Physics-Based Animation (SET09119)

Tutorial 07 - Notes - Momentum & Impulses

1 Momentum & Impulses

1.1 What is momentum?

Momentum is defined as the quantity of motion of a moving body measured as a product of its mass and velocity.
You can calculate momentum using this equation:

$$momentum\ (kg\ m/s) = mass(kg)\ velocity(m/s)$$

Notice that momentum has: (1) magnitude - an amount because it depends on the objects mass, (2) direction - because it depends on the velocity of the object.

- in this tutorial you will work out how to calculate momentum and impulses
- solve simple collision problems using point-masses
- work out the loss of kinetic energy during a collision

If a point-mass (m) is subject to a constant force (F), which results in a constant acceleration (a), we are able to formulate the equations of motion:

$$v = u + at$$
$$f = ma$$

Multiplying the first equation by m and substituting for ma, we get:

$$mv = mu + ft$$

or more importantly:

$$ft = mv - mu$$

The quantity mass times velocity is known as the momentum and has the units Newton-seconds (Ns). The quantity force times time is known as the impulse force and also has the units Newton-seconds (Ns).

$$momentum = (mass)(velocity)$$
$$impulse = (force)(time)$$

The essential equation $mv - mv$ represents the change in momentum for a given impulse.

1.2 Example

A 3kg mass has a velocity of $5ms^{-1}$. What is the momentum?

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solution:
momentum = (3)(5) = 15 Ns

1.3 Example

What is the momentum of a 5 kg object moving at 2 m/s?

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solution:

$$\begin{aligned} momentum &= (mass)(velocity) \\ &= (5kg)(2m/s) \\ &= 10kgm/s \end{aligned} \tag{1}$$

1.4 Example

A car has a mass of 1000kg and is pushed along a level road and acquires a speed of $2ms^{-1}$ from rest in 10 seconds. What is the force pushing it?

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solution:
Change in momentum, we say:

$$(1000)(2) - (1000)(0) = 10F$$

$\therefore$

$$f = 200N$$

Note: An alternative method would be to work out the acceleration first of all (i.e., $0.2ms^{-2}$ - and then use f=ma. However, this change in momentum approach is faster).

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2 Conservation of Momentum and Collisions

The conservation of momentum for collisions states that the total momentum will be the same both before and after the collision. So long as no external forces are acting on the objects involved, the total momentum stays the same in explosions and collisions. We say that momentum is conserved. You can use this idea to work out the mass, velocity or momentum of an object in an explosion or collision.
For example, suppose we have two masses, m_1 and m_2 and they are sliding along a smooth surface with speeds u_1 and u_2. They collide and move off with speeds v_1 and v_2 respectively. Therefore, we can derive the equation:

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

This is an important rule for solving collision problems using impulses.

2.1 Example

A railway truck of mass 1500kg is travelling at $5ms^{-1}$ and hits another truck of mass 1000kg which is stationary. The two trucks couple automatically and go on together. What will be the final speed of the two trucks?

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solution:
using the conservation of momentum:

$$\begin{aligned}(1500)(5) + (1000)(0) &= (2500)(v)\\ \therefore v &= 3\\ hence:\ speed &= 3ms^{-1}\end{aligned}$$

2.2 Example

A bullet with a mass of 0.03 kg leaves a gun at 1000 m/s. If the gun's mass is 1.5 kg, what is the velocity of the recoil on the gun?

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solution:
momentum of bullet = (mass)(velocity)
= (0.03 kg)(1000 m/s)
= 30 kg m/s
Rearrange the equation: velocity = momentum / mass
velocity of recoil on gun = $\frac{30kgm/s}{1.5kg}$
= 20 m/s

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3 Momentum and Force

You need to be able to calculate the force involved in changing the momentum of an object. Here is the equation you need:

$$force = \frac{\text{change in momentum}}{\text{time taken for change}} \tag{2}$$

Force is equal to: dividing the change in momentum by the time taken for the change
The force is measured in newtons, N. The time is measured in seconds, s.

3.1 Example

A 25 kg bicycle is travelling at 12 m/s. What force is needed to bring it to a halt in 5 s?

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solution:
Momentum at start = (25)(12) = 300 kg m/s
Momentum at end = (0)(12) = 0 kg m/s
Change in momentum = (300) - (0) kg m/s

$$force = \frac{\text{change in momentum}}{\text{time taken for change}} = \frac{300}{5} = 60N \tag{3}$$

The force needed is the change in momentum divided by the time taken for the change. So, 300 divided by 5 is 60 newtons.

So a force of 60N is needed.

You should see that, for a given change in momentum, the longer the time taken, the smaller the force needed.

3.2 Example

What is the mass of an object travelling at 2 m/s with 2 J of kinetic energy?

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solution:
1 kg

3.3 Example

A 2 kg object changes from travelling at 10 m/s to 5 m/s in 2 s. What force was needed to do this?

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solution:
5 N

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4 Kinetic Energy and Momentum

Every moving object has kinetic energy. The more mass an object has and the faster it is moving, the more kinetic energy it has. Find out more with this activity.
When a body is in motion, we can represent the kinetic energy as:

$$\frac{1}{2}(mass)(velocity^2)$$

The units for kinetic energy are in joules (J).
Kinetic Energy is NOT Momentum
Remember, momentum is a vector quantity - it has a direction and magnitude in space. Kinetic energy is a scalar quantity - it has no direction and kinetic energies combine like 'regular' numbers.

4.1 Example

A body of mass 5kg is moving with an initial velocity of $7ms^{-1}$ which is reduced to $3ms^{-1}$. Find the loss in kinetic energy?

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solution:
KE before = $\frac{1}{2}(5)(7^2) = 122.5J$
KE after = $\frac{1}{2}(5)(3^2) = 22.5J$
Loss of KE = 122.5 - 22.5 = 100J
KE is an accepted shorthand for kinetic energy as is J for joules.

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5 Summarize

- momentum = (mass)(velocity) and is measured in Ns
- momentum after - momentum before = impulse
- impulse is measured in Ns
- during collisions (i.e., between two or more bodies) there is a conservation of the total momentum, i.e., $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
- kinetic energy = $\frac{1}{2}(mass)(velocity^2)$ and is measured in J
- there is a loss in kinetic energy during any collision