Edinburgh Napier UNIVERSITY

Physics-Based Animation (SET09119)

Tutorial 01 - Mathematics for Physics

1 Question

If $p = \sqrt{3}$ and $s = \sqrt{2}$ evaluate:

$$\sqrt{(5p - 4s)^2 - (4p - 5s)^2}$$

2 Question

Find the set of real numbers λ for which the quadratic equation:

$$x^2 - (\lambda - 3)x + \lambda = 0$$

has distinct, real roots for x.

(Remember: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

3 Question

Let $\mathbf{u} =< -4, -1, 2 >$, $\mathbf{v} =< 1, 3, 2 >$ and $\mathbf{w} =< 0, -2, -4 >$,
Find:

1. $\mathbf{u} + \mathbf{v}$
2. $-3\mathbf{u}$
3. $3\mathbf{u} - 4\mathbf{v}$
4. $2\mathbf{u} + 4\mathbf{v} - 5\mathbf{w}$

4 Question

Let $\mathbf{u} =< 0, 1, 5 >$, $\mathbf{v} =< 1, 1, 5 >$ and $\mathbf{w} =< -0, -1, 2 >$,
Find:

1. $\mathbf{u} \cdot \mathbf{v}$
2. $\mathbf{u} \cdot \mathbf{w}$
3. $\mathbf{u} \times \mathbf{w}$

4. $\mathbf{v} \times \mathbf{w}$
5. $||\mathbf{u}||$ (i.e., the length/magnitude of $\mathbf{u}$)

5 Question

1. Show that if $\vec{a}$ and $\vec{b}$ are two parallel vectors, then $\vec{a} \times \vec{b} = 0$
2. Show that if $\vec{a}$ and $\vec{b}$ are two orthogonal vectors where $\vec{c} = \vec{a} \times \vec{b}$ then

$$||c|| = ||a||\ ||b||$$

3. Find the cross product of the vectors $\vec{a} =< 1, 0, 3 >$ and $\vec{b} =< 9, -3, 1 >$. If $\vec{c} = \vec{a} \times \vec{b}$, verify that $\vec{c}^T \vec{a} = \vec{c}^T \vec{b} = 0$

6 Question

Differentiate:

1. $\frac{d}{dx}(3x^2)$
2. $\frac{d}{dx}(4x^4 - 2)$
3. $\frac{d}{dx}(x + \frac{1}{x})$
4. $\frac{d}{dx}(\sqrt[4]{x})$
5. $\frac{d}{dx}(\sqrt[5]{x} + \frac{5}{\sqrt{x}})$

7 Question

Integrate:

1. $\int 4x^3 dx$
2. $\int 2x^4 dx$
3. $\int x^{-4} dx$
4. $\int 5x^{-3} dx$
5. $\int \frac{6}{x^2} dx$

8 Question

Compute:

1. $\begin{bmatrix} 1 & 6 \\ -3 & 5 \end{bmatrix} \times \begin{bmatrix} 4 & 0 \\ 2 & -1 \end{bmatrix}$
2. $\begin{bmatrix} 1 & 6 \\ -3 & 5 \end{bmatrix} \times \begin{bmatrix} 2 \\ -7 \end{bmatrix}$
3. $\begin{bmatrix} 1 \\ -6 \end{bmatrix} \times \begin{bmatrix} 1 & 6 \\ -3 & 5 \end{bmatrix}$
4. $\begin{bmatrix} 1 \\ 6 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \end{bmatrix}$
5. $\begin{bmatrix} 2 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -6 \end{bmatrix}$

9 Question

Generate the 4×4 transformation matrix for the following transformations:

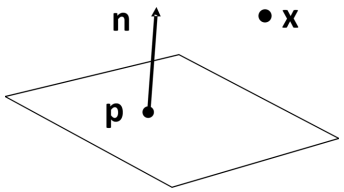
1. A translation of $< 4, 6, 7 >$
2. A scaling of $< 10, 5, 2 >$
3. A z-axis rotation of $\frac{\pi}{2}$ radians
4. A x-axis rotation of π radians
5. A y-axis rotation of $\frac{\pi}{4}$ radians

10 Question

Transform the following vectors using the transformation matrix:

$$\begin{bmatrix} 2.121 & -2.121 & 0 & 10 \\ 2.121 & 2.121 & 0 & 15 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1. $< 4, 5, 10 >$
2. $< 12, 8, 16 >$
3. $< 1, 4, 0 >$
4. $< -10, 12, -4 >$
5. $< 0, 5, -11 >$



11 Question

A plane is described by a point $p < 1, 2, 1 >$ on the plane and a unit normal $n < 0, 1, 0 >$.
Find the distance from point $x < 2, 2, 0 >$ to the plane

12 Question

Find the solution of the following system of algebraic equations:

1.
$$\begin{aligned} -x_1 + 2x_2 - x_3 &= 2 \\ 2x_1 - x_2 &= 1.5 \\ -x_2 + x_3 &= 5 \end{aligned}$$

2.
$$\begin{aligned} -3x_2 + 5x_3 &= 0 \\ -2x_1 + 2x_2 - 3x_3 &= 0 \\ 6x_1 - 2x_2 &= 5.5 \end{aligned}$$

13 Question

A triangle is defined by 3D points $\vec{a}$, $\vec{b}$, and $\vec{c}$
Find the area of the triangle given $\vec{a} < 1, 1, 0 >$, $\vec{b} < 4, 5, 1 >$, and $\vec{c} < 0, 2, 0 >$.