SET08118 - Propositional Logic

These questions are adapted from those in the Russel & Norvig book. For some of them, you will have to read the relevant section of the book.

1. (7.2) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

   > If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

2. (7.4) Which of the following are correct?

   - $\bot \models \top$
   - $\top \models \bot$
   - $(A \wedge B) \models (A \iff B)$
   - $(A \iff B) \models \neg A \vee B$
   - $A \iff B \models A \vee B$

3. (7.6) By constructing models (to prove) or given propositional statements (to provide counterexamples), determine the truth values of the following assertions:

   - If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both), then $(\alpha \wedge \beta) \models \gamma$.
   - If $\alpha \models (\beta \wedge \gamma)$ then $\alpha \models \beta$ and $\beta \models \gamma$.
   - If $\alpha \models (\beta \vee \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$.

4. (7.14) According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable. Which of the following are correct representations of this assertion?

   - $(R \wedge E) \iff C$
   - $R \Rightarrow (E \iff C)$
   - $R \Rightarrow ((C \Rightarrow E) \vee \neg E)$

5. (7.1 - contains references to the book, so you'll need to read those bits) Suppose the agent has progressed to the point shown in Figure 7.4(a), page 239, having perceived nothing in $[1, 1]$, a breeze in $[2, 1]$, and a stench in $[1, 2]$, and is now concerned with the contents of $[1, 3]$, $[2, 2]$ and $[3, 1]$. Each of these can contain a pit, and at most one can contain a wumpus. Following the example of Figure 7.5 (and also the Minesweeper example from the lecture), construct the set of possible worlds. There are 32 of them. Mark the worlds in which the knowledge base is true and those in which each of the following sentences is true:

- α_2 = "There is no pit in $[2, 1]$"
- α_3 = "There is a wumpus in $[1, 3]$"

Hence show that $KB \models \alpha_2$ and $KB \models \alpha_3$.