Chapter 4:
Continuous Random Variables & Probability Distributions

MAS291 - STATISTICS & PROBABILITY

Ly Anh Duong

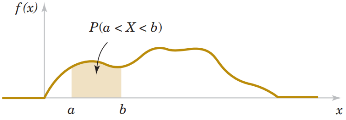
duongla3@fe.edu.vn

Probability Density Function (pdf)

Definition: For a continuous random variable X, a probability density function is a function such that

- $f(x) \geq 0$
- $\displaystyle\int_{-\infty}^{+\infty} f(x)dx = 1$
- $\forall\ a, b \in \mathbb{R}$,

$$P(a \leq X \leq b) = \int_a^b f(x)dx = P(a < X \leq b) = P(a \leq X < b) = P(a < X < b)$$



Note: $P(X = x_0) = 0$ (different from pmf)

Example

Let X be a continuous random variable and suppose that $f(x) = cx^2$, for $-1 < x < 2$, and 0 otherwise is the pdf of X.

a) Find c.

b) Find $P(X \geqslant 0)$.

Let X be a continuous random variable and suppose that $f(x) = cx^2$, for $-1 < x < 2$, and 0 otherwise is the pdf of X.

a) Find c.

b) Find $P(X \geqslant 0)$.

Solution

a) $\displaystyle\int_{-\infty}^{\infty} f(x)dx = \int_{-1}^{2} cx^2 dx = c(8/3 + 1/3) = 3c$

$f(x)$ is the pdf of $X \Longleftrightarrow \displaystyle\int_{-\infty}^{\infty} f(x)dx = 1 \Longleftrightarrow c = 1/3$

b) $P(X \geqslant 0) = \displaystyle\int_{0}^{2} (1/3)\mathrm{x}^2 dx = 8/9$

Example

1 Probability Distributions and Probability Density Functions

The lifetime, in years, of some electronic component is a continuous random variable with the probability density function

$$f(x) = \begin{cases} \dfrac{k}{x^3} & , x > 1 \\ 0 & , \text{otherwise} \end{cases}$$

Find k and compute the probability for the lifetime to exceed 5 years.

The lifetime, in years, of some electronic component is a continuous random variable with the probability density function

$$f(x) = \begin{cases} \dfrac{k}{x^3} & , x > 1 \\ 0 & , \text{otherwise} \end{cases}$$

Find k and compute the probability for the lifetime to exceed 5 years.

Solution

$$\cdot \int_{-\infty}^{\infty} f(x)dx = \int_{1}^{\infty} \frac{k}{x^3}dx = -k(0-1)/2 = 1 \Longleftrightarrow k = 2$$

Table of Contents

Cumulative Distribution Functions (cdf)

Definition: The cumulative distribution function of a continuous random variable X is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)du, \ \forall x \in \mathbb{R}$$

pdf from the cdf: Given $F(x)$, $f(x) = F'(x)$ as long as the derivative exists.

Note:

- $P(a \leq X \leq b) = F(b) - F(a)$.
- $F(x) \leq F(y)$, if $x \leq y$ (increasing).
- $F(x) = P(X \leq x) = P(X < x)$.
- $\lim\limits_{x \to -\infty} F(x) = 0, \ \lim\limits_{x \to +\infty} F(x) = 1$.

Exercise

2 Cumulative Distribution Functions

The time until a chemical reaction is complete (in milliseconds) is approximated by the cumulative distribution function

$$F(x) = \begin{cases} 0 & , x < 0 \\ 1 - e^{-0.01x} & , x \geq 0 \end{cases}$$

a) Determine the probability density function of X.

b) Compute the probability that a reaction completes within 200 milliseconds.

The time until a chemical reaction is complete (in milliseconds) is approximated by the cumulative distribution function

$$F(x) = \begin{cases} 0 & , x < 0 \\ 1 - e^{-0.01x} & , x \geq 0 \end{cases}$$

a) Determine the probability density function of X.
b) Compute the probability that a reaction completes within 200 milliseconds.

Solution

a)

$$f(x) = \begin{cases} 0 & , x < 0 \\ 0.01e^{-0.01x} & , x \geq 0 \end{cases}$$

b) $P(X \leq 200) = F(200) = 1 - e^{-2} = 0.8647.$

Table of Contents

3 Mean and Variance of a Continuous Random Variable

Suppose that X is a continuous random variable with probability density function $f(x)$. Then

- The mean or expected value of X is $\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$
- The variance of X is

$$\sigma^2 = V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx = \int_{-\infty}^{\infty} x^2 f(x) - \mu^2$$

- The standard deviation of X is $\sigma = \sqrt{\sigma^2}$

Expected Value of a Function of a Continuous Random Variable

If X is a continuous random variable with probability density function $f(x)$,

$$E[h(X)] = \int_{-\infty}^{\infty} h(x) f(x) dx$$

Exercise

Given the pdf of X,

$$f(x) = \begin{cases} \dfrac{3}{x^4} \text{ for } x > 1 \\ 0 \text{ otherwise} \end{cases}$$

Find the mean and variance of X.

Exercise

Given the pdf of X,

$$f(x) = \begin{cases} \dfrac{3}{x^4} \text{ for } x > 1 \\ 0 \text{ otherwise} \end{cases}$$

Find the mean and variance of X.

Solution

$$\mathrm{E(X)} = \int_{-\infty}^{\infty} x f(x) dx = \int_{1}^{\infty} \frac{3}{x^3} dx = 1.5$$

$$\mathrm{V(X)} = \mathrm{E}\left(\mathrm{X}^2\right) - \mathrm{E}^2(\mathrm{X}) = 0.75$$

Table of Contents

4 Continuous Uniform Distribution

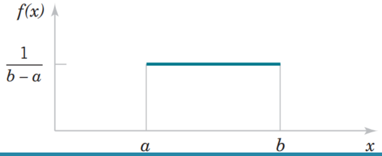
Definition: Continuous uniform distribution is a continuous random variable X with probability density function

$$f(x) = \frac{1}{b-a},\ a \leq x \leq b$$

Mean and Variance

If X is a continuous uniform random variable over $a \leq x \leq b$ then

$$\mu = E(X) = \frac{(a+b)}{2} \text{ and } \sigma^2 = V(X) = \frac{(b-a)^2}{12}$$



Exercise

Let the continuous random variable X denote the current measured in a thin copper wire in milliamperes. Assume the range of X is $[0, 20 \text{ mA}]$, and assume that the probability density function of X is

$$f(x) = 0.05,\ 0 \leq x \leq 20$$

a) Compute the probability that a measurement of current is between 5 and 10 milliamperes.
b) Compute mean and variance of X.

Exercise

4 Continuous Uniform Distribution

Let the continuous random variable X denote the current measured in a thin copper wire in milliamperes. Assume the range of X is $[0, 20 \text{ mA}]$, and assume that the probability density function of X is

$$f(x) = 0.05,\ 0 \leq x \leq 20$$

a) Compute the probability that a measurement of current is between 5 and 10 milliamperes.
b) Compute mean and variance of X.

Solution

a) $P(5 < X < 10) = (10 - 5)(0.05) = 0.25$
b) $E(X) = (a + b)/2 = (0 + 20)/2 = 10$
$V(X) = (b–a)^2/12 = (20–0)^2/12 = 33.33$

Table of Contents

5 Normal Distribution

Definition: A normal random variable X with parameter μ and $\sigma > 0$ if its probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\dfrac{-(x-\mu)^2}{2\sigma^2}}$$

Mean and Variance: $X \sim \mathcal{N}(\mu, \sigma^2) \Rightarrow E(X) = \mu, V(X) = \sigma^2$

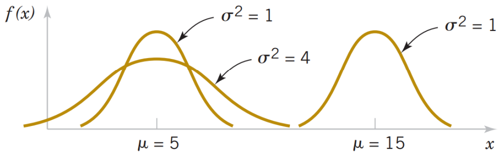


Figure: Normal pdfs for selected values of the parameters μ and σ^2.

Normal Distribution (Gaussian distribution)

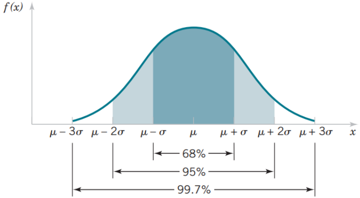


Figure: Probabilities associated with a normal distribution.

For any normal random variable,

$$P(\mu - \sigma < X < \mu + \sigma) = 0.6827$$

$$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9545$$

$$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$$

Standard Normal Random Variable

A normal random variable with

$$\mu = 0,\, \sigma^2 = 1$$

is called a standard normal random variable and is denoted as Z.
The probability density function of Z is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

Notation: $Z \sim \mathcal{N}(0, 1)$
The cumulative distribution function of a standard normal random variable is denoted as

$$\Phi(z) = P(Z \le z)$$

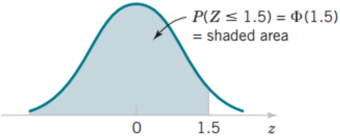
Example 1

5 Normal Distribution

EXAMPLE 4.9 | Standard Normal Distribution

Assume that Z is a standard normal random variable. Appendix Table III provides probabilities of the form $\Phi(z) = P(Z \leq z)$. The use of Table III to find $P(Z \leq 1.5)$ is illustrated in Figure 4.12. Read down the z column to the row that equals 1.5. The probability is read from the adjacent column, labeled 0.00, to be 0.93319.

The column headings refer to the hundredths digit of the value of z in $P(Z \leq z)$. For example, $P(Z \leq 1.53)$ is found by reading down the z column to the row 1.5 and then selecting the probability from the column labeled 0.03 to be 0.93699.



z	0.00	0.01	0.02	0.03
0	0.50000	0.50399	0.50800	0.51197
⋮		⋮		
1.5	0.93319	0.93448	0.93574	0.93699

FIGURE 4.12

Standard normal probability density function.

Example 2

5 Normal Distribution

EXAMPLE 4.10 | Normal Distribution Calculations

The following calculations are shown pictorially in Figure 4.13. In practice, a probability is often rounded to one or two significant digits.

1. $P(Z > 1.26) = 1 - P(Z \leq 1.26) = 1 - 0.89616 = 0.10384$.
2. $P(Z < -0.86) = 0.19490$.
3. $P(Z > -1.37) = P(Z < 1.37) = 0.91465$.
4. $P(-1.25 < Z < 0.37)$. This probability can be found from the difference of two areas, $P(Z < 0.37) - P(Z < -1.25)$. Now,

   $$P(Z < 0.37) = 0.64431$$

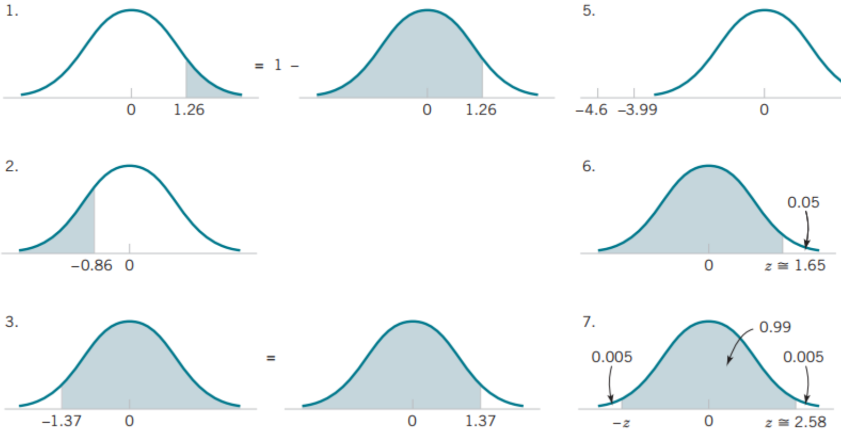
   and

   $$P(Z < -1.25) = 0.10565$$

   Therefore,
5. $P(Z \leq -4.6)$ cannot be found exactly from Appendix Table III. However, the last entry in the table can be used to find that $P(Z \leq -3.99) = 0.00003$. Because $P(Z \leq -4.6) < P(Z \leq -3.99)$, $P(Z \leq -4.6)$ is nearly zero.
6. Find the value z such that $P(Z > z) = 0.05$. This probability expression can be written as $P(Z \leq z) = 0.95$. Now Table III is used in reverse. We search through the probabilities to find the value that corresponds to 0.95. The solution is illustrated in Figure 4.13. We do not find 0.95 exactly; the nearest value is 0.95053, corresponding to $z = 1.65$.
7. Find the value of z such that $P(-z < Z < z) = 0.99$. Because of the symmetry of the normal distribution, if the area of the shaded region in Figure 4.13(7) is to equal 0.99, the area in each tail of the distribution must equal 0.005. Therefore, the value for z corresponds to a probability of 0.995 in Table III. The nearest probability in Table III is 0.99506 when $z = 2.58$.

Example 2

5 Normal Distribution



$$\text{If } X \sim \mathcal{N}(\mu, \sigma^2) \text{ then } Z = \frac{X - \mu}{\sigma} \sim \mathcal{N}(0, 1)$$

Example: Suppose that the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 milliamperes and a variance of 4 $(milliamperes)^2$. What is the probability that a measurement exceeds 13 milliamperes?

Standardizing a Normal Random Variable

$$\text{If } X \sim \mathcal{N}(\mu, \sigma^2) \text{ then } Z = \frac{X - \mu}{\sigma} \sim \mathcal{N}(0,1)$$

Example: Suppose that the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 milliamperes and a variance of 4 $(milliamperes)^2$. What is the probability that a measurement exceeds 13 milliamperes?

Solution

Let X denote the current in milliamperes. The requested probability can be represented as $P(X > 13)$. Let $Z = (X - 10)/2$. The relationship between the several values of X and the transformed values of Z are shown in Figure 4.14. We note that $X > 13$ corresponds to $Z > 1.5$. Therefore, from Appendix Table III,

$$P(X > 13) = P(Z > 1.5) = 1 - P(Z \leq 1.5) = 1 - 0.93319 = 0.06681$$

Solution

5 Normal Distribution

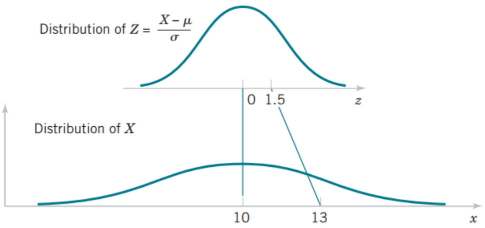


FIGURE 4.14

Standardizing a normal random variable.

Practical Interpretation: Probabilities for any normal random variable can be computed with a simple transform to a standard normal random variable

Standardizing to Calculate a Probability

Suppose that X is a normal random variable with mean μ and variance σ^2. Then,

$$P(X \leq x) = P(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}) = P(Z \leq z)$$

where Z is a standard normal random variable, and $z = \dfrac{x-\mu}{\sigma}$ is the $z - value$ obtained by standardizing X. The probability is obtained by using Appendix Table III with $z = \dfrac{x-\mu}{\sigma}$.

Example (Normally Distributed Current)

5 Normal Distribution

Continuing the previous example.

a) What is the probability that a current measurement is between 9 and 11 milliamperes?

b) Determine the value for which the probability that a current measurement is below this value is 0.98

Example (Normally Distributed Current)

Continuing the previous example.

a) What is the probability that a current measurement is between 9 and 11 milliamperes?

b) Determine the value for which the probability that a current measurement is below this value is 0.98

Solution

a)

$$\begin{aligned} P(9 < X < 11) &= P\left(\frac{9-10}{2} < \frac{X-10}{2} < \frac{11-10}{2}\right) \\ &= P(-0.5 < Z < 0.5) \\ &= P(Z < 0.5) - P(Z < -0.5) \\ &= 0.69146 - 0.30854 = 0.38292 \end{aligned}$$

b)

$$P(X < x) = P\left(\frac{X-10}{2} < \frac{x-10}{2}\right)$$
$$= P\left(Z < \frac{x-10}{2}\right) = 0.98$$

Appendix Table III is used to find the z-value such that $P(Z < z) = 0.98$. The nearest probability from Table III results in

$$P(Z < 2.06) = 0.980301$$

Therefore, $(x-10)/2 = 2.06$, and the standardizing transformation is used in reverse to solve for x. The result is

$$x = 2(2.06) + 10 = 14.1 \text{ mA}$$

Table of Contents

Normal Approximation to the Binomial Distribution

If $X \sim \mathcal{B}(n, p)$ then

$$Z = \frac{X - np}{\sqrt{np(1-p)}} \approx \mathcal{N}(0,1)$$

To approximate a binomial probability with a normal distribution, a continuity correction is applied as follows:

$$P(X \leq x) = P(X \leq x + 0.5) \approx P\left(Z \leq \frac{x + 0.5 - np}{\sqrt{np(1-p)}}\right)$$

$$P(X \geq x) = P(X \geq x - 0.5) \approx P\left(Z \geq \frac{x - 0.5 - np}{\sqrt{np(1-p)}}\right)$$

The approximation is good for $np > 5$ and $n(1-p) > 5$.

Exercise

Assume that in a digital communication channel, the number of bits received in error can be modeled by a binomial random variable, and assume that the probability that a bit is received in error is 10^{-5}. If 16 million bits are transmitted, what is the probability that 150 or fewer errors occur?

Assume that in a digital communication channel, the number of bits received in error can be modeled by a binomial random variable, and assume that the probability that a bit is received in error is 10^{-5}. If 16 million bits are transmitted, what is the probability that 150 or fewer errors occur?

Solution

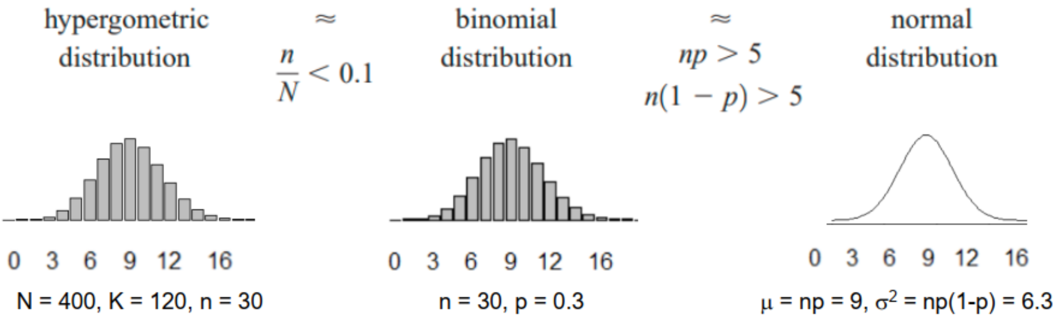
Let the random variable X denote the number of errors. Then X is a binomial random variable with $n = 16 \times 10^6$ and $p = 10^{-5}$, and

$$P(X \leq 150) = \sum_{x=0}^{150} \binom{16,000,000}{x} \left(10^{-5}\right)^x \left(1 - 10^{-5}\right)^{16,000,000-x} \qquad \text{DIFFICULT!}$$

Use normal distribution to approximate

$$P(X \leq 150) = P(X \leq 150.5) = P\left(\frac{X - 160}{\sqrt{160\left(1 - 10^{-5}\right)}} \leq \frac{150.5 - 160}{\sqrt{160\left(1 - 10^{-5}\right)}}\right) \approx P(Z \leq -0.75) = 0.227$$

Conditions for approximating hypergeometric and binomial probabilities



Normal Approximation the Poisson Distribution

If $X \sim \mathcal{P}(\lambda)$ $(E(X) = \lambda$, $V(X) = \lambda$ then

$$Z = \frac{X - \lambda}{\sqrt{\lambda}} \approx \mathcal{N}(0, 1)$$

The approximation is good for $\lambda > 5$.
Normal Approximation to Poisson as follows

$$P(X \leq x) = P(X \leq x + 0.5) \approx P\left(Z \leq \frac{x + 0.5 - \lambda}{\sqrt{\lambda}}\right)$$
$$P(X \geq x) = P(X \geq x - 0.5) \approx P\left(Z \geq \frac{x - 0.5 - \lambda}{\sqrt{\lambda}}\right)$$

Exercise

Assume that the number of asbestos particles in a squared meter of dust on a surface follows a Poisson distribution with a mean of 1000. If a squared meter of dust is analyzed, what is the probability that 950 or fewer particles are found?

Assume that the number of asbestos particles in a squared meter of dust on a surface follows a Poisson distribution with a mean of 1000. If a squared meter of dust is analyzed, what is the probability that 950 or fewer particles are found?

Solution

Let X be the number of particles found. Then $X \sim \mathcal{P}(1000)$

$$P(X \leq 950) = \sum_{x=0}^{950} \frac{e^{-1000} 1000^x}{x!}$$ DIFFICULT

AN APPROXIMATION

$$P(X \leq 950) = P(X \leq 950.5) \approx P(Z \leq \frac{950.5 - 1000}{\sqrt{1000}}) = P(Z \leq -1.57) = 0.058$$

Table of Contents

Definition: The random variable X that equals the distance between successive events from a Poisson process with mean number of events $\lambda > 0$ per unit interval is an exponential random variable with parameter λ. The probability density function of X is

$$f(x) = \lambda e^{-\lambda x},\ x \geq 0$$

Note: $F(x) = P(X \leq x) = 1 - e^{-\lambda x}, x \geq 0$

Mean and Variance

If $X \sim \mathcal{E}(\lambda)$ then $E(X) = \dfrac{1}{\lambda}, V(X) = \dfrac{1}{\lambda^2}$

Exercise

7 Exponential Distribution

In a large corporate computer network, user log-ons to the system can be modeled as a Poisson process with a mean of 25 log-ons per hour.

a) Compute probability that there are no log-ons in an interval of six minutes.

b) Compute the probability that the time until the next log-on is between two and three minutes?

Exercise

7 Exponential Distribution

In a large corporate computer network, user log-ons to the system can be modeled as a Poisson process with a mean of 25 log-ons per hour.
a) Compute probability that there are no log-ons in an interval of six minutes.
b) Compute the probability that the time until the next log-on is between two and three minutes?

Solution

Let X denote the time in hours from the start of the interval until the first log-on. Then X has an exponential distribution with $\lambda = 25$ log-ons per hour.
a) We are interested in the probability that X exceeds 6 minutes = 0.1 hour.

$$P(X > 0.1) = \int_{0.1}^{\infty} 25e^{-25x} dx = e^{-25(0.1)} = 0.082$$

b)Converting 2 minutes = 0.033 hour and 3 minutes = 0.05 hour.

$$P(0.033 < X < 0.05) = \int_{0.033}^{0.05} 25e^{-25x} dx = e^{-25x}\Big|_{0.033}^{0.05} = 0.152$$

Lack of Memory Property

For an exponential random variable X ,

$$P\left(X < t_1 + t_2 \mid X > t_1\right) = P\left(X < t_2\right)$$

Example: The time between the arrival of electronic messages at your computer is exponentially distributed with a mean of two hours $(\lambda = 1/2)$. a/ What is the probability that you do not receive a message during a two-hour period?

$$P(X > 2) = 1 - P(X \leqslant 2) = e^{-2\lambda} = e^{-1} = 0.368$$

b/ If you have not had a message in the last four hours, what is the probability that you do not receive a message in the next two hours?

$$\mathrm{P}(\mathrm{X} > 6 \mid \mathrm{X} > 4) = \mathrm{P}(\mathrm{X} > 2)$$

Table of Contents

8 APPENDIX A (A-8, A-9)

APPENDIX A (A-8, A-9)

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

TABLE III Cumulative Standard Normal Distribution

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00
−3.9	0.000033	0.000034	0.000036	0.000037	0.000039	0.000041	0.000042	0.000044	0.000046	0.000048
−3.8	0.000050	0.000052	0.000054	0.000057	0.000059	0.000062	0.000064	0.000067	0.000069	0.000072
−3.7	0.000075	0.000078	0.000082	0.000085	0.000088	0.000092	0.000096	0.000100	0.000104	0.000108
−3.6	0.000112	0.000117	0.000121	0.000126	0.000131	0.000136	0.000142	0.000147	0.000153	0.000159
−3.5	0.000165	0.000172	0.000179	0.000185	0.000193	0.000200	0.000208	0.000216	0.000224	0.000233
−3.4	0.000242	0.000251	0.000260	0.000270	0.000280	0.000291	0.000302	0.000313	0.000325	0.000337
−3.3	0.000350	0.000362	0.000376	0.000390	0.000404	0.000419	0.000434	0.000450	0.000467	0.000483
−3.2	0.000501	0.000519	0.000538	0.000557	0.000577	0.000598	0.000619	0.000641	0.000664	0.000687
−3.1	0.000711	0.000736	0.000762	0.000789	0.000816	0.000845	0.000874	0.000904	0.000935	0.000968
−3.0	0.001001	0.001035	0.001070	0.001107	0.001144	0.001183	0.001223	0.001264	0.001306	0.001350
−2.9	0.001395	0.001441	0.001489	0.001538	0.001589	0.001641	0.001695	0.001750	0.001807	0.001866
−2.8	0.001926	0.001988	0.002052	0.002118	0.002186	0.002256	0.002327	0.002401	0.002477	0.002555
−2.7	0.002635	0.002718	0.002803	0.002890	0.002980	0.003072	0.003167	0.003264	0.003364	0.003467
−2.6	0.003573	0.003681	0.003793	0.003907	0.004025	0.004145	0.004269	0.004396	0.004527	0.004661
−2.5	0.004799	0.004940	0.005085	0.005234	0.005386	0.005543	0.005703	0.005868	0.006037	0.006210
−2.4	0.006387	0.006569	0.006756	0.006947	0.007143	0.007344	0.007549	0.007760	0.007976	0.008198
−2.3	0.008424	0.008656	0.008894	0.009137	0.009387	0.009642	0.009903	0.010170	0.010444	0.010724
−2.2	0.011011	0.011304	0.011604	0.011911	0.012224	0.012545	0.012874	0.013209	0.013553	0.013903
−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864

TABLE III Cumulative Standard Normal Distribution (*continued*)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621

APPENDIX A (A-8, A-9)

−2.1	0.014262	0.014629	0.015003	0.015386	0.015778	0.016177	0.016586	0.017003	0.017429	0.017864
−2.0	0.018309	0.018763	0.019226	0.019699	0.020182	0.020675	0.021178	0.021692	0.022216	0.022750
−1.9	0.023295	0.023852	0.024419	0.024998	0.025588	0.026190	0.026803	0.027429	0.028067	0.028717
−1.8	0.029379	0.030054	0.030742	0.031443	0.032157	0.032884	0.033625	0.034379	0.035148	0.035930
−1.7	0.036727	0.037538	0.038364	0.039204	0.040059	0.040929	0.041815	0.042716	0.043633	0.044565
−1.6	0.045514	0.046479	0.047460	0.048457	0.049471	0.050503	0.051551	0.052616	0.053699	0.054799
−1.5	0.055917	0.057053	0.058208	0.059380	0.060571	0.061780	0.063008	0.064256	0.065522	0.066807
−1.4	0.068112	0.069437	0.070781	0.072145	0.073529	0.074934	0.076359	0.077804	0.079270	0.080757
−1.3	0.082264	0.083793	0.085343	0.086915	0.088508	0.090123	0.091759	0.093418	0.095098	0.096801
−1.2	0.098525	0.100273	0.102042	0.103835	0.105650	0.107488	0.109349	0.111233	0.113140	0.115070
−1.1	0.117023	0.119000	0.121001	0.123024	0.125072	0.127143	0.129238	0.131357	0.133500	0.135666
−1.0	0.137857	0.140071	0.142310	0.144572	0.146859	0.149170	0.151505	0.153864	0.156248	0.158655
−0.9	0.161087	0.163543	0.166023	0.168528	0.171056	0.173609	0.176185	0.178786	0.181411	0.184060
−0.8	0.186733	0.189430	0.192150	0.194894	0.197662	0.200454	0.203269	0.206108	0.208970	0.211855
−0.7	0.214764	0.217695	0.220650	0.223627	0.226627	0.229650	0.232695	0.235762	0.238852	0.241964
−0.6	0.245097	0.248252	0.251429	0.254627	0.257846	0.261086	0.264347	0.267629	0.270931	0.274253
−0.5	0.277595	0.280957	0.284339	0.287740	0.291160	0.294599	0.298056	0.301532	0.305026	0.308538
−0.4	0.312067	0.315614	0.319178	0.322758	0.326355	0.329969	0.333598	0.337243	0.340903	0.344578
−0.3	0.348268	0.351973	0.355691	0.359424	0.363169	0.366928	0.370700	0.374484	0.378281	0.382089
−0.2	0.385908	0.389739	0.393580	0.397432	0.401294	0.405165	0.409046	0.412936	0.416834	0.420740
−0.1	0.424655	0.428576	0.432505	0.436441	0.440382	0.444330	0.448283	0.452242	0.456205	0.460172
0.0	0.464144	0.468119	0.472097	0.476078	0.480061	0.484047	0.488033	0.492022	0.496011	0.500000

1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967

CASIO 580VN: MENU $\Longrightarrow$ 7 $\Longrightarrow$ 2 $(Lower : -3,9; Upper : z)$

Q&A

Thank you for listening!