Chapter 3
Discrete Random Variables & Probability Distributions

MAS291 - STATISTICS & PROBABILITY

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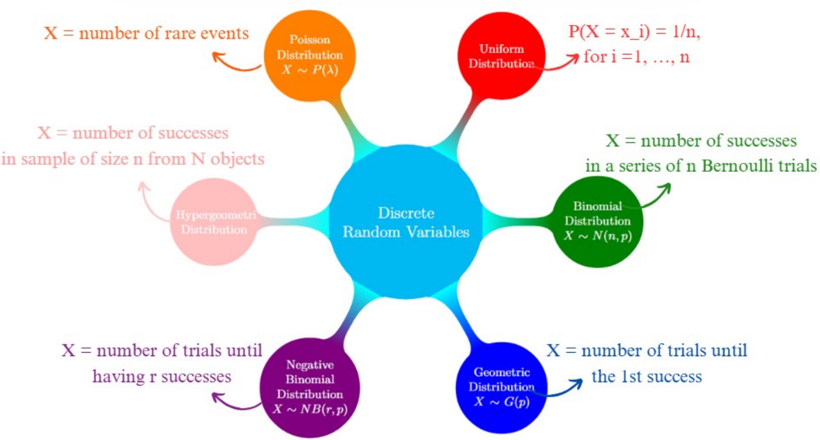
LO

1. Describe a **discrete random variable**.
2. Check if a function is a **probability mass function (pmf)** and use it to calculate probability.
3. Find the **cumulative distribution function (cmf)** of a discrete random variable.
4. Compute the **mean** and **variance** of a discrete random variable.
5. Determine the probability, mean and variance of **uniform, binomial, geometric and negative binomial, hypergeometric & Poisson distributions**.

Introduction

1 Probability Distributions and Probability Mass Functions



Random variable

1 Probability Distributions and Probability Mass Functions

Definition 1.1

A random variable is a function that assigns a real number to each outcome in the sample space of a random experiment.

$$X : S \to \mathbb{R}$$

$$X(\omega) \in \mathbb{R}$$

If $X(S) = \{x_1, x_2, \ldots, x_n\}$ or $X(S) = \{x_1, x_2, \ldots, x_n, \ldots\}$
X is called **Discrete random variable**.

Example 1.1. Flipping a coin twice.

$\to$ The sample space is $S = \{HH, HT, TH, TT\}$.

$X : S \to \mathbb{R}$

$X(\omega) =$ number of heads in each outcome ω

$\omega \in S$	HH	HT	TH	TT
$X(\omega)$	2	1	1	0

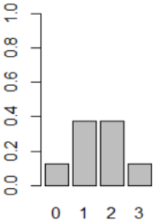
Definition 1.2

The probability distribution of a random variable X is a description of the probabilities associated with the possible values of X.

Example 1.2. Toss a fair coin three times and let **X be the number of Heads** observed, $X(\omega) \in \{0, 1, 2, 3\}$. Then we have the following probabilities

	TTT	HTT THT TTH	HHT HTH THH	HHH
X	0	1	2	3
P(X = x)	1/8	3/8	3/8	1/8

$$\Sigma_x P(x) = 1$$



Probability distribution for number of heads.

Probability Mass Functions (Hàm khối XS)

Definition 1.3

For a discrete random variable X with possible values $x_1, x_2, .., x_n$, a probability mass function (pmf) is a function such that

1. $f(x_i) \geq 0$
2. $\sum_{i=1}^{n} f(x_i) = 1$
3. $f(x_i) = P(X = x_i)$

Example 1.3. Verify that the following function is a pmf $f(x) = \dfrac{2x+1}{25}, x = \overline{0,4}$

1. $f(x) \geqslant 0$
2. $f(x) = P(X = x)$, $P(X = 4) = f(4) = 9/25$
3. $\sum_{i} f(x_i) = f(0) + f(1) + f(2) + f(3) + f(4) = 1$

pmf - Example

Example 1.4. Given the pmf $f(x)$, determine the probabilities.

$$f(x) = \frac{2x+1}{25}, \; x = \overline{0,4}$$

1. $P(X = 4)$
2. $P(X \leqslant 3)$
3. $P(2 \leqslant X \leqslant 4)$
4. $P(X > -3)$

pmf - Example

Example 1.5. Given the pmf $f(x)$, determine the probabilities.

$$f(x) = \frac{2x+1}{25}, \quad x = \overline{0,4}$$

1. $P(X = 4)$
2. $P(X \leqslant 3)$
3. $P(2 \leqslant X \leqslant 4)$
4. $P(X > -3)$

Solution. $P(2 \leqslant X \leqslant 4) = P(X = 2) + P(X = 3) + P(X = 4)$

$$= \frac{(2 \cdot 2 + 1)}{25} + \frac{(2 \cdot 3 + 1)}{25} + \frac{(2 \cdot 4 + 1)}{25} = \frac{21}{25}$$

pmf - Example

Example 1.6. Let the random variable X denote the number of semiconductor wafers that need to be analyzed in order to detect a large particle of contamination. Assume that the probability that a wafer contains a large particle is 0.01 and that the wafers are independent. Determine the probability distribution of X.

pmf - Example

Example 1.7. Let the random variable X denote the number of semiconductor wafers that need to be analyzed in order to detect a large particle of contamination. Assume that the probability that a wafer contains a large particle is 0.01 and that the wafers are independent. Determine the probability distribution of X.

Solution. Let p denote a wafer in which a large particle is present, and let a denote a wafer in which it is absent.

$$S = \{p, ap, aap, aaap, aaaap, \text{ and so forth}\}$$

$$\begin{aligned}
P(X = 1) &= P(p) = 0.01 \\
P(X = 2) &= P(ap) = 0.99(0.01) \\
P(X = 3) &= P(aap) = (0.99)^2(0.01) \\
P(X = x) &= (0.99)^{x-1}(0.01)
\end{aligned}$$

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2 Cumulative Distribution Functions

Cumulative Distribution Functions

Definition 2.1

The cumulative distribution function (cdf) of a discrete random variable X, denoted as $F(x)$ is

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$

For a discrete random variable X, $F(x)$ satisfies the following properties.

1. $F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$
2. $0 \leq F(x) \leq 1$
3. If $x \leq y$ then $F(x) \leq F(y)$.

Example 2.1. Find $F(-1), F(1.9)$.

x	-1	**0**	**1**	**2**	otherwise
f(x)	0.2	0.5	0	0.3	0

Cumulative Distribution Functions

2 Cumulative Distribution Functions

Definition 2.2

The cumulative distribution function (cdf) of a discrete random variable X, denoted as $F(x)$ is

$$F(x) = P(X \le x) = \sum_{x_i \le x} f(x_i)$$

For a discrete random variable X, $F(x)$ satisfies the following properties.

1. $F(x) = P(X \le x) = \sum_{x_i \le x} f(x_i)$
2. $0 \le F(x) \le 1$
3. If $x \le y$ then $F(x) \le F(y)$.

Example 2.2. Find $F(-1), F(1.9)$.

x	-1	0	1	2	otherwise
f(x)	0.2	0.5	0	0.3	0

Solution.

$F(-1) = f(-1) = 0.2$

$F(1.9) = f(-1) + f(0) + f(1) = 0.7$

cmf - Example

Example 2.3. Suppose that a day's production of 850 manufactured parts contains 50 parts that do not conform to customer requirements. Two parts are selected at random, without replacement, from the batch. Let the random variable X equal the number of nonconforming parts in the sample. What is the cumulative distribution function of X?

cmf - Example

Example 2.4. Suppose that a day's production of 850 manufactured parts contains 50 parts that do not conform to customer requirements. Two parts are selected at random, without replacement, from the batch. Let the random variable X equal the number of nonconforming parts in the sample. What is the cumulative distribution function of X?

Solution.

$$P(X = 0) = \frac{800}{850}.\frac{799}{849} = 0.886$$
$$P(X = 1) = 2.\frac{800}{850}.\frac{50}{849} = 0.111$$
$$P(X = 2) = \frac{50}{850}.\frac{49}{849} = 0.003$$

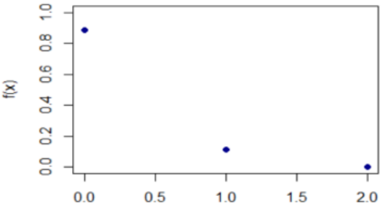
$$F(0) = P(X \leq 0) = 0.866$$
$$F(1) = P(X \leq 1) = 0.886 + 0.111 = 0.997$$
$$F(2) = P(X \leq 2) = 1$$

cmf and pmf

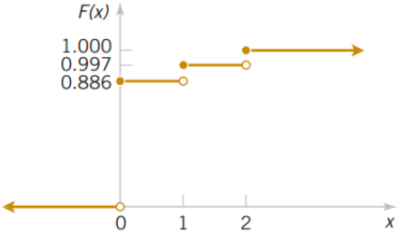
2 Cumulative Distribution Functions

x	0	1	2
f(x)	0.886	0.111	0.003

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.886 & 0 \le x < 1 \\ 0.997 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$$



Probability mass function pmf



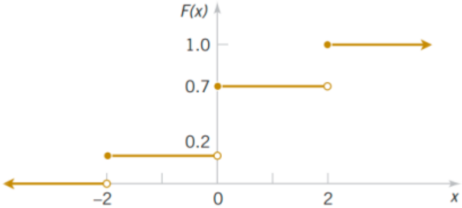
Cumulative distribution function cdf

cdf - Example

2 Cumulative Distribution Functions

Example 2.5. Determine the pmf of X from the following cdf.

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.2 & -2 \le x < 0 \\ 0.7 & 0 \le x < 2 \\ 1 & 2 \le x \end{cases}$$



Find f(x) from F(x):
$f(x) = F(x) - F(x^-)$

x	-3	-2	-1	0	1	2	3
F(x)	0	0.2	0.2	0.7	0.7	1	1
f(x)	0	0.2	0	0.5	0	0.3	0

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Mean and Variance

Definition 3.1

The mean or expected value of X is $\mu = E(X) = \sum_x xP(X = x) = \sum_x xf(x)$

The variance of X is $\sigma^2 = V(X) = E(X-\mu)^2 = \sum_x (x-\mu)^2 f(x) = \sum_x x^2 f(x) - \mu^2$

The standard deviation of X is $\sigma = \sqrt{\sigma^2}$.

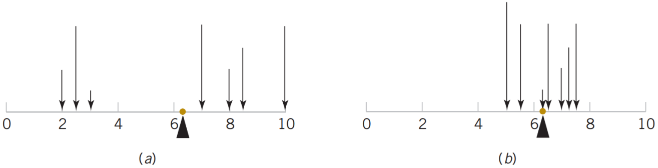

FIGURE 3-5 A probability distribution can be viewed as a loading with the mean equal to the balance point. Parts (a) and (b) illustrate equal means, but part (a) illustrates a larger variance.

Mean and Variance - Example

Example 3.1. The number of e-mail messages per hour has the following distribution:

x = number of messages	10	11	12	13	14	15
$f(x)$	0.08	0.15	0.30	0.20	0.20	0.07

Determine the mean and standard deviation of the number of messages received per hour.

Mean and Variance - Example

Example 3.2. The number of e-mail messages per hour has the following distribution:

x = number of messages	10	11	12	13	14	15
$f(x)$	0.08	0.15	0.30	0.20	0.20	0.07

Determine the mean and standard deviation of the number of messages received per hour.

Solution.

$$E(X) = \sum_x x f(x) = 12.5,\ V(X) = \sum_x x^2 f(x) - E^2(X) = 1.85,$$

$$\sigma = \sqrt{V(X)} = \sqrt{1.85} = 1.36$$

Expected Value of a Function of a Discrete RV

Definition 3.2

If X is a discrete random variable with probability mass function of $f(x)$, then

$$E(h(X)) = \sum_x h(x)f(x)$$

Properties 3.1

1. $E(aX+b) = aE(X)+b$
2. $V(aX+b) = a^2V(X)$
3. $\sigma^2 = \mathrm{V(X)} = \Sigma_{\mathrm{x}}(\mathrm{X}-\mu)^2\mathrm{f(x)} = \mathrm{E}\left[(\mathrm{X}-\mu)^2\right] = \mathrm{E}\left[\mathrm{X}^2 - 2\mu\mathrm{X} + \mu^2\right] = \mathrm{E}\left(\mathrm{X}^2\right) - \mu^2 = \mathrm{E}\left(\mathrm{X}^2\right) - \mathrm{E(X)}^2$

Example 3.3.

- Putting $h(x) = x^2$ then $E(X^2) = \sum_x x^2 f(x)$
- Putting $h(x) = e^x$ then $E(e^X) = \sum_x e^x f(x)$

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4 Discrete Uniform Distribution

Discrete Uniform Distribution

Definition 4.1

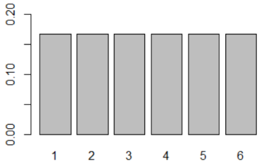
A random variable X has a **discrete uniform distribution** if each of the n values in its range, say, $x_1, x_2, ..., x_n$ has **equal probability**. Then

$$f(x_i) = P(X = x_i) = \frac{1}{n}$$

Mean and Variance. Suppose that X is a discrete uniform random variable on the consecutive integers $a, a+1, a+2, ..., b$ for $a \leq b$. Then

$$\mu = E(X) = \frac{b+a}{2}, \ \sigma^2 = V(X) = \frac{(b-a+1)^2 - 1}{12}$$

Example 4.1. Roll a fair die. Let X be the number shown. Then X is discrete uniform range of 1 to 6.
$f(1) = f(2) = \ldots = f(6) = 1/6$



Discrete Uniform Distribution - Example

Example 4.2. Let the random variable X denote the number of the voice lines that are in use at a particular time. Assume that X is a discrete uniform random variable with a range of 0 to 48. Find $E(X)$, $V(X)$.

Discrete Uniform Distribution - Example

Example 4.4. Let the random variable X denote the number of the voice lines that are in use at a particular time. Assume that X is a discrete uniform random variable with a range of 0 to 48. Find $E(X)$, $V(X)$.

Solution. $E(X) = \dfrac{48+0}{2} = 24,\ V(X) = \sigma^2 = \dfrac{(48-0+1)^2-1}{12} = 200.$

Example 4.5. Suppose the discrete uniform random variable Y has range $5, 10, \ldots, 30$. Let $Y = 5X$, where X has range $1, 2, \ldots, 6$. Find $E(Y)$, $V(Y)$.

Example 4.6. Let the random variable X denote the number of the voice lines that are in use at a particular time. Assume that X is a discrete uniform random variable with a range of 0 to 48. Find $E(X)$, $V(X)$.

Solution. $E(X) = \dfrac{48+0}{2} = 24,\ V(X) = \sigma^2 = \dfrac{(48-0+1)^2-1}{12} = 200.$

Example 4.7. Suppose the discrete uniform random variable Y has range $5, 10, \ldots, 30$. Let $Y = 5X$, where X has range $1, 2, \ldots, 6$. Find $E(Y)$, $V(Y)$.

Solution.

$E(Y) = 5E(X) = 5(1+6)/2 = 17.5$

$V(Y) = 5^2V(X) = 25[(6-1+1)^2-1]/12 = 72.92$

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5 Binomial Distribution

Bernoulli trials

Definition 5.1

Bernoulli trial: A trial with only two possible outcomes (success or failure).

Definition 5.2

Independence: The outcome from one trial has no effect on the outcome to be obtained from any other trial.

Example 5.1. The following random experiments are series of Bernoulli trials:

- Flip a coin 10 times.
- Guess each question of a multiple-choice exam with 50 questions, each with four choices.

Definition 5.3

A random experiment consists of n Bernoulli trials such that

1. The trials are independent.
2. Each trial results in only two possible outcomes, labeled as "success" and "failure".
3. The probability of a success in each trial, denoted as p, remains constant.

The random variable X that equals the number of trials that result in a success is a binomial random variable with parameters $0 < p < 1$ and $n = 1, 2, \ldots$. The probability mass function of X is

$$f(x) = C_n^x p^x (1-p)^{n-x},\ \ x = 0, 1, 2, ..., n$$

Mean and Variance If X is a binomial random variable with parameters p and n,

$$\mu = E(X) = np$$
$$\sigma^2 = V(X) = np(1-p)$$

Binomial Distribution

5 Binomial Distribution

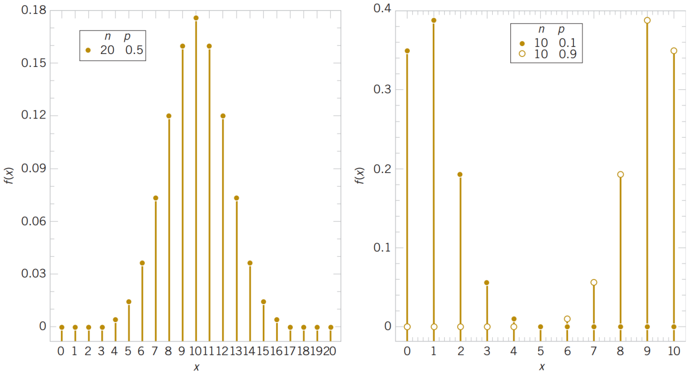

Figure: Binomial distributions for selected values of n and p

Example 5.2. Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant.

a) Find the probability that in the next 18 samples, exactly 2 contain the pollutant.

b) Determine the probability that at least four samples contain the pollutant.

Binomial Distribution - Example

Example 5.3. Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant.
a) Find the probability that in the next 18 samples, exactly 2 contain the pollutant.
b) Determine the probability that at least four samples contain the pollutant.

Solution. Let X be the number of samples that contain the pollutant in the next 18 samples analyzed. Then X is a binomial random variable with p = 0.1 and n = 18.
a) $P(X = 2) = C_{18}^{2}(0.1)^2(0.9)^{16} = 153.(0.1)^2.(0.9)^{16} = 0.284$
b) $P(X \geq 4) = 1 - P(X < 4) = 1 - \sum_{x=0}^{3} P(X = x) = 1 - \sum_{x=0}^{3} C_{18}^{x}(0.1)^x(0.9)^{18-x} = 0.098$

Binomial Distribution - Example

Example 5.4. A quality control engineer tests the quality of produced computers. Suppose that 5defects occur independently of each other. Find the probability of exactly 3 defective computers in a shipment of twenty.

Example 5.6. A quality control engineer tests the quality of produced computers. Suppose that 5defects occur independently of each other. Find the probability of exactly 3 defective computers in a shipment of twenty.

Solution. $P(X = 3) = C_{20}^{3}(0.05)^3(0.95)^{17} = 0.0596$

Example 5.7. The random variable X has a binomial distribution with $n = 10$ and $p = 0.2$. Find $P(X = 4)$, $P(X = 6)$, $P(X \leq 3)$, $P(X \geq 4)$.

Binomial Distribution - Example

Example 5.8. A quality control engineer tests the quality of produced computers. Suppose that 5defects occur independently of each other. Find the probability of exactly 3 defective computers in a shipment of twenty.

Solution. $P(X=3) = C_{20}^{3}(0.05)^3(0.95)^{17} = 0.0596$

Example 5.9. The random variable X has a binomial distribution with $n = 10$ and $p = 0.2$. Find $P(X = 4)$, $P(X = 6)$, $P(X \leq 3)$, $P(X \geq 4)$.

Solution.
$P(X = 4) = f(4) = C_{10}^{4} 0.2^4 (1 - 0.2)^{10-4} = 0.088$
$P(X = 6) = f(6) = C_{10}^{6} 0.2^6 (1 - 0.2)^{10-6} = 0.0055$
$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) \approx 0.879$
$P(X \geq 4) = 1 \breve{} P(X \leq 3) \approx 0.121$

Binomial Distribution - Example

Example 5.10. A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability 0.4, independently of other computers. Find the expected number of computers attacked by this virus.

Binomial Distribution - Example

5 Binomial Distribution

Example 5.11. A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability 0.4, independently of other computers. Find the expected number of computers attacked by this virus.

Solution.

Let X = "number of computers attacked by the virus"

$\Longrightarrow X \sim binom(n = 20, p = 0.4)$

$\Longrightarrow E(X) = np = 20(0.4) = 8$ computers.

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Geometric Distribution

Definition 6.1

In a series of Bernoulli trials (independent trials with constant probability p of a success), the random variable X that equals the number of trials until the first success is a geometric random variable with parameter $0 < p < 1$ and

$$f(x) = (1-p)^{x-1}p$$

Mean and Variance

If X is a geometric random variable with parameter p,

$$\mu = E(X) = \frac{1}{p},\ \sigma^2 = V(X) = \frac{1-p}{p^2}$$

Example 6.1. A search engine goes through a list of sites looking for a given key phrase. Suppose the search terminates as soon as the key phrase is found. The number of sites visited has geometric distribution.

Geometric Distribution

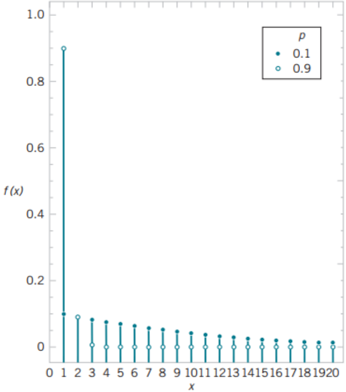

Figure: Geometric distributions for selected values of the parameter p.

Geometric Distribution - Example: Digital Channel

Example 6.2. The probability that a bit transmitted through a digital transmission channel is received in error is 0.1. Assume that the transmissions are independent events. Compute probability that the first four bits are transmitted correctly and the fifth bit is in error.

Geometric Distribution - Example: Digital Channel

Example 6.3. The probability that a bit transmitted through a digital transmission channel is received in error is 0.1. Assume that the transmissions are independent events. Compute probability that the first four bits are transmitted correctly and the fifth bit is in error.

Solution. Let the random variable X denote the number of bits transmitted until the first error.

$P(X = 5) = P(okay\ okay\ okay\ okay\ fail) = P(okay)^4.P(fail) = (0.9)^4(0.1)$

Example 6.4. The probability that a bit transmitted through a digital transmission channel is received in error is 0.1. Assume the transmissions are independent events, and let the random variable X denote the number of bits transmitted until the first error. What is the expected number and variance of bits transmitted until the first error?

Geometric Distribution - Example

Example 6.5. The probability that a bit transmitted through a digital transmission channel is received in error is 0.1. Assume the transmissions are independent events, and let the random variable X denote the number of bits transmitted until the first error. What is the expected number and variance of bits transmitted until the first error?

Solution.
$E(X) = 1/p = 10$ bits.
$\sigma^2 = V(X) = (1-p)/p^2 = (1-0.1)/(0.1)^2$

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7 Negative Binomial Distributions

Negative Binomial Distributions

(Generalization of a geometric distribution)

Definition 7.1

In a series of Bernoulli trials (independent trials with constant probability p of a success), the random variable X that equals the number of trials until r successes occur is a negative binomial random variable with parameters $0 < p < 1$ and $x = r, r+1, r+2, \ldots$

$$f(x) = C_{x-1}^{r-1}(1-p)^{x-r}p^r$$

Mean and Variance

If X is a negative binomial distribution with parameters p and r, then

$$\mu = E(X) = \frac{r}{p},\ \sigma^2 = V(X) = \frac{r(1-p)}{p^2}$$

Negative Binomial Distributions

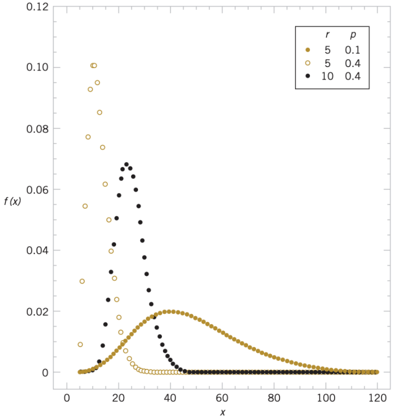

Figure: Negative binomial distributions for selected values of the parameters r and p.

Negative Binomial Distributions - Example

Example 7.1. A website contains three identical computer servers. Only one is used to operate the site, and the other two are spares that can be activated in case the primary system fails. The probability of a failure in the primary computer (or any activated spare system) from a request for service is 0.0005. Assuming that each request represents an independent trial.

a) Compute the probability that all three servers fail within five requests?

b) What is the mean number of requests until failure of all three servers?

Example 7.2. A website contains three identical computer servers. Only one is used to operate the site, and the other two are spares that can be activated in case the primary system fails. The probability of a failure in the primary computer (or any activated spare system) from a request for service is 0.0005. Assuming that each request represents an independent trial.

a) Compute the probability that all three servers fail within five requests?

b) What is the mean number of requests until failure of all three servers?

Solution. a) Let X denotes the number of requests until all three servers fail. Then X has a negative binomial distributions with $p = 0.0005$ and $r = 3$.

$P(X \leq 5) = P(X = 3) + P(X = 4) + P(X = 5) =$

$0.0005^3 + C_3^2 0.0005^3 0.9995 + C_4^2 0.0005^3 (0.9995) = 1.249 \times 10^{-9}$

b) The mean number of requests until failure of all three servers is

$E(X) = 3/0.0005 = 6000(\text{requests})$

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8 Hypergeometric Distribution

Hypergeometric Distribution

Definition 8.1

A set of N objects contains

- K objects classified as success
- $N-K$ objects classified as failures

A sample of size n objects is selected randomly (without replacement) from the N objects, where $K \leq N$ and $n \leq N$.
Let the random variable X denote the number of successes in the sample. Then X is a hypergeometric random variable and

$$f(x) = \frac{C_K^x C_{N-K}^{n-x}}{C_N^n}, \ x = \max\{0, n+K-N\} \text{ to } \min\{K, n\}$$

Mean and Variance If X is a hypergeometric random variable with parameters N, K, n then

$$\mu = E(X) = np, \ \sigma^2 = V(X) = np(1-p)\frac{N-n}{N-1}, \quad p := \frac{K}{N}$$

Here p is interpreted as the proportion of successes in the set of N objects.

Hypergeometric Distribution

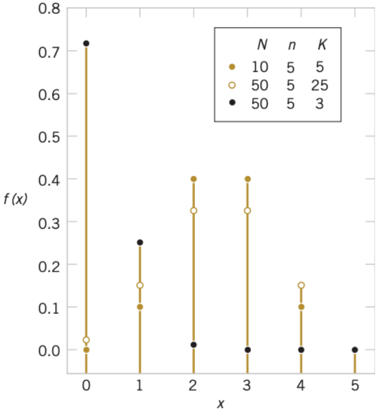

Figure: Hypergeometric distributions for selected values of parameters N, K, and n.

Hypergeometric Distribution - Example

Example 8.1. A batch of parts contains 100 parts from a local supplier of tubing and 200 parts from a supplier of tubing in the next state. Four parts are selected randomly and without replacement.

a) What is the probability they are all from the local supplier?

b) What is the probability that two or more parts in the sample are from the local supplier?

Let X equal the number of parts in the sample from the local supplier. Then X has a hypergeometric distribution.

a) The requested probability is $P(X = 4)$. Consequently,

$$P(X = 4) = \frac{C_{100}^{4} C_{200}^{0}}{C_{300}^{4}} = 0.0119$$

b)

$$\begin{aligned} P(X \geq 2) &= P(X = 2) + P(X = 3) + P(X = 4) \\ &= \frac{C_{100}^{2} C_{200}^{2}}{C_{300}^{4}} + \frac{C_{100}^{3} C_{200}^{1}}{C_{300}^{4}} + \frac{C_{100}^{4} C_{200}^{0}}{C_{300}^{4}} \\ &= 0.298 + 0.098 + 0.0119 = 0.408 \end{aligned}$$

Finite Population Correction Factor

The term in the variance of a hypergeometric random variable

$$\frac{N-n}{N-1}$$

is called the finite population correction factor.

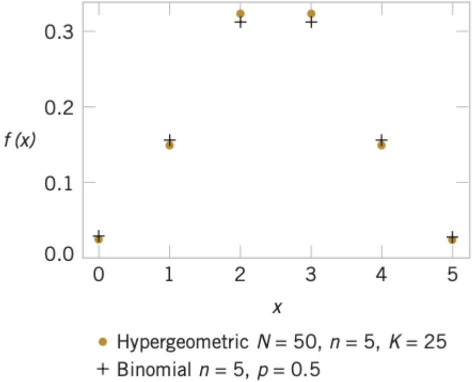

Figure: Comparison of hypergeometric and binomial distributions.

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Poisson Distribution

Definition 9.1

The random variable X that equals the number of events in the interval is a Poisson random variable with parameter $\lambda > 0$, and the probability mass function of X is

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!},\ x = 0, 1, 2, ...$$

Mean and Variance

If X is a Poisson Random Variable then

$$\mu = E(X) = \lambda,\ \sigma^2 = V(X) = \lambda$$

Poisson Distribution

9 Poisson Distribution

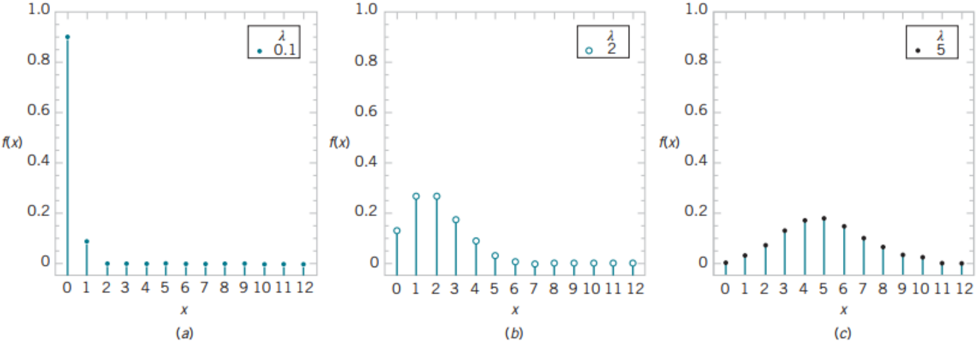

Figure: Poisson distributions for selected values of the parameters.

Example 9.1. Contamination is a problem in the manufacture of optical storage disks. The number of particles per centimeter squared of media surface is 0.1. The area of a disk under study is 100 cm^2.

a) Compute the probability that 12 particles occur in the area of a disk under study.

b) Compute the probability that zero particles in the area of the disk under study.

Poisson Distribution - Example

Example 9.2. Contamination is a problem in the manufacture of optical storage disks. The number of particles per centimeter squared of media surface is 0.1. The area of a disk under study is 100 cm^2.

a) Compute the probability that 12 particles occur in the area of a disk under study.

b) Compute the probability that zero particles in the area of the disk under study.

Solution. Let X denote the number of particles in the area of under study.

a)

$$E(X) = 100 \times 0.1 = 10 \quad \text{(particles)}$$

$$\longrightarrow P(X = 12) = \frac{e^{-10}10^{12}}{12!} = 0.095$$

b) $P(X = 0) = e^{-10} = 4.54 \times 10^{-5}$

Summary

9 Poisson Distribution

Binomial Distribution

n	=	number of trials
X	=	number of successes
p	=	probability of success
P(x)	=	$\binom{n}{x}p^x(1-p)^{n-x}$
E(X)	=	np
V(X)	=	np(1 – p)

Negative Binomial Distribution

X	=	number of trials until r successes
p	=	probability of success
P(x)	=	$\binom{x-1}{r-1}p^r(1-p)^{x-r}$, $x = r, r+1, \ldots$
E(X)	=	r/p
V(X)	=	$r(1-p)/p^2$

Poisson Distribution

λ: frequency, average number of events

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

$\mu = \lambda$

$\sigma^2 = \lambda$

Geometric Distribution

X	=	number of trials
p	=	probability of success
P(x)	=	$p(1-p)^{x-1}$, $x = 1, 2, \ldots$
E(X)	=	1/p
V(X)	=	$(1-p)/p^2$

Hypergeometric Distribution

N	=	number of objects
K	=	number of success-objects
n	=	sample size
P(x)	=	$\dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$
E(X)	=	np, where p = K/N
V(X)	=	$np(1-p)\dfrac{N-n}{N-1}$

Q&A

Thank you for listening!