Visibility Determination

SET09115 - Introduction to Graphics Programming

Dr Kevin Chalmers

School of Computing
Edinburgh Napier University
Edinburgh

k.chalmers@napier.ac.uk

November 6, 2012

Outline

1 Introduction

Outline

1. Introduction

2. Bounding Volume Construction

Outline

1. Introduction
2. Bounding Volume Construction
3. Bounding Volume Tests

Outline

1. Introduction
2. Bounding Volume Construction
3. Bounding Volume Tests
4. Summary

Announcement

We have now covered everything we need for the exam part of the assessment. Everything we cover now is of interest, but will not be formally examined in Decemeber (but might be if you have to do the resit).

Outline

1. Introduction
2. Bounding Volume Construction
3. Bounding Volume Tests
4. Summary

What is a Bounding Volume?

What is a Bounding Volume?

A bounding volume is a 3-dimensional shape that we use to surround our rendered object, thus allowing us to perform such operations as visibility determination and collision detection. For rendering work, we are interested in the first point.

What is a Bounding Volume?

A bounding volume is a 3-dimensional shape that we use to surround our rendered object, thus allowing us to perform such operations as visibility determination and collision detection. For rendering work, we are interested in the first point.

A bounding volume needs to contain all the points that make up our 3D geometry, but also ensure that there is little space left over. This requires volumes that are generally not box shaped, and are not usually axis-aligned.

What is a Bounding Volume?

A bounding volume is a 3-dimensional shape that we use to surround our rendered object, thus allowing us to perform such operations as visibility determination and collision detection. For rendering work, we are interested in the first point.

A bounding volume needs to contain all the points that make up our 3D geometry, but also ensure that there is little space left over. This requires volumes that are generally not box shaped, and are not usually axis-aligned.

Our goal here is to develop techniques that will allow us to perform fast visibility determination. Many of the heavy algorithms are done as a pre-process (i.e. before we run the application).

Principal Component Analysis

Principal Component Analysis

To build a suitable bounding box, aligned to meet the shape of the geometry, we can use a statistical technique called *principal component analysis*. This allows us to build a coordinate space from our vertex positions.

Principal Component Analysis

To build a suitable bounding box, aligned to meet the shape of the geometry, we can use a statistical technique called *principal component analysis*. This allows us to build a coordinate space from our vertex positions.

The first thing we have to do is find the mean (average) position for the coordinates. This is a simple summation operation:

$$\mathbf{m} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{P}_i$$

Principal Component Analysis

To build a suitable bounding box, aligned to meet the shape of the geometry, we can use a statistical technique called *principal component analysis*. This allows us to build a coordinate space from our vertex positions.

The first thing we have to do is find the mean (average) position for the coordinates. This is a simple summation operation:

$$\mathbf{m} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{P}_i$$

We then also construct a 3×3 matrix $\mathbf{C}$ called the *covariance matrix*:

$$\mathbf{C} = \frac{1}{N}\sum_{i=1}^{N}(\mathbf{P}_i - \mathbf{m})(\mathbf{P}_i - \mathbf{m})^T$$

Covariance Matrix

Covariance Matrix

The individual values in the covariance matrix can be calculated as follows:

$$C_{11} = \frac{1}{N}\sum_{i=1}^{N}(x_i - m_x)^2 \qquad C_{12} = C_{21} = \frac{1}{N}\sum_{i=1}^{N}(x_i - m_x)(y_i - m_y)$$

$$C_{22} = \frac{1}{N}\sum_{i=1}^{N}(y_i - m_y)^2 \qquad C_{13} = C_{31} = \frac{1}{N}\sum_{i=1}^{N}(x_i - m_x)(z_i - m_z)$$

$$C_{33} = \frac{1}{N}\sum_{i=1}^{N}(z_i - m_z)^2 \qquad C_{23} = C_{32} = \frac{1}{N}\sum_{i=1}^{N}(y_i - m_y)(z_i - m_z)$$

Covariance Matrix

We won't go any further with this, as we would have to use eigenvalues (which we haven't covered). Essentially from these values we have to generate a matrix that enables us to transform our coordinate space accordingly.

Covariance Matrix

We won't go any further with this, as we would have to use eigenvalues (which we haven't covered). Essentially from these values we have to generate a matrix that enables us to transform our coordinate space accordingly.

What we need to generate is the three axses for computing our bounding volumes, $\mathbf{R}, \mathbf{S}$ and $\mathbf{T}$. Let us consider that these values are:

$$\begin{aligned} \mathbf{R} &= < 1, 0, 0 > \\ \mathbf{S} &= < 0, 1, 0 > \\ \mathbf{T} &= < 0, 0, 1 > \end{aligned}$$

Or in other words a normal axis alignment.

Outline

1 Introduction

2 Bounding Volume Construction

3 Bounding Volume Tests

4 Summary

Bounding Box Construction

Bounding Box Construction

For a bounding box, we need to construct six planes aligned to our axes, which are at the extent (i.e. the furthest point) along these axes.

Bounding Box Construction

For a bounding box, we need to construct six planes aligned to our axes, which are at the extent (i.e. the furthest point) along these axes.

Finding the maximum and minimum is just a case of calculating the dot product of each vertex against the three axses. These can be determined as follows:

$$
\begin{array}{cc}
< \mathbf{R}, -\min_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{R}) > & < -\mathbf{R}, \max_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{R}) > \\
< \mathbf{S}, -\min_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{S}) > & < -\mathbf{S}, \max_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{S}) > \\
< \mathbf{T}, -\min_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{T}) > & < -\mathbf{T}, \max_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{T}) >
\end{array}
$$

Look back at last weeks lecture to recall why this lets us define a plane.

Bounding Box Construction

Next, we need to determine the centre point of the bounding box. We do this by finding the average extents of along the defined axes. We get these by calculating the following values:

$$a = \frac{\min\limits_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{R}) + \max\limits_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{R})}{2}$$

$$b = \frac{\min\limits_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{S}) + \max\limits_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{S})}{2}$$

$$c = \frac{\min\limits_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{T}) + \max\limits_{1 \leq i \leq N}(\mathbf{P}_i \cdot \mathbf{T})}{2}$$

Bounding Box Construction

Next, we need to determine the centre point of the bounding box. We do this by finding the average extents of along the defined axes. We get these by calculating the following values:

$$a = \frac{\min\limits_{1 \leq i \leq N} (\mathbf{P}_i \cdot \mathbf{R}) + \max\limits_{1 \leq i \leq N} (\mathbf{P}_i \cdot \mathbf{R})}{2}$$

$$b = \frac{\min\limits_{1 \leq i \leq N} (\mathbf{P}_i \cdot \mathbf{S}) + \max\limits_{1 \leq i \leq N} (\mathbf{P}_i \cdot \mathbf{S})}{2}$$

$$c = \frac{\min\limits_{1 \leq i \leq N} (\mathbf{P}_i \cdot \mathbf{T}) + \max\limits_{1 \leq i \leq N} (\mathbf{P}_i \cdot \mathbf{T})}{2}$$

This allows us to calculate the centre point of the bounding box, **Q**:

$$\mathbf{Q} = a\mathbf{R} + b\mathbf{S} + c\mathbf{T}$$

Bounding Sphere Construction

Bounding Sphere Construction

For a bounding sphere, we essentially use the same initial technique for finding the three axses. This technique does have the advantage of allowing us to find the axis which has the greatest distance from one point to another - the **R** axis.

Bounding Sphere Construction

For a bounding sphere, we essentially use the same initial technique for finding the three axses. This technique does have the advantage of allowing us to find the axis which has the greatest distance from one point to another - the **R** axis.

Once we have determined the axis **R**, we also need to determine the points $\mathbf{P}_k$ and $\mathbf{P}_l$ that are the furthest extents (minimum and maximum as per bounding box) along the axis. We then define a sphere with the centre **Q** and radius r as follows:

$$\mathbf{Q} = \frac{\mathbf{P}_k + \mathbf{P}}{2}$$

$$r = \|\mathbf{P}_k - \mathbf{Q}\|$$

Bounding Sphere Construction

Our problem now lies in ensuring that all the points for our geometry are enclosed in the bounding sphere. We do this by checking each point and determining if it is inside the bounding sphere. First, we need to determine if a point lies outside our bounding sphere as follows:

$$\|\mathbf{P}_i - \mathbf{Q}\|^2 > r^2$$

Bounding Sphere Construction

Our problem now lies in ensuring that all the points for our geometry are enclosed in the bounding sphere. We do this by checking each point and determining if it is inside the bounding sphere. First, we need to determine if a point lies outside our bounding sphere as follows:

$$\|\mathbf{P}_i - \mathbf{Q}\|^2 > r^2$$

We then have to calculate the direction towards the new centre point. We calculate $\mathbf{G}$ as follows:

$$\mathbf{G} = \mathbf{Q} - r\frac{\mathbf{P}_i - \mathbf{Q}}{\|\mathbf{P}_i - \mathbf{Q}\|}$$

Bounding Sphere Construction

Our problem now lies in ensuring that all the points for our geometry are enclosed in the bounding sphere. We do this by checking each point and determining if it is inside the bounding sphere. First, we need to determine if a point lies outside our bounding sphere as follows:

$$\|\mathbf{P}_i - \mathbf{Q}\|^2 > r^2$$

We then have to calculate the direction towards the new centre point. We calculate $\mathbf{G}$ as follows:

$$\mathbf{G} = \mathbf{Q} - r\frac{\mathbf{P}_i - \mathbf{Q}}{\|\mathbf{P}_i - \mathbf{Q}\|}$$

This point lies on the same line as $\mathbf{Q}$ and the point $\mathbf{P}_i$

Bounding Sphere Construction

We can now calculate our new centre and radius of the sphere as follows:

$$\mathbf{Q}' = \frac{\mathbf{G} + \mathbf{P}_i}{2}$$
$$r' = \|\mathbf{P}_i - \mathbf{Q}'\|$$

Bounding Sphere Construction

We can now calculate our new centre and radius of the sphere as follows:

$$\mathbf{Q}' = \frac{\mathbf{G} + \mathbf{P}_i}{2}$$
$$r' = \|\mathbf{P}_i - \mathbf{Q}'\|$$

There are other bounding volumes in the maths text. However, we will simply work with these.

Outline

1 Introduction

2 Bounding Volume Construction

3 Bounding Volume Tests

4 Summary

Bounding Sphere Test

Bounding Sphere Test

The simplest test we can perform for our view check is the bounding sphere test. The calculation can be described whether the centre of the sphere is within the view frustum, or the centre is within the distance of its radius from the view frustum.

Bounding Sphere Test

The simplest test we can perform for our view check is the bounding sphere test. The calculation can be described whether the centre of the sphere is within the view frustum, or the centre is within the distance of its radius from the view frustum.

Remember last week we defined the view frustum planes as follows:

$$
\begin{aligned}
Near &= < 0, 0, -1, -n > \\
Far &= < 0, 0, 1, f > \\
Left &= < \frac{e}{\sqrt{e^2+1}}, 0, -\frac{1}{\sqrt{e^2+2}}, 0 > \\
Right &= < -\frac{e}{\sqrt{e^2+1}}, 0, -\frac{1}{\sqrt{e^2+2}}, 0 > \\
Bottom &= < 0, \frac{e}{\sqrt{e^2+a^2}}, -\frac{a}{\sqrt{e^2+a^2}}, 0 > \\
Top &= < 0, -\frac{e}{\sqrt{e^2+a^2}}, -\frac{a}{\sqrt{e^2+a^2}}, 0 >
\end{aligned}
$$

Bounding Sphere Test

The simplest test we can perform for our view check is the bounding sphere test. The calculation can be described whether the centre of the sphere is within the view frustum, or the centre is within the distance of its radius from the view frustum.

Remember last week we defined the view frustum planes as follows:

$$
\begin{aligned}
Near &= < 0, 0, -1, -n > \\
Far &= < 0, 0, 1, f > \\
Left &= < \frac{e}{\sqrt{e^2+1}}, 0, -\frac{1}{\sqrt{e^2+2}}, 0 > \\
Right &= < -\frac{e}{\sqrt{e^2+1}}, 0, -\frac{1}{\sqrt{e^2+2}}, 0 > \\
Bottom &= < 0, \frac{e}{\sqrt{e^2+a^2}}, -\frac{a}{\sqrt{e^2+a^2}}, 0 > \\
Top &= < 0, -\frac{e}{\sqrt{e^2+a^2}}, -\frac{a}{\sqrt{e^2+a^2}}, 0 >
\end{aligned}
$$

Bounding Sphere Test

Remember as well we can determine if a homogenous point lies on the positive or negative side of a plane as follows:

$$d = \mathbf{L} \cdot \mathbf{Q}$$

where $\mathbf{L} =< \mathbf{N}, D >$

Bounding Sphere Test

Remember as well we can determine if a homogenous point lies on the positive or negative side of a plane as follows:

$$d = \mathbf{L} \cdot \mathbf{Q}$$

where $\mathbf{L} =< \mathbf{N}, D >$

All we need to do is determine if d is positive or greater than $-r$, then the sphere is within the view frustum. The algorithm is as follows:

intersects $\leftarrow$ *true*
for $i = 1 \rightarrow 6$ **do**
    $d \leftarrow \mathbf{L}_i \cdot \mathbf{Q}$
    if $d \leq -r$ **then**
        intersects $\leftarrow$ *false*
        return *intersects*
    end if
end for
return *intersects*

Bounding Box Test

Bounding Box Test

Testing a bounding box really depends on the shape of the box. Here, we will consider that the box is fairly uniform. The maths text will illustrate the other approach.

Bounding Box Test

Testing a bounding box really depends on the shape of the box. Here, we will consider that the box is fairly uniform. The maths text will illustrate the other approach.

Given a plane normal, **N**, we want to calculate the effective radius of the box in relation to this normal. To do this, we use the following equation:

$$r_{eff} = \frac{1}{2}((\mathbf{R} \cdot \mathbf{N}) + (\mathbf{S} \cdot \mathbf{N}) + (\mathbf{T} \cdot \mathbf{N}))$$

Bounding Box Test

Testing a bounding box really depends on the shape of the box. Here, we will consider that the box is fairly uniform. The maths text will illustrate the other approach.

Given a plane normal, **N**, we want to calculate the effective radius of the box in relation to this normal. To do this, we use the following equation:

$$r_{eff} = \frac{1}{2}((\mathbf{R} \cdot \mathbf{N}) + (\mathbf{S} \cdot \mathbf{N}) + (\mathbf{T} \cdot \mathbf{N}))$$

Then all we need to do is determine the distance of the centre point of the box from the plane, much like we did for the sphere. That is, we need to check if

$$\mathbf{L} \cdot \mathbf{Q} > -r_{eff}$$

Outline

1. Introduction
2. Bounding Volume Construction
3. Bounding Volume Tests
4. Summary

Again, we have only really touched on the topics here. If you want to know more, then chapter 8 of the maths text will help you.