Physics-Based Animation (SET09119)

Tutorial 08 - Notes - Coefficient of Restitution

1 Coefficient of Restitution

1.1 What is the Coefficient of Restitution?

The coefficient of restitution (e) of two colliding objects is typically a positive real number between 0.0 and 1.0 representing the ratio of speeds after and before an impact, taken along the line of the impact.

The Coefficient of Restitution (e) is a variable number with **no units**, with limits from zero to one.

$$0 \leq e \leq 1$$

'e' is a consequence of Newton's Experimental Law of Impact, which describes how the speed of separation of two impacting bodies compares with their speed of approach.

- in this tutorial you will expand the concept of momentum and contacts
- solve simple collision problems using point-masses

When two objects collide momentum is conserved. We can formulate a set of equations that connect the objects velocities before and after the collision.

1.2 Coefficient of Restitution (e)

The coefficient of restitution measures the elasticity of the two bodies and takes a value between 0.0 and 1.0. If e = 0, then the bodies are said to be **perfectly inelastic**, and if e = 1, the bodies are said to be **perfectly elastic**.
One of the simplest examples of this is a moving particle colliding with a fixed surface with velocity u, where the coefficient of restitution between the particle and the surface is e. If the velocity with which the particle rebounds is v, then by Newton's Experimental Law:

$$\frac{\text{Speed of seepration (i.e., relative speed)}}{\text{Speed of approach (i.e., relative speed)}} = \frac{v}{u} = e$$
$$i.e.,$$
$$v = eu$$

1.3 Example

A sphere of mass m is dropped from a height h on to a horizontal floor. Find the height to which the sphere rebounds, if the coefficient of restitution between the sphere and the floor is 0.5.

Solution:
We can find the velocity of the sphere immediately before the collision by equating the loss in potential energy with the gain in kinetic energy.
Initially:
PE = mgh
KE = 0

Ground:
PE = 0
KE = $\frac{1}{2}mu^2$

i.e.,
$\frac{1}{2}mu^2 = mgh$

$\therefore u^2 = 2gh$

$u = \sqrt{2gh}$

The velocity v after the collision is given by eu, i.e.,:
$e\sqrt{2gh}$ or $(0.5)\sqrt{2gh}$, since e = 0.5.

This gives the new kinetic energy as:
$\frac{1}{2}(m)(v^2) = \frac{1}{2}m\left((0.5)\sqrt{2gh}\right)^2 = \frac{1}{2}m(\frac{1}{4}(2)(g)(h) = \frac{mgh}{4}$

For maximum height, this kinetic energy will all be converted into potential energy. If the height is H, the potential energy will be mgH and so:
$mgH = mg\frac{h}{4}$

$\therefore H = \frac{h}{4}$ metres

i.e., the sphere will rebound to one quarter of its original height.

1.4 Example

A 5 kg mass moving at 6 ms^{-1} makes a head-on collision with a 4 kg mass travelling at 3 ms^{-1}. Assuming that there are no external forces acting on the system, what are the velocities of the two masses after impact? (assume coefficient of restitution e = 0.5)

Solution:
u_A = initial speed of 5 kg mass (mass A)= 6 ms^{-1}
u_B = initial speed of 4 kg mass (mass B)= 3 ms^{-1}

$m_A = 5kg$ $m_B = 4kg$

v_A = final speed of mass A v_B = final speed of mass B

momentum before the collision equals momentum after

hence, $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

also coefficient of restitution equation

substituting for e, m_A , u_A , m_B , u_B
we obtain two simultaneous equations
from the conservation of momentum,

$$\begin{aligned}(5)(0.6) + 4(-3) &= 5v_A + 4v_B \\ 3 - 12 &= 5v_A + 4v_B \\ -9 &= 5v_A + 4v_B \\ 5v_A + 4v_B &= -9 \qquad \text{(i} \end{aligned}$$

from the coefficient of restitution expression,

$$\begin{aligned} 0.5 &= \frac{v_B - v_A}{u_A - u_B} \quad \text{coefficient equation} \\ 0.5(u_A - u_B) &= v_B - v_A \\ (0.5)(6) - (0.5(-3)) &= v_B - v_A \\ 3 + 1.5 &= v_B - v_A \\ 4.5 &= v_B - v_A \\ v_B - v_A &= 4.5 \qquad \text{(ii} \end{aligned}$$

multiplying (ii by 5 and adding

$$\begin{aligned} 5v_A + 4v_B &= -9 \\ -5v_A + 5v_B &= 22.5 \\ 9v_B &= 13.5 \\ v_B &= 1.5ms^{-1} \end{aligned}$$

from (ii

$$\begin{aligned} 1.5 - v_A &= 4.5 \\ v_A &= 1.5 - 4.5 \\ v_A &= -3ms^{-1} \end{aligned}$$

Ans. The velocities of the 5 kg and 4 kg masses are -3 ms^{-1} and $1.5ms^{-1}$, respectively.

2 Summarize

- relationship between momentum, impulses, and the collision between objects
- coefficient of restitution ($0 \leq e \leq 1$)