Physics-Based Animation (SET09119)

Tutorial 03 - Projectile Motion

1 Question

A stone is dropped from a bridge reaches the ground in 2 seconds. How high is the bridge (gravity is $9.8ms^{-2}$)?

$$
\begin{aligned}
u &= 0 \\
a &= 9.8 \\
s &= ? \\
t &= 2 \\
&\therefore \\
s &= ut + \frac{1}{2}at^2 \\
s &= 19.6m
\end{aligned}
$$

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2 Question

A grenade, laying on a horizontal surface, is kicked into the air at 7.11 ms^{-1} at 1.15 radians to the horizontal (we assume gravity is 9.8 ms^{-2}).
Find:

1. the time taken to reach its maximum height

Basic trigonometry (we divide the velocity into the horizontal and vertical components):

7.11 cos 1.15= 2.904 ms^{-1}

7.11 sin 1.15= 6.490 ms^{-1}

The grenade reaches a maximum height when the upward velocity is zero (i.e., $v_y = 0$)

Therefore, using $v = u + at$, we substitute in:

$$
\begin{aligned}
v_y &= u_y - at \\
0 &= 6.490 - 9.8t \qquad (1) \\
t &= 0.662s
\end{aligned}
$$

The grenade takes 0.662 seconds to reach its maximum height.

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2. the maximum height in metres

To find the maximum height, we want to know when s_y is greatest.
We use $s = \frac{u+v}{2}t$, and substitute in:

$$\begin{aligned} s_y &= \frac{v_y + u_y}{2}t = \frac{6.490 + 0}{2}t \\ &= 2.148\ m \end{aligned} \tag{2}$$

The maximum height is 2.148 m.

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3. the range

The grenade takes as long to go up as it does to come down, hence, its time of flight is $2 \times 0.662 = 1.324$ second.
The range is when s_x is at its maximum. We use $s = \frac{u+v}{2}t$.

$$\begin{aligned} s &= \frac{2.904 + 2.904}{2}1.324 \\ &= 3.845\ m \end{aligned} \tag{3}$$

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4. what is the speed, and what direction is it moving after $\frac{1}{2}$ second?

After $\frac{1}{2}$ second, the vertical velocity v_y is given by $v_y = 6.490 - (9.8 \times 0.5) = 1.590$ ms^{-1}.
Since the horizontal velocity is constant (i.e., $v_x = 2.904\ ms^{-1}$).
After half a second the grenade is moving at: $3.311\ ms^{-1}$ with and angle of $0.501\ radians$ to the horizon.

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3 Question

A balloon which is stationary starts to rise with an acceleration of $2ms^{-2}$. What is its velocity 10 seconds later (gravity is $9.8ms^{-2}$)?

If ballast is dropped at the end of 10 seconds, what will be the velocity of the ballast after another 10 seconds?

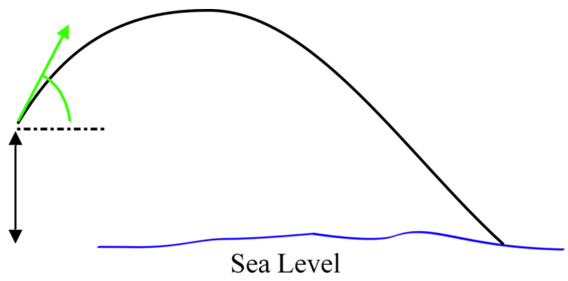
(a)

$$
\begin{aligned}
u &= 0 \\
v &= ? \\
a &= 2 \\
t &= 10 \\
&\therefore \\
v &= u + at \\
v &= 20ms^{-1}
\end{aligned}
$$

(b) Ballast has initial velocity of $20ms^{-1}$ upwards and is pulled downwards by gravity

$$
\begin{aligned}
u &= -20 \\
v &= ? \\
a &= 9.8 \\
t &= 10 \\
\text{The initial velocity downards is } -20ms^{-1} \\
&\therefore \\
v &= u + at \\
v &= 78ms^{-1}
\end{aligned}
$$

4 Question



A cannon is on a mountain, 123.69 m above the sea level. The cannon is fired with velocity 81.06 ms^{-1} at 1.06 $radians$.
Find:

1. the maximum height of the cannon above the sea

   Basic trigonometry (we divide the velocity into the horizontal and vertical components):

   81.06 cos 1.06= 39.628 ms^{-1}

   81.06 sin 1.06= 70.713 ms^{-1}

The vertical height is maximum when $v_y = 0$

Use $v^2 = u^2 + 2as$

substitute in:

$$\begin{aligned} 0^2 &= 70.713^2 + (2)(-9.8)(s) \\ s &= \frac{70.713^2}{(2)(9.8)} \\ &= 255.119 \end{aligned} \tag{4}$$

The cannon's maximum height above the sea is $= 123.69 + 255.119 = 378.809ms^{-1}$.

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2. the time taken to hit the sea

We can find the time taken to reach the maximum height and then add to it the time taken to fall back down to the sea. However, there is a quicker way.

The initial velocity of the cannon can be regarded as 70.713 ms^{-1} upwards.

Using $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned} 0 &= (0.5)(9.8)t^2 - 70.713t - 123.69 \\ t &= \frac{70.713 \pm \sqrt{(-70.713)^2 - (4)(0.5)(9.8)(-123.69)}}{(2)(0.5)(9.8)} \quad \text{[using the quadratic formula]} \end{aligned} \tag{5}$$

$t = 16.008\ s$ or $t = -1.577\ s$

Hence, the cannon takes 16.008 seconds to fly up to its maximum point and then fall back down to the sea.

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3. the horizontal distance from the cannon to the position where the bomb hits the sea

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We search for the horizontal velocity v_x using:

$s = \frac{u+v}{2}t$

we get:

$$\begin{aligned} s &= \frac{39.628 + 39.628}{2} 16.008 \\ &= 634.365\ m \end{aligned} \tag{6}$$

We have a 634.365 m range.

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4. the velocity magnitude and direction of the cannon when it hits the sea

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We want to find the downward vertical velocity v_y, using $v = u + at$:

$$\begin{aligned} v &= -70.713 + (9.8)(16.008) \\ &= 86.165 \end{aligned} \tag{7}$$

Since the horizontal velocity is constant (i.e., $39.628ms^{-1}$) we have:

$v_{hit} = \sqrt{86.165^2 + 39.628^2} = 94.841$

$\tan\theta = \frac{86.165}{39.628}$, therefore, $\theta = 1.140$

The cannon enters the sea at 94.841 ms^{-1} at an angle of 1.140 $radians$.

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5 Question

A jet of water leaves a hose-pipe with horizontal and vertical velocities of $15ms^{-1}$ and $25ms^{-1}$. Find for how long each particle of water is in the air, and how far the jet reaches.

Firstly:
Time taken to reach its maximum height (i.e., when final velocity is zero):
$v = u + at$
$0 = 25 - 9.8t$
$\therefore\ t = 2.55$ seconds to reach its maximum height.

Secondly:
Time of flight $= (2)(2.55) = 5.1$ seconds

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6 Question

A tank is on a mountain, 122.34 m above the sea level. The tank is fired with velocity 60.85 ms^{-1} at 0.92 $radians$.
Find:

1. the maximum height of the tank above the sea

Basic trigonometry (we divide the velocity into the horizontal and vertical components):

60.85 cos 0.92= 36.864 ms^{-1}

60.85 sin 0.92= 48.412 ms^{-1}

The vertical height is maximum when $v_y = 0$

Use $v^2 = u^2 + 2as$

substitute in:

$$\begin{aligned} 0^2 &= 48.412^2 + (2)(-9.8)(s) \\ s &= \frac{48.412^2}{(2)(9.8)} \\ &= 119.578 \end{aligned} \tag{8}$$

The tank's maximum height above the sea is $= 122.34 + 119.578 = 241.918 ms^{-1}$.

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2. the time taken to hit the sea

We can find the time taken to reach the maximum height and then add to it the time taken to fall back down to the sea. However, there is a quicker way.
The initial velocity of the tank can be regarded as 48.412 ms^{-1} upwards.
Using $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned} 0 &= (0.5)(9.8)t^2 - 48.412t - 122.34 \\ t &= \frac{48.412 \pm \sqrt{(-48.412)^2 - (4)(0.5)(9.8)(-122.34)}}{(2)(0.5)(9.8)} \quad \text{[using the quadratic formula]} \end{aligned} \tag{9}$$

$t = 11.966\ s$ or $t = -2.086\ s$
Hence, the tank takes 11.966 seconds to fly up to its maximum point and then fall back down to the sea.

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3. the horizontal distance from the tank to the position where the bomb hits the sea

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We search for the horizontal velocity v_x using:
$s = \frac{u+v}{2}t$
we get:

$$\begin{aligned} s &= \frac{36.864 + 36.864}{2} 11.966 \\ &= 441.115\ m \end{aligned} \tag{10}$$

We have a 441.115 m range.

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4. the velocity magnitude and direction of the tank when it hits the sea

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We want to find the downward vertical velocity v_y, using $v = u + at$:

$$\begin{aligned} v &= -48.412 + (9.8)(11.966) \\ &= 68.855 \end{aligned} \tag{11}$$

Since the horizontal velocity is constant (i.e., $36.864 ms^{-1}$) we have:
$v_{hit} = \sqrt{68.855^2 + 36.864^2} = 78.102$
$\tan\theta = \frac{68.855}{36.864}$, therefore, $\theta = 1.079$
The tank enters the sea at 78.102 ms^{-1} at an angle of 1.079 $radians$.

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7 Question

A frog, laying on a horizontal surface, is kicked into the air at 6.69 ms^{-1} at 1.37 radians to the horizontal (we assume gravity is 9.8 ms^{-2}).
Find:

1. the time taken to reach its maximum height

Basic trigonometry (we divide the velocity into the horizontal and vertical components):

6.69 cos 1.37= 1.334 ms^{-1}

6.69 sin 1.37= 6.556 ms^{-1}

The frog reaches a maximum height when the upward velocity is zero (i.e., $v_y = 0$)

Therefore, using $v = u + at$, we substitute in:

$$\begin{aligned} v_y &= u_y - at \\ 0 &= 6.556 - 9.8t \\ t &= 0.669s \end{aligned} \tag{12}$$

The frog takes 0.669 seconds to reach its maximum height.

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2. the maximum height in metres

To find the maximum height, we want to know when s_y is greatest.

We use $s = \frac{u+v}{2}t$, and substitute in:

$$\begin{aligned} s_y &= \frac{v_y + u_y}{2}t = \frac{6.556 + 0}{2}t \\ &= 2.193\ m \end{aligned} \tag{13}$$

The maximum height is 2.193 m.

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3. the range

The frog takes as long to go up as it does to come down, hence, its time of flight is $2 \times 0.669 = 1.338$ second.

The range is when s_x is at its maximum. We use $s = \frac{u+v}{2}t$.

$$\begin{aligned} s &= \frac{1.334 + 1.334}{2}1.338 \\ &= 1.785\ m \end{aligned} \tag{14}$$

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4. what is the speed, and what direction is it moving after $\frac{1}{2}$ second?

After $\frac{1}{2}$ second, the vertical velocity v_y is given by $v_y = 6.556 - (9.8 \times 0.5) = 1.656$ ms^{-1}.

Since the horizontal velocity is constant (i.e., $v_x = 1.334$ ms^{-1}).

After half a second the frog is moving at: 2.126 ms^{-1} with and angle of 0.893 $radians$ to the horizon.