Physics-Based Animation (SET09119)

Tutorial 09 - Notes - Springs & Elastic Strings

1 Springs & Elastic Strings

1.1 What is a spring?

A spring is an example of an elastic object - when stretched, it exerts a restoring force which tends to bring it back to its original length. This restoring force is generally proportional to the amount of stretch, as described by Hooke's Law.

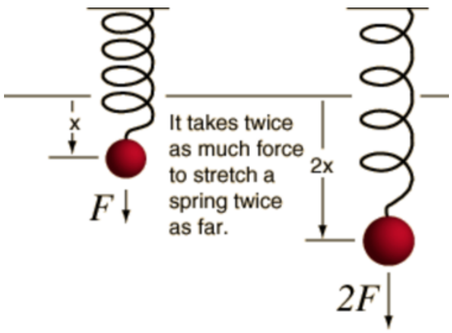

Figure 1: Relationship between force and displacement for a linear spring.

In this tutorial, you will work out the energy of a spring and solve simple spring-energy problems.

1.2 Elastic Energy

Elastic potential energy is the energy stored in elastic materials as the result of their stretching or compressing. Elastic potential energy can be stored in rubber bands, bungee chords, trampolines, springs, an arrow drawn into a bow, etc. The amount of elastic potential energy stored in such a device is related to the amount of stretch of the device - the more stretch, the more stored energy.

Springs are a special instance of a device that can store elastic potential energy due to either compression or stretching. A force is required to compress a spring; the more compression there is, the more force that is required to compress it further. For certain springs, the amount of force is directly proportional to the amount of stretch or compression (x); the constant of proportionality is known as the spring constant (k).

$$
\begin{aligned}
F_{spring} &= k\ x \\
&= \left(\frac{\lambda}{l}\right)\ x
\end{aligned}
$$

When an elastic object - such as a spring - is stretched, the increased length is called its extension. The extension of an elastic object is directly proportional to the force applied to it.

$$
\begin{aligned}
\text{Elastic Energy} = EE &= k\ \frac{1}{2}\ x^2 \\
&= \left(\frac{\lambda}{l}\right)\frac{1}{2}\ x^2
\end{aligned}
$$

where EE is the elastic energy, k is the spring constant, and x is the amount of compression (i.e., relative to the rest position), l is the rest length, and λ is called the **modulus of elasticity**.

The principle of conservation of energy, says that the sum of the energies of a system must remain constant. Hence, the sum of kinetic, potential, and elastic energies at any position of the system is constant.

1.3 Question

A mass of 4kg is suspended on one end of a spring and produces an extension of 2m. Find the modulus of elasticity if the rest length is 5m.

Solution:

$F = 4g$ and also $F = \frac{\lambda}{l}\ x$

$x = 2$ and $l = 5$, hence:

$4g = \frac{(\lambda)(2)}{5}$

$\lambda = 10g$, i.e., the modulus is 10g.

1.4 Question

An elastic string of rest length 2m has a modulus of elasticity of 3g. Find the extension when a particle of mass $4kg$ is suspended from one end.

Solution:
The system is in equilibrium, hence:

$$
\begin{aligned}
F - 4g &= 0 \\
F &= 4
\end{aligned}
$$

Since $F = \frac{\lambda}{l}\ x$, $\lambda = 3g$ and, $l = 2$

$$4g = \frac{3gx}{2}$$
$$x = \frac{8}{3}$$

1.5 Question

A car's shock absorbers make the ride more comfortable, by using a spring that absorbs energy when the car goes over a bump. One of these springs, placed next to a wheel, needs to store 250 J of energy when compressed by a distance of 10 cm.
(a) What value of force constant is required for the spring?
(b) How much energy would be stored if it were compressed by 20 cm?

Solution:

(a)
Since EE $= \frac{1}{2}kx^2$
$250 = \frac{1}{2}(k)(0.102)$
k $= \frac{(250(2)}{0.102} = 50000Nm^1$

(b) The extent of compression has been doubled. Provided the spring is still elastic, the energy is proportional to the square of the extension. Twice the extension will therefore equal four times the energy.

energy stored = (250)(4) = 1000 J

1.6 Question

A buffer consists of a spring that requires a force of 200000 newtons per metre for compression. A train of mass 300000 kg hits the buffers when travelling at 3.0 ms^1.

Calculate (a) the kinetic energy of the train, (b) the work done on the spring by the train before

it stops, (c) the distance the spring is compressed by the train.

2 Summarize

- the energy stored in a spring (i.e., $\left(\frac{\lambda}{l}\right)\frac{1}{2}\,x^2$)
- Hooke's law can be written in terms of stress and strain ($stress \propto strain$)
- the spring constant k is now $k = \frac{\lambda}{l}$
- λ is derived from Young's modulus and is a measure of an object's stiffness