2024

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Содержание

1 Nilpotency considerations

Let $\mathcal{J}_k$ be nilpotent matrix of order k. It looks like

$$\begin{pmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & ... \\ ... & 0 & ... & ... & 0 \\ ... & ... & ... & 0 & 1 \\ 0 & ... & ... & 0 & 0 \end{pmatrix}$$

The main feature is that nilpotent matrices are degrees of each other. Let's prove the main thing: that $\mathcal{J}_k$ is a nilpotent matrix of order k.
Proof:
Base case: the $\mathcal{J}_1 = \begin{pmatrix} 0 \end{pmatrix}$ is nilpotent matrix of order 1.
Induction step:
Consider matrices of order n. Notice that $\mathcal{J}_m * \mathcal{J}_n = \mathcal{J}_{m-1},\ m \leq n$ (because the last row of $\mathcal{J}_m$ always gives zero, and all other rows match the columns of larger matrix).

$$\begin{pmatrix} 0 & 0 & 0 & \color{red}{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \color{red}{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \color{red}{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \color{red}{1} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & \color{red}{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \color{red}{1} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$(\mathcal{J}_k * \mathcal{J}_k) * ... * \mathcal{J}_k = \mathcal{J}_{k-1} * ... * \mathcal{J}_k = ...$$

$\mathcal{J}_k^k = \mathcal{J}_{k-1} * \mathcal{J}_k^{k-2} = \mathcal{J}_{k-(d-1)} * \mathcal{J}_k^{k-d} = \mathcal{J}_1 * \mathcal{J}_k^0 \ \ (d = k) = 0 * E = 0 \ \ \forall k > 0,\ q.e.d.$