Chapter 6: Descriptive Statistics

MAS291 - STATISTICS & PROBABILITY

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- Time Sequence Plots

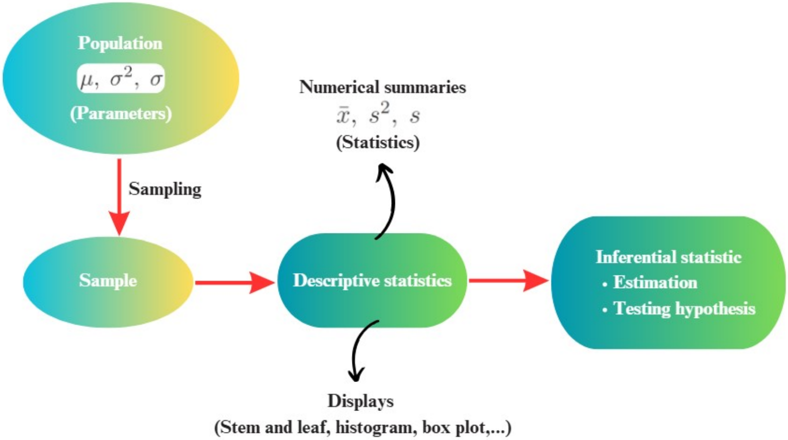
LO

1 Numerical Summaries of Data

1. Find numeric summaries of data; describe data using stem-andleaf diagrams, frequency distributions, histograms, time series plots.
2. Determine quartiles and construct the box plot for a data; identify outliers.

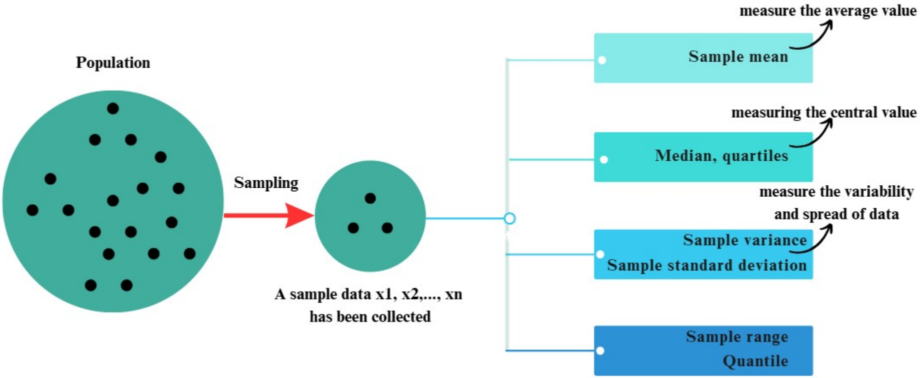
Introduction

1 Numerical Summaries of Data



Descriptive statistics

1 Numerical Summaries of Data


Population
Sampling
A sample data x1, x2,..., xn has been collected
measure the average value
Sample mean
measuring the central value
Median, quartiles
measure the variability and spread of data
Sample variance
Sample standard deviation
Sample range
Quantile

Numerical Summaries of Data

If the n observations in a sample are denoted by $x_1, x_2, ..., x_n$ then

- The **sample mean** is $\overline{x} = \dfrac{x_1 + x_2 + ... + x_n}{n}$
- The **sample variance** is $s^2 = \dfrac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}$
- The **sample range** is $r = \max(x_i) - \min(x_i)$

When the population is finite and consists of N values then

- The **population mean** is $\mu = \dfrac{x_1 + x_2 + ... + x_N}{N}$
- The **population variance** is $\sigma^2 = \dfrac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N}$

Sample mean

1 Numerical Summaries of Data

Definition 1.1

If the n observations in a sample are denoted by $x_1, x_2, ..., x_n$ then the **sample mean** is

$$\overline{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{\displaystyle\sum_{i=1}^{n} x_i}{n}$$

Example 1.1. Let's consider the eight observations on pull-off force collected from the prototype engine connectors from Chapter 1. The eight observations are $x_1 = 12.6, x_2 = 12.9, x_3 = 13.4,\ x_4 = 12.3, x_5 = 13.6, x_6 = 13.5, x_7 = 12.6$, and $x_8 = 13.1$. The sample mean is

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_{i=1}^{8} x_i}{8} = \frac{12.6 + 12.9 + \cdots + 13.1}{8} = \frac{104}{8} = 13.0 \text{ pounds}$$

Sample mean

1 Numerical Summaries of Data

A physical interpretation of the sample mean as a measure of location is shown in the dot diagram of the pull-off force data. See Figure 6.1. Notice that the sample mean $\bar{x} = 13.0$ can be thought of as a balance point. That is, if each observation represents 1 pound of mass placed at the point on the x-axis, a fulcrum located at $\bar{x}$ would balance this system of weights exactly.

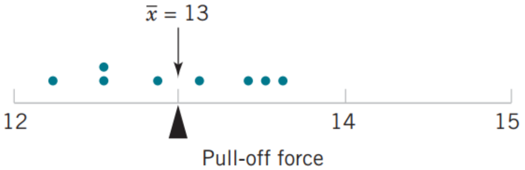


FIGURE 6.1

Dot diagram showing the sample mean as a balance point for a system of weights.

Sample median

Definition 1.2

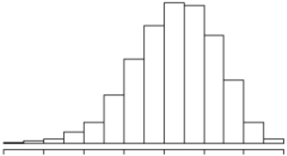
1. The value that lies in the middle of the data when the data set is ordered.
2. Measures the center of an ordered data set by dividing it into two equal parts.
3. If the data set has an ($L = \dfrac{n+1}{2}$)
   - even number of entries: median is the mean of the two middle data entries.
   - odd number of entries: median is the middle data entry.

Example 1.2. The female students in an undergraduate engineering core course at ASU self-reported their heights to the nearest inch. The data follow. Calculate the sample median of height.

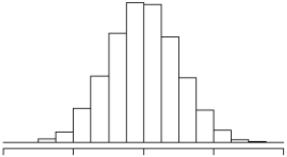
62 64 61 67 65 68 61 65 60 65 64 63 59 68 64 66 68 69 65 67 62 66 68 67 66 65 69 65 69 65 67 67 65 63 64 67 65

Solution. The 19th smallest observation = 65 is the sample median.
(Sort the data. n = 37 is odd and (n + 1)/2 = 19.)

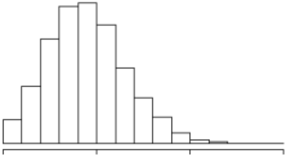
Mean vs Median



Mean < Median
Left-skewed
Negative skew



Median = Mean
Symmetric



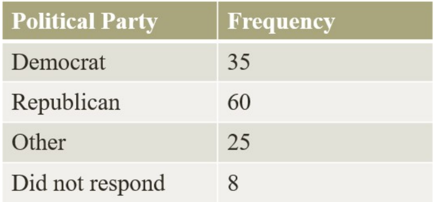
Median < Mean
Right-skewed
Positive skew

Sample mode

Definition 1.3

1. The data entry that occurs with the greatest frequency.
2. If no entry is repeated the data set has no mode.
3. If two entries occur with the same greatest frequency, each entry is a mode (bimodal).

Example 1.3. At a political debate a sample of audience members was asked to name the political party to which they belong. Their responses are shown in the table. What is the mode of the responses?



Political Party	Frequency
Democrat	35
Republican	60
Other	25
Did not respond	8

Sample Variance and Standard Deviation

Definition 1.4

The **sample variance** is

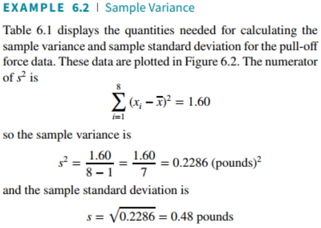
$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}$$

The **sample standard deviation**, s, is the positive square root of the sample variance.

Example 1.4. The female students in an undergraduate engineering core course at ASU self-reported their heights to the nearest inch. The data follow. Calculate the sample variance and the standard deviation.
62 64 61 67 65 68 61 65 60 65 64 63 59 68 64 66 68 69 65 67 62 66 68 67 66 65 69 65 69 65 67 67 65 63 64 67 65

Example - Sample Variance

1 Numerical Summaries of Data



EXAMPLE 6.2 | Sample Variance

Table 6.1 displays the quantities needed for calculating the sample variance and sample standard deviation for the pull-off force data. These data are plotted in Figure 6.2. The numerator of s^2 is

$$\sum_{i=1}^{8} (x_i - \overline{x})^2 = 1.60$$

so the sample variance is

$$s^2 = \frac{1.60}{8-1} = \frac{1.60}{7} = 0.2286 \text{ (pounds)}^2$$

and the sample standard deviation is

$$s = \sqrt{0.2286} = 0.48 \text{ pounds}$$

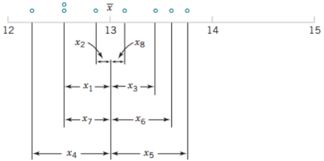


FIGURE 6.2

How the sample variance measures variability through the deviations $x_i - \overline{x}$.

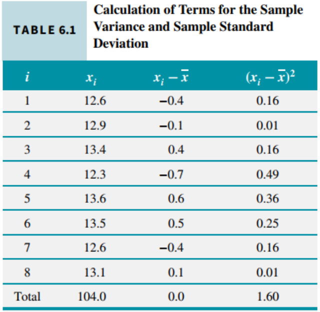


TABLE 6.1 **Calculation of Terms for the Sample Variance and Sample Standard Deviation**

i	x_i	$x_i - \overline{x}$	$(x_i - \overline{x})^2$
1	12.6	−0.4	0.16
2	12.9	−0.1	0.01
3	13.4	0.4	0.16
4	12.3	−0.7	0.49
5	13.6	0.6	0.36
6	13.5	0.5	0.25
7	12.6	−0.4	0.16
8	13.1	0.1	0.01
Total	104.0	0.0	1.60

Sample range

1 Numerical Summaries of Data

Definition 1.5

1. The difference between the maximum and minimum data entries in the set.
2. The data must be quantitative.
3. If the n observations in a sample are denoted by $x_1, x_2, \cdots, x_n$, the sample range is

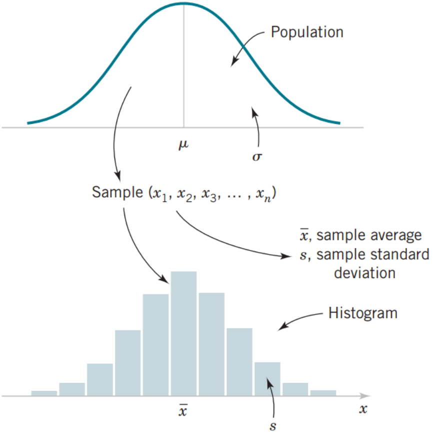
$$r = \max(x_i) - \min(x_i)$$

Remark 1.1. Casio 580VN

MENU $\Longrightarrow$ 6 $\Longrightarrow$ 1 $\Longrightarrow$ INPUT DATA $\Longrightarrow$ AC

OPTION $\Longrightarrow$ 2

Relationship between a population and a sample


Population
μ
σ
Sample $(x_1, x_2, x_3, \dots, x_n)$
$\bar{x}$, sample average
s, sample standard deviation
Histogram
$\bar{x}$
s
x

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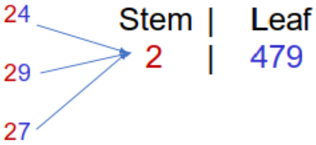
Stem-and-Leaf diagrams

Steps to construct a stem-and-leaf diagram

(1) Divide each number xi into two parts: a stem, consisting of one or more of the leading digits, and a leaf, consisting of the remaining digit.
(2) List the stem values in a vertical column.
(3) Record the leaf for each observation beside its stem.
(4) Write the units for stems and leaves on the display.



Stem-and-Leaf diagrams - Example

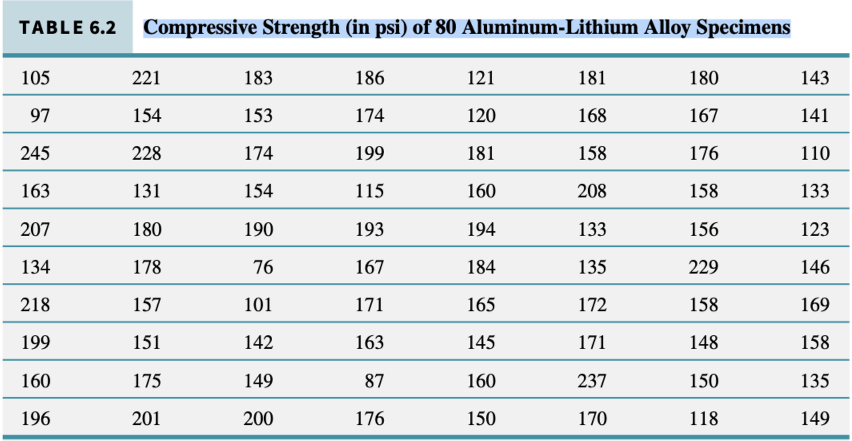


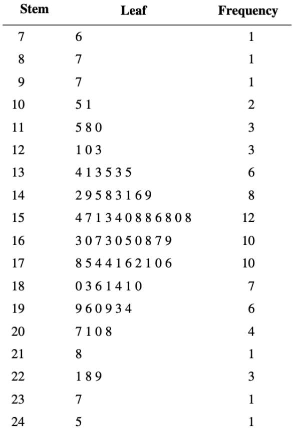
TABLE 6.2 **Compressive Strength (in psi) of 80 Aluminum-Lithium Alloy Specimens**

105	221	183	186	121	181	180	143
97	154	153	174	120	168	167	141
245	228	174	199	181	158	176	110
163	131	154	115	160	208	158	133
207	180	190	193	194	133	156	123
134	178	76	167	184	135	229	146
218	157	101	171	165	172	158	169
199	151	142	163	145	171	148	158
160	175	149	87	160	237	150	135
196	201	200	176	150	170	118	149

Stem-and-Leaf diagrams - Example

2 Stem-and-Leaf diagrams

Stem-and-leaf diagram for the compressive strength data in Table 6.2.



Stem	Leaf	Frequency
7	6	1
8	7	1
9	7	1
10	5 1	2
11	5 8 0	3
12	1 0 3	3
13	4 1 3 5 3 5	6
14	2 9 5 8 3 1 6 9	8
15	4 7 1 3 4 0 8 8 6 8 0 8	12
16	3 0 7 3 0 5 0 8 7 9	10
17	8 5 4 4 1 6 2 1 0 6	10
18	0 3 6 1 4 1 0	7
19	9 6 0 9 3 4	6
20	7 1 0 8	4
21	8	1
22	1 8 9	3
23	7	1
24	5	1

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Frequency Distributions

3 Frequency Distributions and Histograms

Frequency Distributions

- More compact than a stem-and-leaf diagram
- The range of the data is divided into intervals (class intervals, cells, bins)

A frequency histogram consists of columns, one for each bin, whose height is determined by the number of observations in the bin.

A relative frequency histogram has the same shape but a different vertical scale. Its column heights represent the proportion of all data that appeared in each bin.

Histograms

3 Frequency Distributions and Histograms

- The histogram is a visual display of the frequency distribution.
- Histograms have a shape similar to the pmf or pdf of data, especially in large samples.
- Histograms are stable and reliable for large data sets, preferably of size 75 to 100 or more.

Constructing a Histogram (Equal Bin Widths)

(1) Label the bin (class interval) boundaries on a horizontal scale.

(2) Mark and label the vertical scale with the frequencies or the relative frequencies.

(3) Above each bin, draw a rectangle where height is equal to the frequency (or relative frequency) corresponding to that bin.

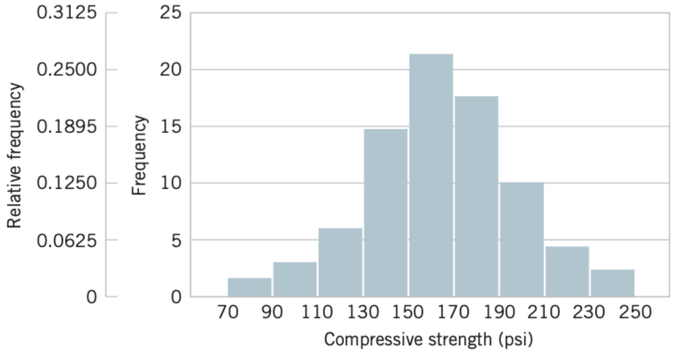

FIGURE 6.7

Histogram of compressive strength for 80 aluminum-lithium alloy specimens.

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4 Box Plots

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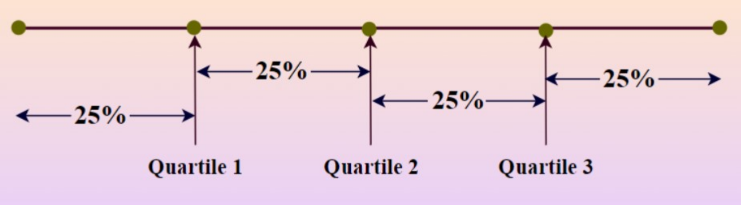
Box Plots

The box plot is a graphical display that simultaneously describes several important features of a data set, such as center, spread, departure from symmetry, and identification of unusual observations or outliers.

Quartiles

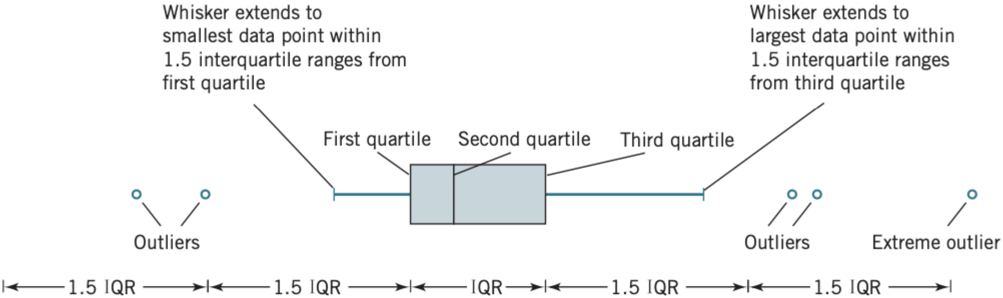
4 Box Plots



The quartiles q_1, q_2, q_3 of the sample data are the values at positions: $0.25(n+1)$, $0.5(n+1)$, $0.75(n+1)$, respectively.
The interquartile (IQR) is defined by $IQR = q_3 - q_1$

Box Plots

4 Box Plots



Whisker extends to smallest data point within 1.5 interquartile ranges from first quartile
Whisker extends to largest data point within 1.5 interquartile ranges from third quartile
First quartile
Second quartile
Third quartile
Outliers
Outliers
Extreme outlier
1.5 IQR
1.5 IQR
IQR
1.5 IQR
1.5 IQR

FIGURE 6.13

Description of a box plot.

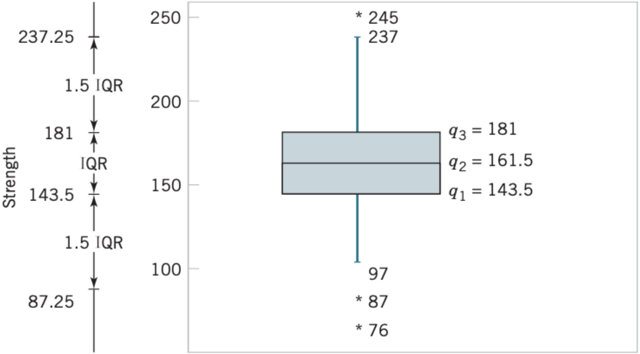


FIGURE 6.14

Box plot for compressive strength data in Table 6.2.

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Time Sequence Plots

A time series or time sequence is a data set in which the observations are recorded in the order in which they occur.

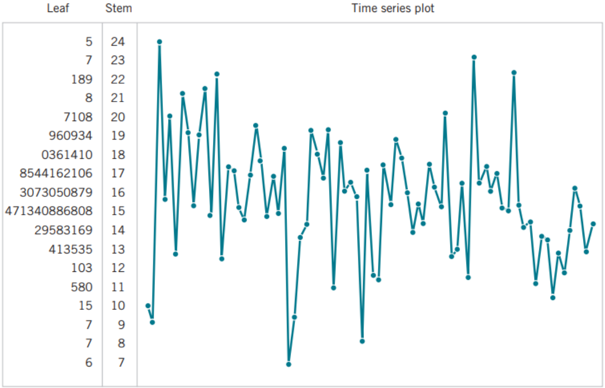
A time series plot

- the vertical axis denotes the observed value
- the horizontal axis denotes the time

In a time series plot, we often see

- trends
- cycles
- or other broad features of the data

Time Sequence Plots



Leaf	Stem
5	24
7	23
189	22
8	21
7108	20
960934	19
0361410	18
8544162106	17
3073050879	16
471340886808	15
29583169	14
413535	13
103	12
580	11
15	10
7	9
7	8
6	7

FIGURE 6.17

A digidot plot of the compressive strength data in Table 6.2.

Q&A

Thank you for listening!