Gödel's Incompleteness Theorems

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Kurt Gödel

A quick reminder of Axioms

Definition (Axiom)

Statements that are true without a formal proof of them.
For example:

$$x = y \land y = z \implies x = z$$

"It is possible to draw a straight line from any point to any other point"

- Any mathematical system starts out with a set of axioms

Completeness

Definition (Complete)

A set of axioms is (syntactically, or negation-) complete if, for any statement in the axioms' language, that statement or its negation is provable from the axioms. [1]

Gödel's First Incompleteness Theorem

- If axioms do not contradict each other and are computably enumerable some statements are true, but cannot be proofed.

Definition (Computably Enumerable Language)

A recursively enumerable language is a formal language for which there exists a Turing machine which will enumerate all valid strings of the language.

Gödel's First Incompleteness Theorem

- The status quo is that we have some axioms that are unprofable. Wouldn't it just make sense to add these axioms to our system, that we have a complete system now?

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//Sample Code
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References

A. Avron, "Peter smith. an introduction to gödel's theorems. cambridge introductions to philosophy, cambridge university press, 2007, xiv+ 362 pp.," *Bulletin of Symbolic Logic*, vol. 15, no. 2, pp. 218–222, 2009.