Блядское бдз v15 сука ненавижу тфкп

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1 Найти $|z|, Re(z), Im(z)$

1)

$$z = \left(\frac{-3 - i\sqrt{3}}{1 - i}\right)^{96} = (z^*)^{96}$$

$$z^* = \frac{-3 - i\sqrt{3}}{1 - i} = \frac{3 + i\sqrt{3}}{i - 1} = \frac{(3 + i\sqrt{3})(i + 1)}{(i - 1)(i + 1)} = \frac{3i - \sqrt{3} + 3 + i\sqrt{3}}{-1 - 1} =$$

$$= \frac{3 - \sqrt{3}}{-2} + i\ \frac{3 + \sqrt{3}}{-2} = \frac{-3 - \sqrt{3}}{2} + i\ \left(-\frac{3 + \sqrt{3}}{2}\right)$$

$$|z^*| = \sqrt{\left(-\frac{3 - \sqrt{3}}{2}\right)^2 + \left(-\frac{3 + \sqrt{3}}{2}\right)^2} = \sqrt{\frac{9 - 6\sqrt{3} + 3}{4} + \frac{9 + 6\sqrt{3} + 3}{4}} = \sqrt{\frac{24}{4}} = \sqrt{6}$$

$$z^* = \sqrt{6}\left(-\frac{3 - \sqrt{3}}{2\sqrt{6}} + i\left(-\frac{3 + \sqrt{3}}{2\sqrt{6}}\right)\right)$$

$$\cos\phi = -\frac{3 - \sqrt{3}}{2\sqrt{6}},\ \sin\phi = -\frac{3 + \sqrt{3}}{2\sqrt{6}}$$

$$\operatorname{tg}\phi = \frac{\sin\phi}{\cos\phi} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} + 1)^2}{3 - 1} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$$

$$-\frac{\pi}{2} < \arctan\phi < \frac{\pi}{2}$$

$$\phi = \arctan(\tan\phi) - \pi = \arctan(2 + \sqrt{3}) - \pi$$

$$z^* = \sqrt{6}\ [\ \cos\ (\arctan(2 + \sqrt{3}) - \pi) +\ i\ \sin\ (\arctan(2 + \sqrt{3}) - \pi)\ ]$$

$$|z| = (z^*)^{96} = (\sqrt{6})^{96} = 6^{48}$$

$$z = (z^*)^{96} = 6^{48} * \cos(96[\arctan(2+\sqrt{3}) - \pi]) + \ i * 6^{48} * \sin(96[\arctan(2+\sqrt{3}) - \pi])$$
$$= Re(z) + Im(z)$$

2)

$$z = \sqrt[3]{8-8i} \to z^3 = z^* = 8 - 8i$$
$$|z^*| = \sqrt{8^2+8^2} = \sqrt{2*8^2} = \sqrt{2}*8 = 8\sqrt{2}$$
$$z^* = 8\sqrt{2}\left(\frac{1}{\sqrt{2}} + \ i(-\frac{1}{\sqrt{2}})\right)$$
$$\cos\phi = \frac{1}{\sqrt{2}}, \ \sin\phi = -\frac{1}{\sqrt{2}} \to \phi = -\frac{\pi}{4}$$
$$z^* = 8\sqrt{2} \ (\ \cos(-\frac{\pi}{4}) + \ i \ \sin(-\frac{\pi}{4}) \ )$$
$$|z| = \sqrt[3]{|z^*|} = \sqrt[3]{8\sqrt{2}} = 2\sqrt[6]{2}$$

$$z = \sqrt[3]{z^*} = 2\sqrt[6]{2}\left[\cos\left(\frac{-\frac{\pi}{4}+2\pi k}{n}\right)\right] + i\left[\sin\left(\frac{-\frac{\pi}{4}+2\pi k}{n}\right)\right]$$
$$n = 3 \to k \in \{0;1;2\}$$

$k = 0:$

$$z_0 = 2\sqrt[6]{2}\left(\cos\frac{-\frac{\pi}{4}+2\pi*0}{3} + i \ * \sin\frac{-\frac{\pi}{4}+2\pi*0}{3}\right) =$$
$$= 2\sqrt[6]{2} \ \cos(-\frac{\pi}{12}) + i * 2\sqrt[6]{2} * \sin(-\frac{\pi}{12})$$

$k = 1:$

$$z_0 = 2\sqrt[6]{2}\left(\cos\frac{-\frac{\pi}{4}+2\pi*1}{3} + i \ * \sin\frac{-\frac{\pi}{4}+2\pi*1}{3}\right) =$$
$$= 2\sqrt[6]{2} \ \cos\left(\frac{7\pi}{12}\right) + i * 2\sqrt[6]{2} * \sin\left(\frac{7\pi}{12}\right)$$

$k = 2:$

$$z_0 = 2\sqrt[6]{2}\left(\cos\frac{-\frac{\pi}{4}+2\pi*2}{3} + i \ * \sin\frac{-\frac{\pi}{4}+2\pi*2}{3}\right) =$$
$$= 2\sqrt[6]{2} \ \cos\left(\frac{15\pi}{12}\right) + i * 2\sqrt[6]{2} * \sin\left(\frac{15\pi}{12}\right)$$

3) Ебучая пизда с гиперболическими функциями. Важные формулы:

$$\cosh z = \frac{e^z + e^{-z}}{2} \to \cosh^2 z = \frac{e^{2z} + e^{-2z} + 2}{4}$$

$$\sinh z = \frac{e^z - e^{-z}}{2} \to \sinh^2 z = \frac{e^{2z} + e^{-2z} - 2}{4}$$

$$\cosh^2 z - \sinh^2 z = 1$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} \to \cos^2 z = \frac{e^{2iz} + e^{-2iz} + 2}{4}$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} \to \sin^2 z = -\frac{e^{2iz} + e^{-2iz} - 2}{4}$$

$$\cos^2 z + \sin^2 z = 1$$

$\cosh iz = \cos z = \cos(-z)$
$-i \sinh(-z) = i \sinh z = \sin iz$
$2\cosh^2 z - 1 = 2\frac{e^{2z}+2+e^{-2z}}{4} - \frac{2}{2} = \frac{e^{2z}+e^{-2z}}{2} = \cosh 2z$

$$z = \tan(2+i) = \frac{\sin(2+i)}{\cos(2+i)}$$

$$\sin(2+i) = \sin 2 \cos i + \sin i \cos 2 = \sin 2 \cosh 1 + i \sinh 1 \cos 2$$

$$\cos(2+i) = \cos 2 \cos i - \sin 2 \sin i = \cos 2 \cosh 1 - i \sin 2 \sinh 1$$

$$z = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i \frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2}$$

$$= \frac{\sin 2 \cosh 1 \cos 2 \cosh 1 + \sinh 1 \cos 2(-\sin 2 \sinh 1)}{\cos^2 2 \cosh^2 1 + \sin^2 2 \sinh^2 1}$$

$$+i \, \frac{\cos 2 \cosh 1 \sinh 1 \cos 2 - \sin 2 \cosh 1(-\sin 2 \sinh 1)}{\cos^2 2 \cosh^2 1 + \sin^2 2 \sinh^2 1}$$

$$= \frac{\sin 2 \cos 2(\cosh^2 1 - \sinh^2 1)}{\cos^2 2 \cosh^2 1 + (1 - \cos^2 2) \sinh^2 1}$$

$$+i \, \frac{\cosh 1 \sinh 1(\cos^2 2 + \sin^2 2)}{\cos^2 2 \cosh^2 1 + (1 - \cos^2 2) \sinh^2 1}$$

$$= \frac{\sin 2 \cos 2}{\cos^2 2(\cosh^2 1 - \sinh^2 1) + \sinh^2 1}$$

$$+i \, \frac{\cosh 1 \sinh 1}{\cos^2 2(\cosh^2 1 - \sinh^2 1) + \sinh^2 1}$$

$$= \frac{\sin 2 \cos 2 + i \ \cosh 1 \sinh 1}{\cos^2 2 + \sinh^2 1}$$

$$= \frac{\sin 4 + i \ \sinh 2}{2\cos^2 2 + 2(\cosh^2 1 - 1)}$$

$$= \frac{\sin 4 + i \ \sinh 2}{2\cos^2 2 - 1 + 2\cosh^2 1 - 1}$$

$$= \frac{\sin 4 + i \ \sinh 2}{\cos 4 + \cosh 2}$$

$$= \frac{\sin 4}{\cos 4 + \cosh 2} + i \ \frac{\sinh 2}{\cos 4 + \cosh 2}$$

$$|z| = \sqrt{\frac{\sin^2 4 + \sinh^2 2}{(\cos 4 + \cosh 2)^2}} = \frac{\sqrt{\sin^2 4 + \sinh^2 2}}{|\cos 4 + \cosh 2|}$$