

Physics-Based Animation (SET09119)

Tutorial 05 - Centre of Mass & Moments

1 Question

An L-shaped 'card' of uniform density is shown below in Figure 2. Find the coordinates of the centre of mass (i.e., solid shape):

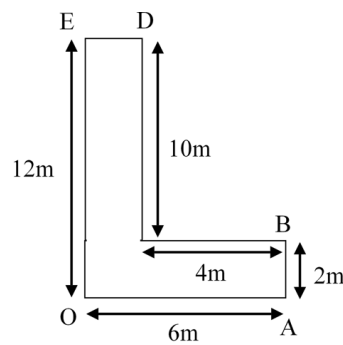


Figure 1:

$$\begin{aligned}x_1 &= 1 \\x_2 &= 3 \\y_1 &= 7 \\y_2 &= 1 \\m_1 &= 20 \\m_2 &= 12\end{aligned}$$

$$\begin{aligned}x_{cm} &= \frac{(20)(1) + (12)(3)}{20 + 12} = 1.75m \\y_{cm} &= \frac{(20)(7) + (12)(1)}{20 + 12} = 4.75m\end{aligned}$$

The centre of mass has the coordinates (1.75, 4.75).

2 Question

Find the centre of mass of a length of “wire” of uniform density bent into the shape of an L, as shown below in Figure 2 (i.e., hollow shape):

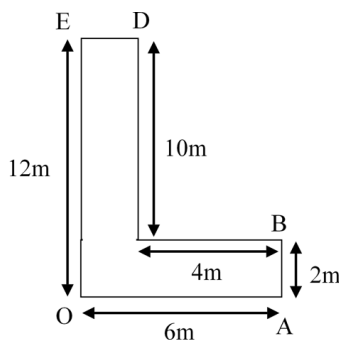


Figure 2:

We're concerned with the edges - each edge will constitute a mass.

Going clockwise from the left edge - the list of masses:

$$m_1 = 12 \quad m_2 = 2 \quad m_3 = 10 \quad m_4 = 4 \quad m_5 = 2 \quad m_6 = 6$$

x positions for the edges

$$x_1 = 0 \quad x_2 = 1 \quad x_3 = 2 \quad x_4 = 4 \quad x_5 = 6 \quad x_6 = 3$$

similarly find the y positions.

$$\begin{aligned} x_{cm} &= \frac{(12)(0) + (2)(1) + (10)(2) + (4)(4) + (2)(6) + (6)(3)}{12 + 2 + 10 + 4 + 2 + 6} \\ &= \frac{68}{36} \\ &= 1.888 \quad \text{or} \quad 1\frac{8}{9} \end{aligned}$$

$$x_{cm} = 1\frac{8}{9}m$$

$$y_{cm} = 4\frac{8}{9}m$$

The centre of mass has the coordinates $(1\frac{8}{9}, 4\frac{8}{9})$.

3 Question

A light rod, shown below in Figure 3, has forces acting on it. Given an angle of 30 degrees:

- Find the total anti-clockwise moment about A
- Find the total anti-clockwise moment about C

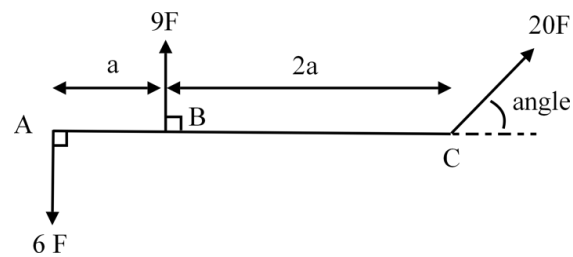


Figure 3:

Solution:

(a)

The counter-clockwise moment about A: $(9F)(a) + [(20F)\sin(30)](3a) = 39Fa$

(b)

The counter-clockwise moment about C: $(6F)(3a) - (9F)(2a) = (18Fa) - (18Fa) = 0$

4 Question

Find the coordinates of the centre of mass of the F shape shown in Figure 4 (assume a uniform solid thin material with the units in metres).

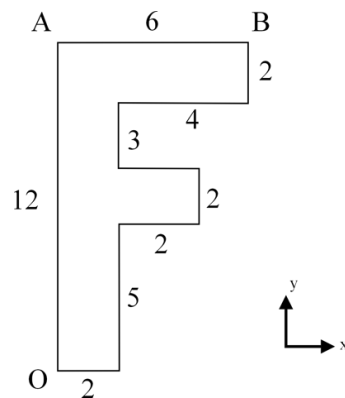


Figure 4:

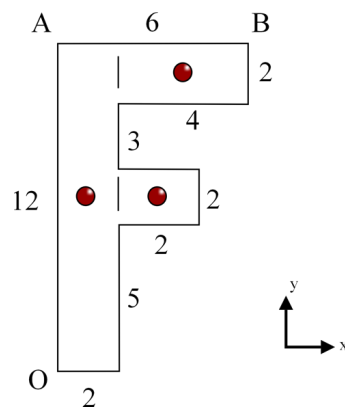


Figure 5:

overall centre of mass is:

$$x_{cm} = 1\frac{8}{9}$$

$$y_{cm} = 7\frac{1}{9}$$

5 Question

Find the coordinates of the centre of mass of the F shape shown in Figure 4, if the shape is made of a uniform 'wire'.

overall centre of mass is:

$$x_{cm} = 2\frac{1}{10}$$

$$y_{cm} = 7$$

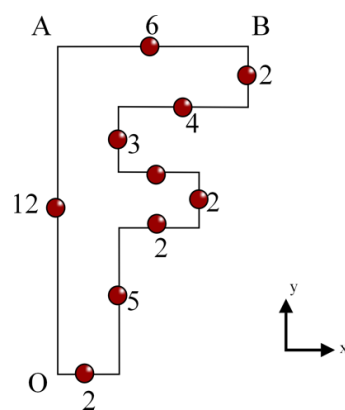


Figure 6: If F shape is made of a uniform 'wire'

6 Question

Find the length of OA for the L shape 'wire' given in Figure 7 below, if the centre of mass at $\langle \frac{17}{9}, \frac{44}{9} \rangle$. The wire has a uniform density.

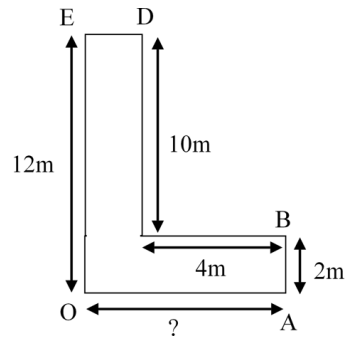


Figure 7:

length OA = 6m
