Lighting Part 2

Computer Graphics - SET08116

EDINBURGH NAPIER UNIVERSITY

Outline

Review - Vector Mathematics

How do we represent a vector?

Vector Representation

We have already shown how a vector can be represented in tuple form:

$$\mathbf{V} = < V_1, V_2, \ldots, V_n >$$

It is also common in graphics work to think of a vector in its matrix form, which is a single column with **n** rows:

$$\mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

However, to save space, it is common to write a vector as a single row. This is actually the transpose of the column vector, and should be written:

$$\mathbf{V}^T = \begin{bmatrix} V_1 & V_2 & \cdots & V_n \end{bmatrix}$$

Review - Vector Mathematics

How do we do simple vector operations?
(e.g., adding/subtracting/dot/cross)

Vector Operations

Vector addition (and subtraction) works by adding each individual component of the vectors together:

$$\mathbf{P} + \mathbf{Q} = < P_1 + Q_1, P_2 + Q_2, \ldots, P_n + Q_n >$$

So, for example:

$$< 2, 3, 4 > + < 1, 5, 7 > = < 3, 8, 11 >$$

For scaling a vector by a real number, we simply scale each individual value:

$$a\mathbf{V} = < aV_1, aV_2, \ldots, aV_n >$$

So, for example:

$$5 < 1, 2, 3 > = < 5, 10, 15 >$$

Vector Operations

Another common operation is calculating the length of a vector.

The length is considered the distance from the end of the vector, to the origin $(0, 0, \ldots, 0)$.

The length (magnitude) of the vector is denoted as follows:

$$\|\mathbf{V}\| = \sqrt{\sum_{i=1}^{n} V_i^2}$$

The $\|\mathbf{V}\|$ being the magnitude operator (you will see this operator in many places).

Vector Operations

It is most likely that we will only get **lengths** of 3-dimensional vectors. This means that we can simplify the previous definition to the following:

$$\|\mathbf{V}\| = \sqrt{{V_x}^2 + {V_y}^2 + {V_z}^2}$$

We are also usually interested in creating a vector that only has unit length (that is, its magnitude is 1). This operation is called **normalization**, and is performed as follows:

$$\hat{\mathbf{V}} = \frac{\mathbf{V}}{\|\mathbf{V}\|}$$

That is, we divide the individual components of the vector by the length of the vector. You can also write the equation as follows:

$$\hat{\mathbf{V}} = \frac{1}{\|\mathbf{V}\|}\mathbf{V}$$

Dot Product

We have seen how to multiply a vector by a scalar, but what about multiplying vectors together. Well, there are two methods to do this:

- **Dot** Product (scalar)
- **Cross** Product (vector)

The dot product is also known as the **scalar product** or **inner product**. It is useful as a measure between the difference in directions of two vectors.

For any two **n**-dimensional vectors, the dot product is defined as follows:

$$\mathbf{P} \cdot \mathbf{Q} = \sum_{i=1}^{n} P_i Q_i$$

Dot Product

For a 3-dimensional vector, we essentially have the following equation:

$$\mathbf{P} \cdot \mathbf{Q} = P_x Q_x + P_y Q_y + P_z Q_z$$

The dot product also has another property, which allows us to measure the angle between two vectors. The dot product $\mathbf{P} \cdot \mathbf{Q}$ satisfies the following equation:

$$\mathbf{P} \cdot \mathbf{Q} = \|\mathbf{P}\| \|\mathbf{Q}\| \cos \alpha$$

Where α is the planar angle between the lines connecting the origin to the points represented by **P** and **Q**. There is a proof of why this is the case in the maths text.

Dot Product

The relation between the dot product and the cosine of the angle between them provides us with a useful observation. If the dot product is equal to 0, then the vectors must be perpendicular. That is:

$$\mathbf{P} \cdot \mathbf{Q} = 0 \Rightarrow \mathbf{P} \perp \mathbf{Q}$$

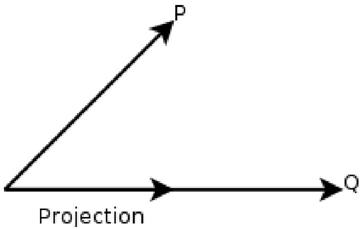
We can also use the sign of the dot product to determine how close two vectors are to pointing in the same direction. We can consider **P** to be orthogonal to a plane passing through the origin. Then, any vector lying on the same side of the plane has a positive dot product, and any vector lying on the other side of the plane has a negative dot product. That is:

$$\mathbf{P} \cdot \mathbf{Q} > 0 \Rightarrow \angle \mathbf{PQ} < \frac{\pi}{2} rad$$

$$\mathbf{P} \cdot \mathbf{Q} < 0 \Rightarrow \angle \mathbf{PQ} > \frac{\pi}{2} rad$$

Projection Vector

Sometimes we wish to work out the projection of one vector onto another. A vector projected onto another has the same direction as the second vector, but has a length equal to the distance along the second that the first vector reaches. The following diagram should help:



The equation for working out the projection of one vector onto another is as follows:

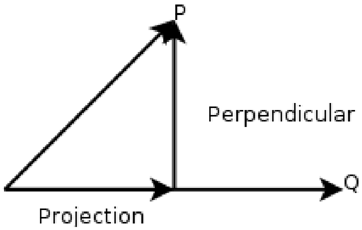
$$proj_q \mathbf{P} = \frac{\mathbf{P} \cdot \mathbf{Q}}{\|\mathbf{Q}\|^2} \mathbf{Q}$$

Perpendicular Component

The perpendicular component of **P** in regards to **Q** is the vector left when we subtract the projection vector from the original vector. That is:

$$
\begin{aligned}
perp_q\mathbf{P} &= \mathbf{P} - proj_q\mathbf{Q} \\
&= \mathbf{P} - \frac{\mathbf{P} \cdot \mathbf{Q}}{\|\mathbf{Q}\|^2}\mathbf{Q}
\end{aligned}
$$

The following diagram indicates the two values we have just calculated:



Cross Product

The cross product only holds properties of interest for 3-dimensional vectors. The cross product, sometimes called the **vector product**, is defined as follows:

$$\mathbf{P} \times \mathbf{Q} = < P_y Q_z - P_z Q_y, P_z Q_x - P_x Q_z, P_x Q_y - P_y Q_x >$$

There is a simple way to remember this:

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

The cross product generates a vector that is perpendicular to the original two vectors. This also allows us to make the following observation:

$$(\mathbf{P} \times \mathbf{Q}) \cdot \mathbf{P} = 0$$

$$(\mathbf{P} \times \mathbf{Q}) \cdot \mathbf{Q} = 0$$

Cross Product

As the dot product, the cross product has a relationship to a trigonometric value:

$$\|\mathbf{P} \times \mathbf{Q}\| = \|\mathbf{P}\| \|\mathbf{Q}\| \sin \alpha$$

Where α is the planar angle between the two lines connecting the origin to the two points.

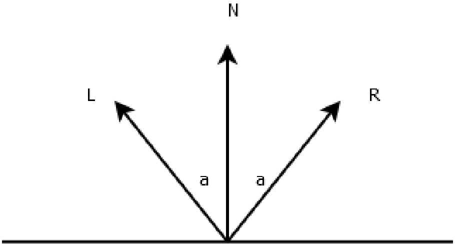
We can also use the cross product to calculate the **area of a triangle**. Given a triangle defined by the points $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$, we can use the following equation to calculate the area, **A**:

$$A = \frac{1}{2} \|(\mathbf{V}_2 - \mathbf{V}_1) \times (\mathbf{V}_3 - \mathbf{V}_1)\|$$

The maths text illustrates why this is the case.

Reflection Vector

We often wish to determine the direction of light reflecting from a shiny surface. This is actually a simple concept - we just want the angle of incidence to equal the angle of reflection.

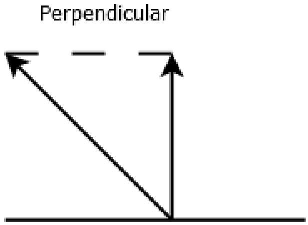


We are just saying that the angle between the light vector **L** and the normal **N** is equal to the angle between the normal **N** and the reflected vector **R**.

Reflection Vector

We assume that **N** and **L** have been normalized (are unit length). We need to define an equation for calculating the reflection vector **R**.

First we need to calculate the perpendicular component of **L** to **N**.



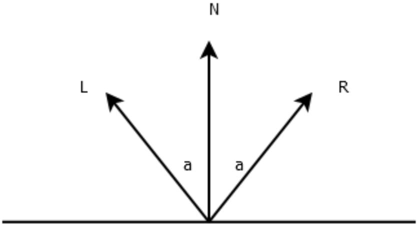
This is given by the equation:

$$perp_N\mathbf{L} = \mathbf{L} - (\mathbf{N} \cdot \mathbf{L})\mathbf{N}$$

Reflection Vector

The reflection vector lies twice the distance from **L** as does its projection onto the normal vector **N**. We can calculate **R** as follows:

$$
\begin{aligned}
\mathbf{R} &= \mathbf{L} - 2 perp_{\mathbf{N}}\mathbf{L} \\
&= \mathbf{L} - 2(\mathbf{L} - (\mathbf{N} \cdot \mathbf{L})\mathbf{N}) \\
&= 2(\mathbf{N} \cdot \mathbf{L})\mathbf{N} - \mathbf{L}
\end{aligned}
$$



Reflection Vector

For example, if $\mathbf{L} =< 0.0, 1.0, 0.0 >$ and $\mathbf{N} =< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 >$, then our calculation is:

$$\begin{aligned}
\mathbf{R} &= 2(\mathbf{N} \cdot \mathbf{L})\mathbf{N} - \mathbf{L} \\
&= 2(\frac{1}{\sqrt{2}}) < \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 > - < 0.0, 1.0, 0.0 > \\
&= \frac{2}{\sqrt{2}} < \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 > - < 0.0, 1.0, 0.0 > \\
&=< \frac{2}{2}, \frac{2}{2}, 0 > - < 0.0, 1.0, 0.0 > \\
&=< 1.0, 0.0, 0.0 >
\end{aligned}$$

Review - Colour

- What is colour?
- How do we represent colour?
- How do we mix and modify colours?

Colour

As mentioned, we treat **colour as a vector**. This allows us to perform standard vector operations on our colours.
A colour $\mathcal{C}$ can be multiplied by a scalar s:

$$s\mathcal{C} =< s\mathcal{C}_r, s\mathcal{C}_g, s\mathcal{C}_b >$$

Two colours $\mathcal{C}$ and $\mathcal{D}$ can be added:

$$\mathcal{C} + \mathcal{D} =< \mathcal{C}_r + \mathcal{D}_r, \mathcal{C}_g + \mathcal{D}_g, \mathcal{C}_b + \mathcal{D}_b >$$

They can also be multiplied:

$$\mathcal{C}\mathcal{D} =< \mathcal{C}_r\mathcal{D}_r, \mathcal{C}_g\mathcal{D}_g, \mathcal{C}_b\mathcal{D}_b >$$

Review - Ambient Lighting

- What is ambient light?

Ambient Light

Ambient light in a scene is light that is applied equally **across all objects** in the scene.

Ambient light basically gives the appearance of coming from every direction with equal intensity. Thus, an object is lit uniformly across its surface.

We will denote the ambient light colour of the scene using the following symbol:

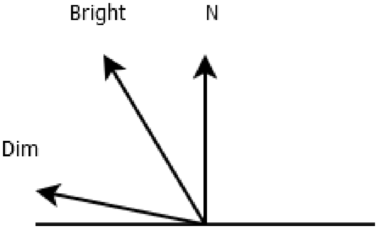
$$\mathcal{A}$$

Review - Diffuse Lighting

- What is diffuse light?

Diffuse Reflection

As mentioned we want to work out the **intensity of a light based on the angle** that the light direction makes with the normal of the surface.



We use the dot product to work this out. Remember, **L** and **N** are unit length. Thus:

$$
\begin{aligned}
&\|\mathbf{L}\|\|\mathbf{N}\| \cos\theta \\
=&(1)(1)(\cos\theta) \\
=&\cos\theta
\end{aligned}
$$

Calculating Normals

We need a simple solution to allow us to calculate surface normals for our objects.

We have already discussed how to do this with the cross product. For example, consider that we have three vectors V_0, V_1 and V_2 that form a triangle. We can work out the three normals as follows:

$$N_0 = normalize((V_1 - V_0) \times (V_2 - V_0))$$
$$N_1 = normalize((V_2 - V_1) \times (V_0 - V_1))$$
$$N_2 = normalize((V_0 - V_2) \times (V_1 - V_2))$$

Handedness of space also matters here. $V_0 \times V_1 \neq V_1 \times V_0$

Example

Consider that we have the following definitions:

$V_0 =< 0.0, 1.0, 0.5 >, V_1 =< 1.0, 0.0, 0.0 >, V_2 =< -1.0, -1.0, -0.5 >$

Then our normals are calculated as:

$$
\begin{aligned}
N_0 &= normalize(< 1.0, -1.0, -0.5 > \times < -1.0, -2.0, -1.0 >) \\
&= normalize(< 0, 1.5, -3 >) =< 0, \frac{1.5}{\sqrt{11.25}}, \frac{-3}{\sqrt{11.25}} > \\
N_1 &= normalize(< -2.0, -1.0, -0.5 > \times < -1.0, 1.0, 0.5 >) \\
&= normalize(< 0, 1.5, -3 >) =< 0, \frac{1.5}{\sqrt{11.25}}, \frac{-3}{\sqrt{11.25}} > \\
N_2 &= normalize(< 1.0, 2.0, 1.0 > \times < 2.0, 1.0, 0.5 >) \\
&= normalize(< 0, 1.5, -3 >) =< 0, \frac{1.5}{\sqrt{11.25}}, \frac{-3}{\sqrt{11.25}} >
\end{aligned}
$$

Ambient & Diffuse Reflection

- ✓ We have Ambient light
- ✓ We have Diffuse Reflection light

- How do we combine them?

Diffuse Reflection

Let us now build a formula that determines $\mathcal{K}$, the colour of the light reflected towards the viewer from a particular point **Q** on the surface of the object.

We will work with the diffuse reflection (material) colour of the surface, **D**. Our diffuse light calculation is as follows:

$$\mathcal{K}_{diffuse} = \mathcal{D}\mathcal{A} + \mathcal{D}\mathcal{C}\,max(\mathbf{N} \cdot \mathbf{L}, 0)$$

Where $\mathcal{C}$ is the colour of the light. We can actually have multiple lights by applying a simple change:

$$\mathcal{K}_{diffuse} = \mathcal{D}\mathcal{A} + \mathcal{D}\sum_{i=1}^{n} \mathcal{C}_i\,max(\mathbf{N} \cdot \mathbf{L}_i, 0)$$

Diffuse Reflection - Example

Let us consider the following definitions:

$$\mathcal{A} = < 0.6, 0.6, 0.6 >$$
$$\mathcal{D} = < 0.8, 0.1, 0.2 >$$
$$\mathcal{C} = < 0.7, 0.9, 0.9 >$$
$$\mathbf{N} = < 0.0, 1.0, 0.0 >$$
$$\mathbf{L} = < \frac{3}{5}, \frac{4}{5}, 0.0 >$$

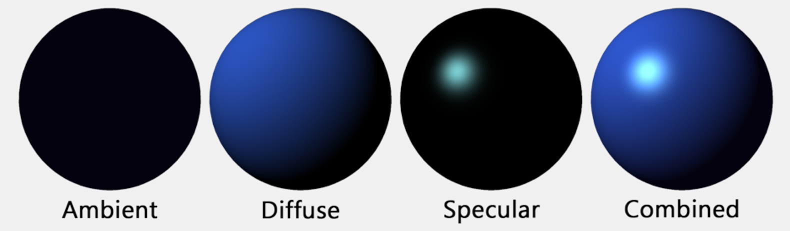
Then our diffuse reflection calculation becomes:

$$\begin{aligned}\mathcal{K}_{diffuse} &= < 0.48, 0.06, 0.12 > + < 0.56, 0.09, 0.18 > max(\frac{4}{5}, 0) \\ &= < 0.48, 0.06, 0.12 > + < 0.448, 0.072, 0.144 > \\ &= < 0.928, 0.132, 0.264 >\end{aligned}$$

Lighting

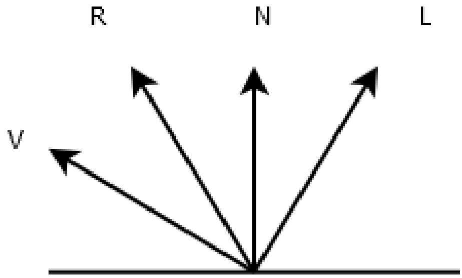
- We have Diffuse & Ambient lighting
- What else is missing?

Lighting (Ambient, Diffuse, & Specular)



Specular Reflection

Specularity requires us to also have the position of the viewer taken into account.



Here, the vector **V** is a unit length vector pointing to the viewer. The angle between **R** and **V** provides the specular intensity value.

Specular Reflection

The model equation we use provides a realistic look, but has no physical basis. We have the specular reflection colour of the surface, $\mathcal{S}$, and also the specular exponent, m.

We only calculate the value when the dot product of **N** and **L** is greater than 0 (in other words, the point **Q** is lit). The equation is:

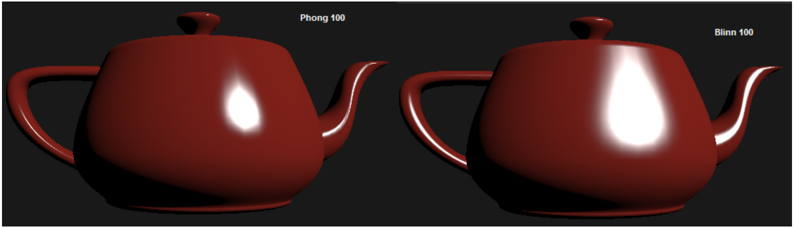
$$\mathcal{K}_{specular} = \mathcal{S}\mathcal{C} max(\mathbf{R} \cdot \mathbf{V}, 0)^m (\mathbf{N} \cdot \mathbf{L} > 0)$$

Lighting

Why is H (blinn) used instead of R (phong) in **specular shading**?

Phong vs. Blinn

Why is H (blinn) used instead of R (phong) in **specular shading**?



Gouraud Shading

Gouraud shading is a **per-vertex model of light**. That is, we calculate the light on a per-vertex basis.

The Gouraud shading equation is as follows:

$$
\begin{aligned}
\mathcal{K}_{primary} &= \mathcal{D}\mathcal{A} + \mathcal{D}\mathcal{C}\, max(\mathbf{N} \cdot \mathbf{L}, 0) \\
\mathcal{K}_{secondary} &= \mathcal{S}\mathcal{C}\, max(\mathbf{R} \cdot \mathbf{V}, 0)^m (\mathbf{N} \cdot \mathbf{L} > 0)
\end{aligned}
$$

Review - Why the ‘max’ test?

Recommended Reading

Chapter 7 in the maths book goes into far more detail than we have here. In particular, calculations for point lights and specular lights are also available in the chapter. I strongly recommend you look at these.

The chapter also goes into more detail and shows other approaches to calculating some of the light values. Our implementation in the practical is slightly different. If you feel confident, change to the model provided in the book. It uses slightly fewer values, and slightly fewer calculations.