Блядское бдз v15 сука ненавижу тфкп

Я нахуй, а кто ещё

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1 Найти $|z|, Re(z), Im(z)$

1)

$$z = \left(\frac{-3-i\sqrt{3}}{1-i}\right)^{96} = (z^*)^{96}$$

$$z^* = \frac{-3-i\sqrt{3}}{1-i} = \frac{3+i\sqrt{3}}{i-1} = \frac{(3+i\sqrt{3})(i+1)}{(i-1)(i+1)} = \frac{3i-\sqrt{3}+3+i\sqrt{3}}{-1-1} =$$

$$= \frac{3-\sqrt{3}}{-2} + i\ \frac{3+\sqrt{3}}{-2} = \frac{-3-\sqrt{3}}{2} + i\ \left(-\frac{3+\sqrt{3}}{2}\right)$$

$$|z^*| = \sqrt{\left(-\frac{3-\sqrt{3}}{2}\right)^2 + \left(-\frac{3+\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9-6\sqrt{3}+3}{4} + \frac{9+6\sqrt{3}+3}{4}} = \sqrt{\frac{24}{4}} = \sqrt{6}$$

$$z^* = \sqrt{6}\left(-\frac{3-\sqrt{3}}{2\sqrt{6}} + i\left(-\frac{3+\sqrt{3}}{2\sqrt{6}}\right)\right)$$

$$\cos\phi = -\frac{3-\sqrt{3}}{2\sqrt{6}},\ \sin\phi = -\frac{3+\sqrt{3}}{2\sqrt{6}}$$

$$\operatorname{tg}\phi = \frac{\sin\phi}{\cos\phi} = \frac{3+\sqrt{3}}{3-\sqrt{3}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{3-1} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

$$-\frac{\pi}{2} < \arctan\phi < \frac{\pi}{2}$$

$$\phi = \arctan(\tan\phi) - \pi = \arctan(2+\sqrt{3}) - \pi$$

$$z^* = \sqrt{6}\ [\ \cos\ (\arctan(2+\sqrt{3}) - \pi) +\ i\ \sin\ (\arctan(2+\sqrt{3}) - \pi)\ ]$$

$$|z| = (z^*)^{96} = (\sqrt{6})^{96} = 6^{48}$$

$$z = (z^*)^{96} = 6^{48} * \cos(96[\arctan(2+\sqrt{3}) - \pi]) + \ i * 6^{48} * \sin(96[\arctan(2+\sqrt{3}) - \pi])$$
$$= Re(z) + Im(z)$$

2)

$$z = \sqrt[3]{8-8i} \rightarrow z^3 = z^* = 8 - 8i$$
$$|z^*| = \sqrt{8^2+8^2} = \sqrt{2*8^2} = \sqrt{2}*8 = 8\sqrt{2}$$
$$z^* = 8\sqrt{2}\left(\frac{1}{\sqrt{2}} + \ i(-\frac{1}{\sqrt{2}})\right)$$
$$\cos\phi = \frac{1}{\sqrt{2}}, \ \sin\phi = -\frac{1}{\sqrt{2}} \rightarrow \phi = -\frac{\pi}{4}$$
$$z^* = 8\sqrt{2} \ (\ \cos(-\frac{\pi}{4}) + \ i \ \sin(-\frac{\pi}{4}) \ )$$
$$|z| = \sqrt[3]{|z^*|} = \sqrt[3]{8\sqrt{2}} = 2\sqrt[6]{2}$$

$$z = \sqrt[3]{z^*} = 2\sqrt[6]{2}\left[\cos\left(\frac{-\frac{\pi}{4}+2\pi k}{n}\right)\right] + i\left[\sin\left(\frac{-\frac{\pi}{4}+2\pi k}{n}\right)\right]$$
$$n = 3 \rightarrow k \in \{0;1;2\}$$

$k = 0:$

$$z_0 = 2\sqrt[6]{2}\left(\cos\frac{-\frac{\pi}{4}+2\pi*0}{3} + i \ * \sin\frac{-\frac{\pi}{4}+2\pi*0}{3}\right) =$$
$$= 2\sqrt[6]{2} \ \cos(-\frac{\pi}{12}) + i * 2\sqrt[6]{2} * \sin(-\frac{\pi}{12})$$

$k = 1:$

$$z_0 = 2\sqrt[6]{2}\left(\cos\frac{-\frac{\pi}{4}+2\pi*1}{3} + i \ * \sin\frac{-\frac{\pi}{4}+2\pi*1}{3}\right) =$$
$$= 2\sqrt[6]{2} \ \cos\left(\frac{7\pi}{12}\right) + i * 2\sqrt[6]{2} * \sin\left(\frac{7\pi}{12}\right)$$

$k = 2:$

$$z_0 = 2\sqrt[6]{2}\left(\cos\frac{-\frac{\pi}{4}+2\pi*2}{3} + i \ * \sin\frac{-\frac{\pi}{4}+2\pi*2}{3}\right) =$$
$$= 2\sqrt[6]{2} \ \cos\left(\frac{15\pi}{12}\right) + i * 2\sqrt[6]{2} * \sin\left(\frac{15\pi}{12}\right)$$

3) Ебучая пизда с гиперболическими функциями. Важные формулы:

$$\cosh z = \frac{e^z + e^{-z}}{2} \to \cosh^2 z = \frac{e^{2z} + e^{-2z} + 2}{4}$$

$$\sinh z = \frac{e^z - e^{-z}}{2} \to \sinh^2 z = \frac{e^{2z} + e^{-2z} - 2}{4}$$

$$\cosh^2 z - \sinh^2 z = 1$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} \to \cos^2 z = \frac{e^{2iz} + e^{-2iz} + 2}{4}$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} \to \sin^2 z = -\frac{e^{2iz} + e^{-2iz} - 2}{4}$$

$$\cos^2 z + \sin^2 z = 1$$

$\cosh iz = \cos z = \cos(-z)$
$-i\sinh(-z) = i\sinh z = \sin iz$
$2\cosh^2 z - 1 = 2\frac{e^{2z}+2+e^{-2z}}{4} - \frac{2}{2} = \frac{e^{2z}+e^{-2z}}{2} = \cosh 2z$

$$z = \tan(2+i) = \frac{\sin(2+i)}{\cos(2+i)}$$

$$\sin(2+i) = \sin 2\cos i + \sin i \cos 2 = \sin 2\cosh 1 + i\sinh 1\cos 2$$

$$\cos(2+i) = \cos 2\cos i - \sin 2\sin i = \cos 2\cosh 1 - i\sin 2\sinh 1$$

$$z = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}$$

$$= \frac{\sin 2\cosh 1\cos 2\cosh 1 + \sinh 1\cos 2(-\sin 2\sinh 1)}{\cos^2 2\cosh^2 1 + \sin^2 2\sinh^2 1}$$

$$+ i\,\frac{\cos 2\cosh 1\sinh 1\cos 2 - \sin 2\cosh 1(-\sin 2\sinh 1)}{\cos^2 2\cosh^2 1 + \sin^2 2\sinh^2 1}$$

$$= \frac{\sin 2\cos 2(\cosh^2 1 - \sinh^2 1)}{\cos^2 2\cosh^2 1 + (1-\cos^2 2)\sinh^2 1}$$

$$+ i\,\frac{\cosh 1\sinh 1(\cos^2 2 + \sin^2 2)}{\cos^2 2\cosh^2 1 + (1-\cos^2 2)\sinh^2 1}$$

$$= \frac{\sin 2\cos 2}{\cos^2 2(\cosh^2 1 - \sinh^2 1) + \sinh^2 1}$$

$$+ i\,\frac{\cosh 1\sinh 1}{\cos^2 2(\cosh^2 1 - \sinh^2 1) + \sinh^2 1}$$

$$= \frac{\sin 2 \cos 2 + i \ \cosh 1 \sinh 1}{\cos^2 2 + \sinh^2 1}$$

$$= \frac{\sin 4 + i \ \sinh 2}{2\cos^2 2 + 2(\cosh^2 1 - 1)}$$

$$= \frac{\sin 4 + i \ \sinh 2}{2\cos^2 2 - 1 + 2\cosh^2 1 - 1}$$

$$= \frac{\sin 4 + i \ \sinh 2}{\cos 4 + \cosh 2}$$

$$= \frac{\sin 4}{\cos 4 + \cosh 2} + i \ \frac{\sinh 2}{\cos 4 + \cosh 2}$$

$$|z| = \sqrt{\frac{\sin^2 4 + \sinh^2 2}{(\cos 4 + \cosh 2)^2}} = \frac{\sqrt{\sin^2 4 + \sinh^2 2}}{|\cos 4 + \cosh 2|}$$

4)

$$z = \ln\left(\frac{1+i}{1-i\sqrt{3}}\right) = \ln z^*$$

$$z^* = \frac{(1+i)(1+i\sqrt{3})}{4} = \frac{(1-\sqrt{3}) + i \ (1+\sqrt{3})}{4}$$

$$|z^*| = \sqrt{\frac{(1-\sqrt{3})^2 + (1+\sqrt{3})^2}{4^2}} = \frac{1}{\sqrt{2}}$$

$$z^* = \frac{1}{\sqrt{2}}\left(\frac{1-\sqrt{3}}{2\sqrt{2}} + i \ \frac{1+\sqrt{3}}{2\sqrt{2}}\right)$$

$$\tan\phi = -\frac{1+\sqrt{3}}{1-\sqrt{3}} = -\frac{(1+\sqrt{3})^2}{-2} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3},$$

$$\phi = \pi - \arctan(\tan\phi) = \pi - \arctan(2+\sqrt{3})$$

$$z^* = \frac{1}{\sqrt{2}}\left[\cos\left(\pi - \arctan\left(2+\sqrt{3}\right)\right) + i \ \sin\left(\pi - \arctan\left(2+\sqrt{3}\right)\right)\right]$$

$\ln z = \ln(\rho * e^{i\phi}) = \ln\rho + i\phi$
$\ln\frac{1}{\sqrt{2}} = \ln(2^{-\frac{1}{2}}) = -\frac{1}{2}\ln 2$

$$z = \ln z^* = -\frac{1}{2}\ln 2 + i \ \left(\pi - \arctan\left(2+\sqrt{3}\right)\right)$$

$$|z| = \sqrt{\frac{1}{4}\ln^2 2 + \left(\pi - \arctan\left(2+\sqrt{3}\right)\right)^2}$$

Замечание: эту конскую ебанину можно упростить, если решать другим способом. Напиши, если тебя это интересует.

5)

$$z = (1+i\ \sqrt{3})^{1-2i} = e^{(Ln(1+i\ \sqrt{3}))*(1-2i)}$$

$$Ln(1+i\ \sqrt{3}) = Ln\left(2\left(\frac{1}{2}+i\ \frac{\sqrt{3}}{2}\right)\right) = \ln 2 + i\ (\frac{\pi}{3}+2\pi k)$$

$$z = e^{(\ln 2 + i\ (\frac{\pi}{3}+2\pi k))(1-2i)} = e^{\ln 2 + 2(\frac{\pi}{3}+2\pi k) + i((\frac{\pi}{3}+2\pi k) - 2\ln 2)} =$$

$$2e^{\frac{2\pi}{3}+4\pi k}(\cos(\frac{\pi}{3}+2\pi k - \ln 4)) + i\ 2e^{\frac{2\pi}{3}+4\pi k}(\sin(\frac{\pi}{3}+2\pi k - \ln 4)),\ |z| = 2e^{\frac{2\pi}{3}+4\pi k}$$