Z \leq z)$. Read down the z column to the row that equals 1.5. The probability $P(Z \leq 1.5)$ is read from the adjacent column, labeled 0.00, to be 0.93319.



z	0.00	0.01	0.02	0.03
0	0.50000	0.50399	0.50398	0.51197
⋮		⋮		
1.5	0.93319	0.93448	0.93574	0.93699

Example

$$\Phi(z) = \boxed{P(Z \leq z)} = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$

Using Table III in the text book to find probabilities of Normal distribution

$\Phi(1.34) = P(Z \leqslant 1.34) = 0.909877$

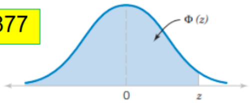
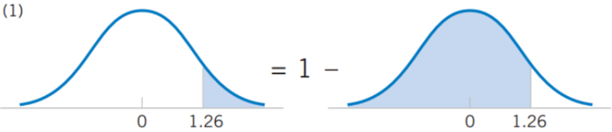
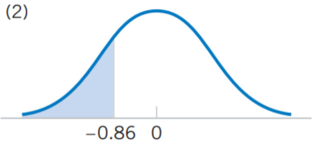


Table III Cumulative Standard Normal Distribution (*continued*)

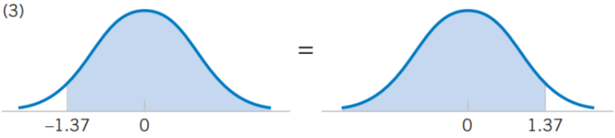
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083



$$P(Z > 1.26) = 1 - P(Z \le 1.26) = 1 - 0.89616$$
$$= 0.10384$$



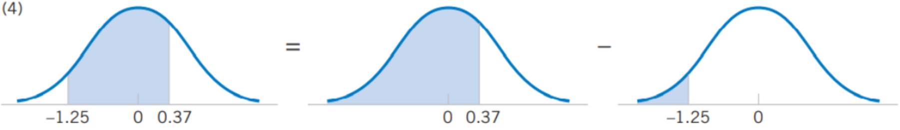
$$P(Z < -0.86) = 0.19490.$$



$$P(Z > -1.37) = P(Z < 1.37) = 0.91465$$

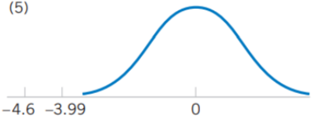
Example

(4)



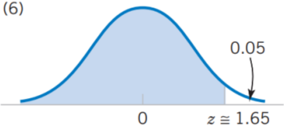
P(-1.25 < Z < 0.37) = P(Z < 0.37) − P(Z < -1.25)

(5)



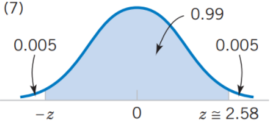
(5) P(Z < -4.6) cannot be found exactly from Appendix Table III. The last entry in the table can be used to find that P(Z < -3.99) = 0.00003. So, 0 ≈ P(Z < -4.6) < P(Z < -3.99).

(6)



(6) Find z such that P(Z > z) = 0.05. Equivalently, find z such that P(Z ⩽ z) = 0.95

(7)



(7) Find z such that P(-z < Z < z) = 0.99.
➔ Find z such that P(Z ⩽ z) = 0.995

Example

5 Normal Distribution

Example 5.2. Use Appendix Table III to determine the following probability for the standard normal random variable Z:

a. $P(Z < 1.32)$
b. $P(Z > 1.45)$
c. $P(Z > -2.15)$
d. $P(-2.34 < Z < 1.76)$

Example 5.3. Assume Z has a standard normal distribution. Use Appendix Table III to determine the value for z that solves each of the following:

a. $P(Z > z) = 0.9$
b. $P(Z < z) = 0.9$
c. $P(-1.24 < Z < z) = 0.8$

Standardizing a Normal Random Variable

Example 5.4. Suppose that the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 milliamperes and a variance of 4 $(milliamperes)^2$. What is the probability that a measurement exceeds 13 milliamperes?

Example 5.5. Suppose that the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 milliamperes and a variance of 4 $(milliamperes)^2$. What is the probability that a measurement exceeds 13 milliamperes?

Solution. Let X denote the current in milliamperes.

Let $Z = (X - 10)/2$.
We note that $X > 13$ corresponds to $Z > 1.5$. Therefore, from Appendix Table III,
$P(X > 13) = P(Z > 1.5) = 1 - P(Z \leq 1.5) = 1 - 0.93319 = 0.06681$

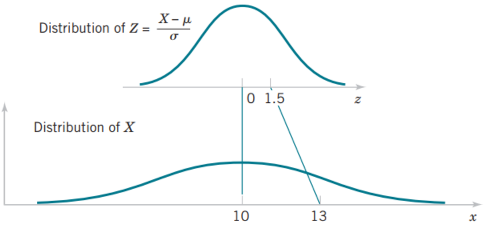


FIGURE 4.14

Standardizing to Calculate a Probability

Suppose that X is a normal random variable with mean μ and variance σ^2. Then,

$$P(X \leq x) = P(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}) = P(Z \leq z)$$

where Z is a standard normal random variable, and $z = \dfrac{x-\mu}{\sigma}$ is the $z-value$ obtained by standardizing X. The probability is obtained by using Appendix Table III with $z = \dfrac{x-\mu}{\sigma}$.

Example 5.6. (Normally Distributed Current) Continuing the previous example.

a. What is the probability that a current measurement is between 9 and 11 milliamperes?

b. Determine the value for which the probability that a current measurement is below this value is 0.98

Example 5.7. (Normally Distributed Current) Continuing the previous example.

a. What is the probability that a current measurement is between 9 and 11 milliamperes?

b. Determine the value for which the probability that a current measurement is below this value is 0.98

Solution.

a) $P(9 < X < 11) = P(-0.5 < Z < 0.5) = P(Z < 0.5) - P(Z < -0.5) = 0.38292$

b) $P(X < x) = P\left(\frac{X-10}{2} < \frac{x-10}{2}\right) = P\left(Z < \frac{x-10}{2}\right) = 0.98$ The nearest probability from Table III results in

$$P(Z < 2.06) = 0.980301$$

Therefore, $(x - 10)/2 = 2.06$, and the standardizing transformation is used in reverse to solve for x. The result is

Table of Contents

Normal Approximation to the Binomial Distribution

6 Normal Approximation to the Binomial and Poisson Distribution

If $X \sim \mathcal{B}(n, p)$ then

$$Z = \frac{X - np}{\sqrt{np(1-p)}} \approx \mathcal{N}(0, 1)$$

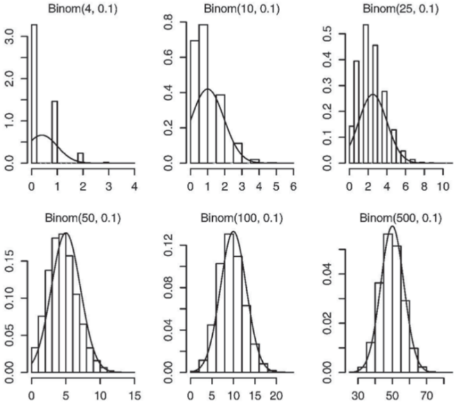
Good approximation with n and p such that $np > 5$ and $n(1-p) > 5$.

Normal Approximation of Binomial as follow

$$P(X \leqslant x) = P(X \leqslant x + 0.5) = P\left(Z \leqslant \frac{x + 0.5 - np}{\sqrt{np(1-p)}}\right)$$

continuity correction

$$P(x \leqslant X) = P(x - 0.5 \leqslant X) = P\left(\frac{x - 0.5 - np}{\sqrt{np(1-p)}} \leqslant Z\right)$$



Example 6.1. Assume that in a digital communication channel, the number of bits received in error can be modeled by a binomial random variable, and assume that the probability that a bit is received in error is 10^{-5}. If 16 million bits are transmitted, what is the probability that 150 or fewer errors occur?

Example 6.2. Assume that in a digital communication channel, the number of bits received in error can be modeled by a binomial random variable, and assume that the probability that a bit is received in error is 10^{-5}. If 16 million bits are transmitted, what is the probability that 150 or fewer errors occur?

Solution. Let the random variable X denote the number of errors. Then X is a binomial random variable with $n = 16 \times 10^6$ and $p = 10^{-5}$, and

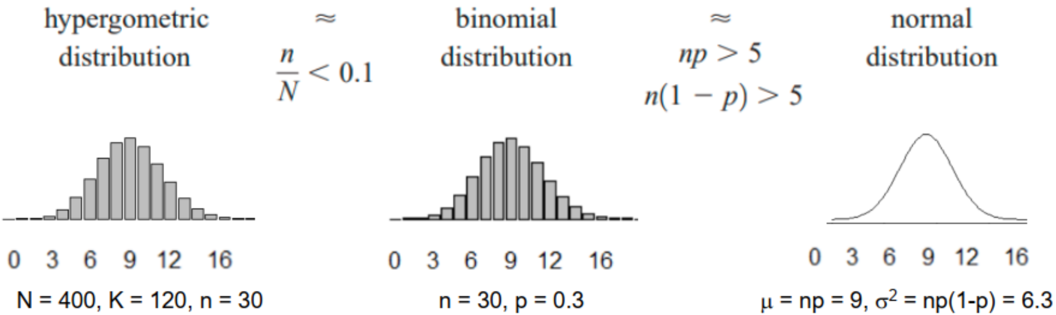
$$P(X \leq 150) = \sum_{x=0}^{150} \binom{16,000,000}{x} \left(10^{-5}\right)^x \left(1 - 10^{-5}\right)^{16,000,000-x} \qquad \text{DIFFICULT!}$$

Use normal distribution to approximate

$$P(X \leq 150) = P(X \leq 150.5) = P\left(\frac{X - 160}{\sqrt{160\left(1 - 10^{-5}\right)}} \leq \frac{150.5 - 160}{\sqrt{160\left(1 - 10^{-5}\right)}}\right) \approx P(Z \leq -0.75) = 0.227$$

Conditions for approximating hypergeometric and binomial probabilities

6 Normal Approximation to the Binomial and Poisson Distribution



Normal Approximation the Poisson Distribution

If $X \sim \mathcal{P}(\lambda)$ $(E(X) = V(X) = \lambda$ then $Z = \dfrac{X-\lambda}{\sqrt{\lambda}} \approx \mathcal{N}(0,1)$

The approximation is good for $\lambda > 5$.

Normal Approximation to Poisson as follows

$$P(X \le x) = P(X \le x + 0.5) \approx P\left(Z \le \frac{x + 0.5 - \lambda}{\sqrt{\lambda}}\right)$$

$$P(X \ge x) = P(X \ge x - 0.5) \approx P\left(Z \ge \frac{x - 0.5 - \lambda}{\sqrt{\lambda}}\right)$$

Example

6 Normal Approximation to the Binomial and Poisson Distribution

Example 6.3. Assume that the number of asbestos particles in a squared meter of dust on a surface follows a Poisson distribution with a mean of 1000. If a squared meter of dust is analyzed, what is the probability that 950 or fewer particles are found?

Example 6.4. Assume that the number of asbestos particles in a squared meter of dust on a surface follows a Poisson distribution with a mean of 1000. If a squared meter of dust is analyzed, what is the probability that 950 or fewer particles are found?

Solution. Let X be the number of particles found. Then $X \sim \mathcal{P}(1000)$

$$P(X \leq 950) = \sum_{x=0}^{950} \frac{e^{-1000} 1000^x}{x!} \text{ DIFFICULT}$$

AN APPROXIMATION

$$P(X \leq 950) = P(X \leq 950.5) \approx P(Z \leq \frac{950.5 - 1000}{\sqrt{1000}}) = P(Z \leq -1.57) = 0.058$$

Table of Contents

Exponential Distribution $(X \sim \mathcal{E}(\lambda))$

Definition 7.1

The random variable X that equals the distance between successive events from a Poisson process with mean number of events $\lambda > 0$ per unit interval is an exponential random variable with parameter λ. The probability density function of X is

$$f(x) = \lambda e^{-\lambda x}, \ x \geq 0$$

Mean and Variance: If $X \sim \mathcal{E}(\lambda)$ then $E(X) = \dfrac{1}{\lambda}$, $V(X) = \dfrac{1}{\lambda^2}$

Remark 7.1. $F(x) = P(X \leq x) = 1 - e^{-\lambda x}, \ x \geq 0$

Remark 7.2. Often used to model waiting time between rare events: time between telephone calls, accidents, etc. When the number of events is Poisson, the time between events is Exponential.

Example

Example 7.1. In a large corporate computer network, user log-ons to the system can be modeled as a Poisson process with a mean of 25 log-ons per hour.

a) Compute probability that there are no log-ons in an interval of six minutes.

b) Compute the probability that the time until the next log-on is between two and three minutes?

Example 7.2. In a large corporate computer network, user log-ons to the system can be modeled as a Poisson process with a mean of 25 log-ons per hour.
a) Compute probability that there are no log-ons in an interval of six minutes.
b) Compute the probability that the time until the next log-on is between two and three minutes?

Solution.
Let X denote the time in hours from the start of the interval until the first log-on. Then X has an exponential distribution with $\lambda = 25$ log-ons per hour.
a) We are interested in the probability that X exceeds 6 minutes = 0.1 hour.

$$P(X > 0.1) = \int_{0.1}^{\infty} 25e^{-25x}dx = e^{-25(0.1)} = 0.082$$

b) Converting 2 minutes = 0.033 hour and 3 minutes = 0.05 hour.

$$P(0.033 < X < 0.05) = \int^{0.05} 25e^{-25x}dx = e^{-25x}\Big|^{0.05} = 0.152$$

Lack of Memory Property

For an exponential random variable X ,

$$P\left(X < t_1 + t_2 \mid X > t_1\right) = P\left(X < t_2\right)$$

Example 7.3. The time between the arrival of electronic messages at your computer is exponentially distributed with a mean of two hours ($\lambda = 1/2$).

a) What is the probability that you do not receive a message during a two-hour period?

$$P(X > 2) = 1 - P(X \leqslant 2) = e^{-2\lambda} = e^{-1} = 0.368$$

b) If you have not had a message in the last four hours, what is the probability that you do not receive a message in the next two hours?

$$\mathrm{P}(\mathrm{X} > 6 \mid \mathrm{X} > 4) = \mathrm{P}(\mathrm{X} > 2)$$

Table of Contents

8 APPENDIX A (A-8, A-9)

APPENDIX A (A-8, A-9)

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

TABLE III Cumulative Standard Normal Distribution

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00
−3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048
−3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072
−3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108
−3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159
−3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233
−3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337
−3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483
−3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687
−3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968
−3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350
−2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866
−2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555
−2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467
−2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661
−2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210
−2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198
−2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724
−2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903
−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

TABLE III Cumulative Standard Normal Distribution *(continued)*

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621

APPENDIX A (A-8, A-9)

8 APPENDIX A (A-8, A-9)

−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864
−2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750
−1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717
−1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930
−1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565
−1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799
−1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807
−1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757
−1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801
−1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070
−1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666
−1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655
−0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060
−0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855
−0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964
−0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253
−0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538
−0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578
−0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089
−0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740
−0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000

1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

CASIO 580VN: MENU $\Longrightarrow 7 \Longrightarrow 2\ (Lower : -3,9; Upper : z)$

Q&A

Thank you for listening!