Physics-Based Animation (SET09119)

Tutorial 03 - Notes - Projectile Motion

1 Projectile Motion

1.1 What do we mean by Projectile Motion?

Projectile motion is a form of motion in which an object or particle (called a projectile) is thrown near the earth's surface, and it moves along a curved path under the action of gravity only. The only force of significance that acts on the object is gravity, which acts downward to cause a downward acceleration. There are no horizontal forces needed to maintain the horizontal motion consistent with the concept of inertia.

- Constant acceleration
- Five variables (u,v,s,t,a)

  s=distance

  v=velocity

  u=initial velocity

  a=acceleration

  t=time
- Four equations (very important)

  $s = \frac{t(u+v)}{2}$

  $v = u + at$

  $s = ut + \frac{1}{2}a(t^2)$

  $v^2 = u^2 + 2as$

Important - the four fundamental equations of projectile motion are essential (see Figure 1.1 below).

- uniform acceleration means constant acceleration
- retardation means negative acceleration
- you should be able to state the four constant acceleration formulas
- you should be able to work out the identity $18kmh^{-1} = 5ms^{-1}$ (i.e., 18000/(60)(60))
- unless otherwise stated gravity due to acceleration is constant (i.e., $9.8ms^{-2}$)

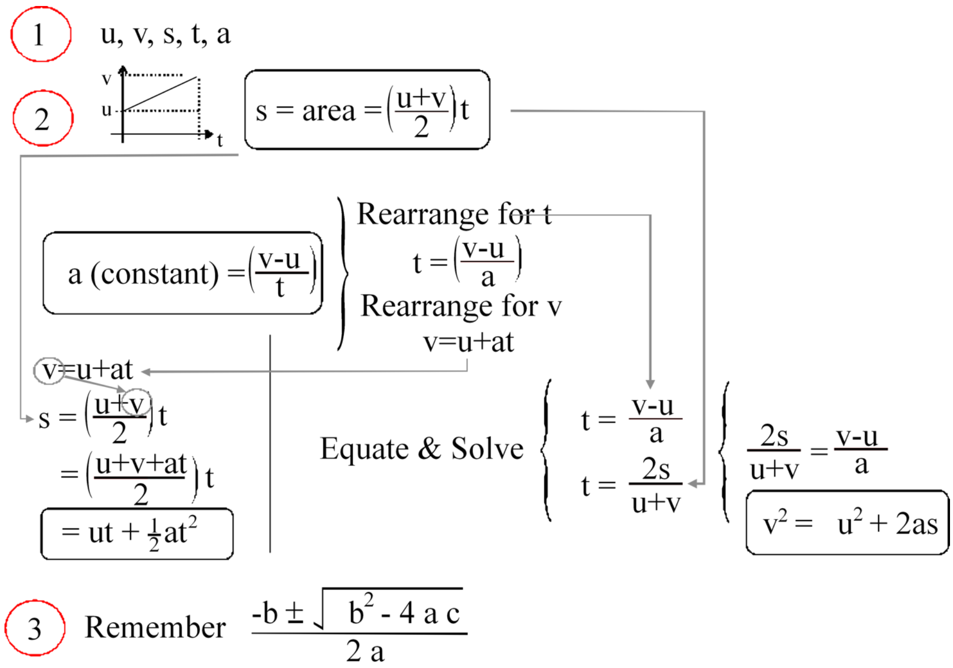

Figure 1: Relationship between the equations for projection motion of a particle. Key observations - the gradient of the distance-time graph gives you the velocity; gradient of the velocity-time graph gives you acceleration (i.e., change in distance with time is velocity and change in velocity with distance is acceleration). The area under a velocity-time graphic gives you the distance covered.

We explore what is meant by distance, displacement, speed, velocity, and acceleration. We apply the four constant acceleration equations to calculate particle body projectile motion.

2 Example Question

A duck, laying on a horizontal surface, is kicked into the air at 7.11 ms^{-1} at 1.15 radians to the horizontal (we assume gravity is 9.8 ms^{-1}). Find:

Part A

the time taken to reach its maximum height

Basic trigonometry (we divide the velocity into the horizontal and vertical components):
7.11 cos 1.15= 2.904 ms^{-1}
7.11 sin 1.15= 6.490 ms^{-1}
The duck reaches a maximum height when the upward velocity is zero (i.e., $v_y = 0$)
Therefore, using $v = u + at$, we substitute in:

$$\begin{aligned} v_y &= u_y - at \\ 0 &= 6.490 - 9.8t \\ t &= 0.662s \end{aligned} \tag{1}$$

The duck takes 0.662 seconds to reach its maximum height.

Part B

the maximum height in metres

To find the maximum height, we want to know when s_y is greatest.
We use $s = \frac{u+v}{2}t$, and substitute in:

$$\begin{aligned} s_y &= \frac{v_y + u_y}{2}t = \frac{6.490 + 0}{2}t \\ &= 2.148\ m \end{aligned} \tag{2}$$

The maximum height is 2.148 m.

Part C

the range

The duck takes as long to go up as it does to come down, hence, its time of flight is $2 \times 0.662 = 1.324$ second.
The range is when s_x is at its maximum. We use $s = \frac{u+v}{2}t$.

$$\begin{aligned} s &= \frac{2.904 + 2.904}{2}1.324 \\ &= 3.845\ m \end{aligned} \tag{3}$$

Part D

What is the speed, and what direction is it moving after $\frac{1}{2}$ second?

After $\frac{1}{2}$ second, the vertical velocity v_y is given by $v_y = 6.490 - (9.8 \times 0.5) = 1.590\ ms^{-1}$.
Since the horizontal velocity is constant (i.e., $v_x = 2.904\ ms^{-1}$).
After half a second the duck is moving at: 3.311 ms^{-1} with and angle of 0.501 *radians* to the horizon.

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3 Example Question

A tank is on a mountain, 150 m above the sea level. The tank fires a bomb with velocity 80 ms^{-1} at 1.046 *radians*. Find:

Part A

the maximum height of the bomb above the sea

Basic trigonometry (we divide the velocity into the horizontal and vertical components):
80 cos 1.046= 40.083 ms^{-1}
80 sin 1.046= 69.234 ms^{-1}
The vertical height is maximum when $v_x = 0$
Use $v^2 = u^2 + 2as$
substitute in:

$$\begin{aligned} 0^2 &= 69.234^2 + (2)(-9.8)(s) \\ s &= \frac{69.234^2}{(2)(9.8)} \\ &= 244.559 \end{aligned} \tag{4}$$

The bomb's maximum height above the sea is $= 150 + 244.559 = 394.559 ms^{-1}$.

Part B

the time taken to hit the sea

We can find the time taken to reach the maximum height and then add to it the time taken to fall back down to the sea. However, there is a quicker way.
The initial velocity of the bomb can be regarded as 69.234 ms^{-1} downwards.
Using $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned} 0 &= (0.5)(9.8)t^2 - 69.234t - 150 \\ t &= \frac{69.234 \pm \sqrt{69.234^2 - (4)(0.5)(9.8)(150)}}{(2)(0.5)(9.8)} \quad \text{[using the quadratic formula]} \end{aligned} \tag{5}$$

$t = 11.458$ s or $t = 2.672$ s
Hence, the bomb takes 11.458 seconds to fly up to its maximum point and then fall back down to the sea.

Part C

the horizontal distance from the tank to the position where the bomb hits the sea

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We search for the horizontal velocity v_x using:
$s = \frac{u+v}{2}t$
we get:

$$\begin{aligned} s &= \frac{40.083 + 40.083}{2} 11.458 \\ &= 459.271\ m \end{aligned} \tag{6}$$

We have a 459.271 m range.

Part D

the velocity magnitude and direction of the bomb when it hits the sea

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We want to find the downward vertical velocity v_y, using $v = u + at$:

$$\begin{aligned} v &= -69.234 + (9.8)(11.458) \\ &= 43.054 \end{aligned} \tag{7}$$

Since the horizontal velocity is constant (i.e., $40.083ms^{-1}$) we have:
$v_{hit} = \sqrt{43.054^2 + 40.083^2} = 58.824$
$\tan\theta = \frac{43.054}{40.083}$, therefore, $\theta = 0.821$
The bomb enters the sea at 58.824 ms^{-1} at an angle of 0.821 $radians$.
