Opl Res. Q., Pergamon Press 1977. Vol. 28, 1, ii, pp. 167 to 179. Printed in Great Britain

Hospital Layout as a Quadratic Assignment Problem*

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The problem of locating hospital departments so as to minimize the total distance travelled by patients can be formulated as a Quadratic Assignment Problem. In this paper we discuss some practical considerations of the problem, its formulation, we present a heuristic procedure to solve it and comment on our computational experience with the heuristic.

THE GROWING concern of governments over their health care programmes has resulted in an increasing number of hospitals being established and operated on various degrees of complexity, such developments frequently being accompanied by the use of Operational Research techniques throughout the stages of planning, construction and operation. Hospitals nowadays are very large and complex organizations which have high initial capital costs and enormous running expenses but which, however, should be regarded as investments that will pay economic dividends in the form of reduced unemployment as a direct consequence of reductions in sickness and incapacity and an increase in fitness. Furthermore, while planning for a hospital it is essential to take into account the human factors as well as the economic since the convenience to the patients must be a prime consideration in any hospital system.

In this paper we consider the problem of the relative location of clinics within a hospital department. The objective is to decide upon the location of the various clinics so as to reduce the effort spent by the patients while moving from one clinic to another. Thus the objective is to locate the clinics, within the given building, so as to minimize the total distance (in patient–metres) travelled per year.

STATEMENT OF THE PROBLEM

The hospital concerned (the Ahmed Maher Hospital) is located in a rather densely populated part of Cairo. It is composed of six major departments: Out-patient, In-patient, Dental Research, Accident and Emergency, Physiotherapy and Housekeeping and Maintenance, each department occupying a separate building. In recent years the centre of gravity of activity within the hospital has been moving steadily from the wards towards the Out-patient

*This paper was presented at the ORS Annual Conference, Loughborough, 23–26 Sept. 1975.

department. As a result, this latter department has been becoming more and more overcrowded with the average daily number of patients now exceeding 700, and with these patients having to move among the 17 clinics in the department. The locations of the clinics relative to each other has been criticized for causing too much travelling for patients and for causing bottlenecks and serious delays. It was therefore decided to conduct a study aimed at an improvement in the layout of the department leading to a reduction in the total distance travelled by patients and hence in the frequency of bottlenecks and congestions.

PROBLEM FORMULATION AND SURVEY OF SOME PAST WORK

In this situation we are concerned with locating n facilities in n given locations. The yearly flow (f_{ik}) between each pair of facilities (i and k) is known and so is the distance (d_{jq}) between each pair of locations (j and q). Each location can house only one facility and each facility occupies only one location. The objective is to locate the facilities so as to minimize the total distance travelled by patients per year. The problem can be formulated as follows:

$$\begin{aligned}
\text{Minimize } \phi = \; & \Sigma_{i,j}\Sigma_{k,q} f_{ik}\, d_{jq}\, y_{ij}\, y_{kq} \\
\text{such that } \; & \Sigma_{j\epsilon J} \;\; y_{ij} = 1 \;\forall i\epsilon I \\
& \Sigma_{i\epsilon I} \;\; y_{ij} = 1 \;\forall j\epsilon J \\
& y_{ij} \begin{cases} = 1 & \text{if facility } i \text{ is located at } j \\ = 0 & \text{otherwise} \end{cases}
\end{aligned}$$

where I is the set of all facilities to be located and J is the set of all locations. The nature of the cost elements in the formulation is such that the objective function is composed of terms in which the location variables of the two facilities involved appear together, so that each term is quadratic in nature. Consequently, this formulation of the problem has been termed the *Quadratic Assignment Problem* (QAP).

Interest in this problem, and its name, dates back to the work of Koopmans and Beckmann.[1] They recognized it in relation to locating indivisible plants. Since then, the problem has received continuing interest and a number of generalized versions have been suggested. Lawler[2] has introduced the multi-product generalization and Graves and Whinston[3] have pointed out the possibility that other cost items could appear in some applications; these two extensions led to Pierce and Crowston's more generalized formulation.[4]

The QAP has many applications, including: i) minimizing total wire length in electronic and electrical assemblies,[5] ii) locating machines, departments or offices within a plant so as to minimize transportation efforts,[6] iii) arranging the indicators and controls in a control room so as to minimize eye fatigue,[7] iv) laying out offices in a building or operating theatres in a hospital,[8] and v) relocating civil service departments with the monetary objective of reducing the cost of office accommodation and the social objective of providing employment in development areas.[9]

Several solution techniques for the QAP have been suggested in the literature. Armour and Buffa,[6] Gaschutz and Ahrens,[10] Graves and Whinston,[3] Hillier,[11] Hillier and Connors.[12] Nugent *et al.*,[13] Steinberg[5] and Whitehead and Elders[8] have presented heuristics to obtain good, but not necessarily optimal, solutions to the problem. Pierce and Crowston[4] have presented a unifying framework for some of the above approaches and described some new algorithms to handle the generalized case.

Since the treatment of the QAP in the literature has been rather extensive and has been reviewed in more than one context in the past[2,4,12,13], we feel that it is unnecessary to discuss it further. From the computational point of view, it has been discovered that the problem is extremely difficult to solve optimally[14] and that for an optimising approach to be feasible there is a need for some breakthrough in the theory or computational methods of combinatorial programming. In fact, no breakthrough in either area is yet evident. However, the approach of resorting to the physical properties to find alternative, and more tractable, formulations seems promising as Beale and Tomlin[9] have demonstrated.

Meanwhile, the QAP is continuing to demonstrate its potential applicability in real world situations, Lundqvist,[14] for example, this should be developed as an integrated dynamic model to determine an optimal combination of building stock allocation (to various urban zones) and transport network structure.

FEATURES OF THE SOLUTION PROCEDURE

Since exact solutions of the QAP are rather difficult to obtain in a reasonable amount of computational time the hospital layout problem was solved by a heuristic rather than an optimising algorithm. The heuristic was developed by M. S. Bazaraa and the author. It is in two parts: Part A develops an initial layout and Part B improves it. In this section we explain the strategies for building the initial layout and for improving it. Then in the following section the steps of the heuristic are given.

A. STRATEGIES FOR BUILDING THE INITIAL SOLUTION

Selecting the next facility and location:

Strategy 1. We use a rank L_i in terms of the number of facilities having interactions with facility i and the total number of interactions between it and the other facilities. We also rank the locations in terms of R_j, the sum of distances from j to all other locations. Then at any general step k we choose, from the unassigned facilities, the one with the greatest L_i and locate it at the location, amongst the vacant ones, with least R_j.

It is obvious that a complete initial solution could be obtained by assigning the facilities, ranked in a descending order of L_i, to the locations, ranked in an ascending order of R_j.

Strategy 2. At any general step k we choose, from the unassigned facilities, the one with the maximum number of interactions with the most recently located facility. Then we locate it at that vacant location which causes a minimum increase in the total cost. A complete initial solution could be obtained by repeating this step until all facilities are assigned.

Either of the above two strategies could be adopted for the development of the initial solution. The first one has the advantage of locating the facility with the maximum weight, regarding interactions, in the most central available location. However, this is done regardless of what effect this would have on the facilities which have been located so far. On the other hand, the second strategy chooses the facility with the maximum number of interactions with the one most recently located and locates it so as to result in a minimum increase in the objective function. However, the choice of the facility does not take into consideration either its interactions with all other facilities or the probable strategic importance for the chosen location with respect to the facilities which have not been located so far. Thus we have combined the two strategies as shown through the steps of the procedure, explained in the following section.

Improving any partial solution:

At any general step k once a facility i is located at j, we test whether the assignment at hand is the best for the facilities and the locations involved in it or not. If it is, we proceed to step $k + 1$, otherwise we replace it by a better one and then proceed to step $k + 1$. This test is performed through exchanging the most recently located facility with each of the previously located ones in turn and by replacing the original assignment by the one which results in maximum reduction in the objective function.

B. STRATEGIES FOR IMPROVING THE INITIAL SOLUTION

Once a complete assignment is available, it is possible to seek improvements by interchanging the locations of two facilities. Once such an improvement has been achieved, it is possible either to repeat the process in a search for further improvements or to stop. This process can, indeed, be repeated until we get to the stage where we find that no further improvements are available through single exchanges to the pattern on hand. At each stage, it is possible to choose either the first exchange which will result in an improvement or the one which yields the maximum improvement.

We have chosen the strategy of testing the effect of interchanging each facility i, in turn, with every other facility, storing the best resulting pattern (p_i) and performing an interchange if this best pattern provides a reduction in the total cost. Otherwise the process is repeated for the next facility and so on. At any stage, if two facilities are eventually interchanged, we restart. However, if we fail to achieve any further improvement by interchanging the location of two facilities in the best assignment we take the next best assignment, chosen from amongst the patterns p_i's and repeat the whole procedure.

The termination criterion is simple. We stop when we have repeated the overall procedure, each time starting with a different pattern for a given number of times without achieving any improvement.

Like most heuristics in the literature of the QAP, this heuristic neither guarantees the attainment of an optimal solution nor gives an indication of the goodness of the solution as compared to the true optimum.

STEPS OF THE SOLUTION PROCEDURE

Part A: Build the initial solution

1. Rank the locations as follows:
   (i) Calculate R_j, the sum of distances from location j to all other locations
   (ii) Rank the locations in ascending order of R_j.
2. Rank the facilities as follows:
   (i) Calculate E_i^1, the number of facilities having interactions with facility i.
   (ii) Calculate E_i^2, the total interactions, i.e. number of passengers travelling to and from facility i.
   (iii) Rank the facilities in descending order of E_i^1.
   (iv) Rank the facilities in descending order of E_i^2.
   (v) Find the combined index L_i for each facility, by adding its two rankings in iii) and iv).
   (vi) Rank the facilities in ascending order of L_i.
3. At any general step k, check whether any facility remains unassigned, if so proceed to Step 4, otherwise go to Step 7.
4. Pick the next facility in the rank, locate it at the first available location in the rank, set key = 1 and proceed to Step 6.

5. Pick the facility with maximum number of interactions with the most recently located facility, if none are available, go to Step 7. Otherwise find the location to which this facility can be assigned at a minimal increase in the objective function. Assign the facility to it, set key = 2 and proceed to Step 6.
6. Exchange the facility which has just been located with each of the previously located facilities in turn, each time calculating the resulting change in the objective function. Maintain the partial assignment which causes the maximum reduction, if any; finally replace the original assignment with that one. If key = 1, return to Step 5, otherwise return to Step 3.
7. Terminate as all facilities have been assigned.

Part B: Improve the initial solution

1. Set KOUNT = 0, the number of patterns investigated without improvement, declare the list of accumulated patterns empty.
2. Set $i = 0$, the index of the facility to be exchanged with other facilities.
3. Replace i by $i + 1$
4. If $i \leqslant n - 1$ (n is the total number of facilities), go to step 6. Otherwise proceed to step 5.
5. Replace KOUNT by KOUNT + 1 and go to step 11.
6. Set $j = i + 1$, the index of the facility to be exchanged with i. Set COMP = ∞
7. Interchange i and j. Calculate the resulting difference in the objective function DIF. If DIF > COMP go to step 9. Otherwise proceed to step 8.
8. Put COMP = DIF and KEEP = j.
9. Exchange the values of j and i, if $j = n$ go to step 10. Otherwise replace j by $j + 1$ and go to step 7.
10. If COMP < 0, interchange i & KEEP, erase any accumulated patterns, set KOUNT = 0 and go to step 2. Otherwise add this pattern p_i to the list of accumulated patterns and go to step 3.
11. If KOUNT $\leqslant m$ (m is the maximum number of patterns to be considered without any improvement), select from the list of accumulated patterns ($p_i's$) the one with the least value of the objective function and go to step 2. Otherwise proceed to step 12.
12. Declare the pattern in hand as the best solution found and terminate.

ANALYSIS OF THE DEPARTMENT'S LAYOUT

The outpatient department is composed of a receiving and recording room, a waiting room and 17 clinics. There is also an administration section, a lecture room, a staff housing facility and stairs between the floors. The flow of patients is, however, confined between the receiving and recording room, and the 17 clinics, i.e. 18 facilities in total. Thus, it was decided to fix the other sections at their original location and investigate the relative location of the 18 facilities. All the facilities needed roughly the same area with the exception of the Minor Operation section which occupied nearly double the space necessary for any other facility. Thus it was split in two pseudo facilities which have to exist beside each other. As a result, the total number of facilities is 19 and they are displayed in Exhibit I.

EXHIBIT I: DISPLAY OF THE FACILITY CODE AND FUNCTION

Facility's code	Facility's Function	Facility's code	Facility's Function
1	Receiving and Recording	11	X-Ray
2	General Practitioner	12	Orthopedic
3	Pharmacy	13	Psychiatric
4	Gynecological & Obstetric	14	Squint
5	Medicine	15	Minor Operations
6	Paediatric	16	Minor Operations
7	Surgery	17	Dental
8	Ear, Nose & Throat	18	Dental Surgery
9	Urology	19	Dental Prosthetic
10	Laboratory		

DESCRIPTION OF THE DATA

Estimates of the patient flows between clinics were available on a yearly basis. Entries in the flow matrix were obtained by averaging the flow between each pair of clinics, thus generating a symmetric matrix.[15] The distances between locations were actually measured by tracing the paths taken by patients while moving from one location to another. Whenever the movement involved a change in floors, the corresponding vertical distance was multiplied by a subjective factor of 3. It was noticed that a patient, after being through a sequence of visits to more than one clinic, must return to the first clinic he visited to mark off his card. In doing so he traces, more or less, the same path he has taken in his forward trip because all the clinics are in the same building and there is only one main corridor per floor. Thus the distance matrix can also be taken to be symmetric, even for pairs of locations on two different floors. Since the flow and distance data are symmetrical, the data can be compactly presented as in table 1, where the elements below the diagonal represent the flows and those above the diagonal represent the distances. The flow between pseudo facilities 15 and 16 is put equal to an extremely large number so as to force them to be in two adjacent locations in the final solution.

COMPUTATIONAL EXPERIENCE

We started by fixing facilities 15 & 16 in locations 18 & 17 as they are nearest to each other. Then the heuristic was applied in order to locate the remaining facilities. The steps through which the heuristic proceeded are shown in Exhibit II. The heuristic iterated, as appropriate, through steps 4, 5 and 6 until all the facilities were located. Then improvements were sought by interchanging the locations of pairs of facilities. The heuristic proceeded as follows: once no further improvements could be found by pairwise interchanges of all the facilities of a certain pattern, another pattern was chosen and the process repeated. The choice was made among the patterns which had been generated at the immediately preceeding step in search of better solutions, but which, having been proven to be worse, had been stored in ascending cost order. When no further improvement was found after considering 50 patterns (i.e. $m = 50$), the procedure terminated.

The cost of the best solution obtained is 11,281,887 as compared to the cost of the original layout[15] which is 13,973,298, i.e. a resulting decrease of approximately 19.2%. The full solution required 136 seconds CPU time on an IBM 360/40, the initial solution being obtained after 44 seconds CPU time with the remaining 92 seconds being taken by part B. The initial solution was 16.4% better than the original layout, a further 2.8% improvement being obtained from part B.

As with most heuristics, a major consideration is how good the solutions obtained by the heuristic are and how well they compare with the solutions obtained by other heuristics. The heuristic was compared with these due to Hillier,[11] Hillier and Connors,[12] Armour and Buffa,[6] Nugent *et al*[13] and Neghabat.[10] We could not obtain comparative results on the problem of current interest as the codes for the other heuristics were not available to us. However, the heuristic described here was used to solve the 8 test problems of Nugent *et al* as there exists comparative computational experience on these. Table 2 shows the results of solving these problems by our heuristic and the others.

From table 2 we notice that in general the heuristic compares with the others reasonably well. In the case of problem 7 it led to a solution better than the best one previously mentioned in the literature. We also know that the qualities of solutions obtained as a result of applying part A only are good for small and medium size problems. Part B leads to significant improvements for large size problems.

CONCLUSIONS

An exact procedure to solve a quadratic assignment problem of the given size in a reasonable amount of computational time is not available at the present time. However, the heuristic described in this paper has led to an

TABLE 1. DISTANCES AND FLOWS MATRIX

Distances

0	12	36	28	52	44	110	126	94	63	130	102	65	98	132	132	126	120	126
76687	0	24	75	82	75	108	70	124	86	93	106	58	124	161	161	70	64	70
0	40951	0	47	71	47	110	73	126	71	95	110	46	127	163	163	73	67	73
415	4118	3848	0	42	34	148	111	160	52	94	148	49	117	104	109	111	105	111
545	5767	2524	256	0	42	125	136	102	22	73	125	32	94	130	130	136	130	136
819	2055	3213	0	0	0	148	111	162	52	96	148	49	117	152	152	111	105	111
135	1917	2072	0	0	0	0	46	46	136	47	30	108	51	79	79	46	47	41
1368	2746	4225	0	0	0	0	0	69	141	63	46	119	68	121	121	27	24	36
819	1097	566	0	47	0	196	0	0	102	34	45	84	23	80	80	69	64	51
5630	5712	0	829	1655	926	1538	0	1954	0	64	118	29	95	131	131	141	135	141
0	0	0	128	287	161	196	301	418	0	0	47	56	54	94	94	63	46	24
3432	0	404	0	0	0	0	0	0	282	1686	0	100	51	89	89	46	40	36
9082	0	9372	0	42	0	0	0	0	0	0	0	0	77	113	113	119	113	119
1503	268	0	0	0	0	0	0	0	0	0	0	0	0	79	79	68	62	51
0	0	972	0	0	0	0	0	0	0	0	0	0	0	0	10	113	107	119
0	1373	0	0	0	0	0	0	0	0	0	0	0	0	99999	0	113	107	119
13732	268	13538	0	226	0	0	0	0	0	226	0	0	0	0	0	0	6	24
1368	0	1368	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12
1783	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Flows

EXHIBIT II: STEPS OF THE SOLUTION

	Locations at which facilities are assigned																				
Heuristic's step no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Cost	Remarks
Initial															18	17					
4	11														18	17					
5	11	19													18	17					
6	11	19													18	17					No improvement is possible
4	11	19	14												18	17					
6	11	19	14												18	17					No improvement is possible
5	11	19	14												18	17	9			5414129	
6	11	9	14												18	17	19			5337900	Improvement due to exchange
4	11	9	4							13					18	17	19				
6	11	9	14							13					18	17	19				No improvement is possible
5	11	9	14						10	13					18	17	19				
6	11	9	14						10	13					18	17	19				No improvement is possible
4	11	9	14		12				10	13					18	17	19				
6	11	9	14		12				10	13					18	17	19				No improvement is possible
5	11	9	14		12				10	13	7				18	17	19			7075494	
6	11	9	14		12				10	7	13				18	17	19			6893988	Improvement due to exchange
4	11	9	14	8	12				10	7	13				18	17	19				
6	11	9	14	8	12				10	7	13				18	17	19				No improvement is possible
5	11	9	14	8	12				10	7	13			5	18	17	19			7668136	
6	11	9	14	8	12				5	7	13			10	18	17	19			7641503	Improvement due to exchange
4	11	9	14	8	12		3		5	7	13			10	18	17	19			8351126	
6	11	9	14	8	12		13		5	7	3			10	18	17	19			8153915	Improvement due to exchange
5	11	9	14	8	12		13		5	7	3			10	18	17	19		2	8319734	
6	11	9	14	8	12		13		2	7	3			10	18	17	19		5	8292907	Improvement due to exchange
4	11	9	14	8	12		13		2	7	3		1	10	18	17	19		5	10396304	
6	11	9	14	8	12		1		2	7	3		13	10	18	17	19		5	9592112	Improvement due to exchange
5	11	9	14	8	12		1		2	7	3		13	10	18	17	19	16	5		
6	11	9	14	8	12		1		2	7	3		13	10	18	17	19	16	5		No improvement is possible
4	11	9	14	8	12	4	1		2	7	3		13	10	18	17	19	16	5	10755096	
6	11	9	14	8	12	16	1		2	7	3		13	10	18	17	19	4	5	10475368	Improvement due to exchange
5	11	9	14	8	12	16	1		2	7	3	6	13	10	18	17	19	4	5	10973088	
6	11	9	14	8	12	16	1		2	7	3	10	13	6	18	17	19	4	5	10955632	Improvement due to exchange

4	11 9 14 8 12 16 1 15 2 7 3 10 13 6 18 17 19 4 5	11686744	Best Initial Solution's Cost
6	11 9 14 8 12 16 1 15 2 7 3 10 13 6 18 17 19 4 5		No improvement is possible
Exchange 2 facilities	14 9 11 8 12 16 1 15 2 7 3 10 13 6 18 17 19 4 5	11586149	Improvement due to exchange
Exchange 2 facilities	14 9 11 8 7 16 1 15 2 12 3 10 13 6 18 17 19 4 5	11575736	Improvement due to exchange
Exchange 2 facilities	14 9 11 8 7 16 3 15 2 12 1 10 13 6 18 17 19 4 5	11569067	Improvement due to exchange
Exchange 2 facilities	14 9 11 12 7 16 3 15 2 8 1 10 13 6 18 17 19 4 5	11562590	Improvement due to exchange
Exchange 2 facilities	14 9 11 7 12 16 3 15 2 8 1 10 13 6 18 17 19 4 5	11558109	Improvement due to exchange
Exchange 2 facilities	14 9 11 13 12 16 3 15 2 8 1 10 7 6 18 17 19 4 5	11509272	Improvement due to exchange
Exchange 2 facilities	14 9 11 15 12 16 3 13 2 8 1 10 7 6 18 17 19 4 5	11474803	Improvement due to exchange
Exchange 2 facilities	14 9 11 15 12 16 3 13 2 8 5 10 7 6 18 17 19 4 1	11447279	Improvement due to exchange
Exchange 2 facilities	14 9 11 15 12 16 3 13 2 8 4 10 7 6 18 17 19 5 1	11429396	Improvement due to exchange
Exchange 2 facilities	14 9 11 16 12 15 3 13 2 8 4 10 7 6 18 17 19 5 1	11429231	Improvement due to exchange
Exchange 2 facilities	14 9 11 16 12 15 2 13 3 8 4 10 7 6 18 17 19 5 1	11421851	Improvement due to exchange
Exchange 2 facilities	14 9 11 16 12 15 2 13 3 8 4 5 7 6 18 17 19 10 1	11392841	Improvement due to exchange
Exchange 2 facilities	14 9 11 16 12 10 2 13 3 8 4 5 7 6 18 17 19 15 1	11384065	Improvement due to exchange
Exchange 2 facilities	14 9 11 15 12 10 2 13 3 8 4 5 7 6 18 17 19 16 1	11383425	Improvement due to exchange
Exchange 2 facilities	14 9 11 15 12 10 2 13 3 8 4 5 7 16 18 17 19 6 1	11359055	Improvement due to exchange
Exchange 2 facilities	14 9 11 15 12 10 2 13 3 8 4 5 7 1 18 17 19 6 16	11357487	Improvement due to exchange
Exchange 2 facilities	14 9 11 15 12 10 2 13 3 8 4 5 7 1 18 17 19 6 16	11357487	No improvement is possible
Start from stored pattern	14 9 11 15 12 10 2 13 3 8 4 1 7 16 18 17 19 6 5	11362255	
Exchange 2 facilities	14 9 11 15 12 10 2 13 3 8 4 1 7 16 18 17 19 5 6	11340366	Improvement due to exchange
Exchange 2 facilities	14 9 11 15 12 10 2 13 3 8 4 1 7 16 18 17 19 5 6	11340366	No improvement possible
Start from stored patterns and exchange facilities	No improvement possible after considering 21 patterns		
Start from stored pattern	14 9 11 13 12 5 2 10 3 8 4 1 7 16 18 17 19 15 6	11342693	
Exchange 2 facilities	14 9 11 8 12 5 2 10 3 13 4 1 7 16 18 17 19 15 6	11339236	Possible improvement due to exchange
Exchange 2 facilities	14 9 11 10 12 5 2 8 3 13 4 1 7 16 18 17 19 15 6	11312433	Possible improvement due to exchange
Exchange 2 facilities	14 9 11 10 12 5 2 8 3 13 4 1 7 16 18 17 19 6 15	11299399	Possible improvement due to exchange
Exchange 2 facilities	14 9 11 10 12 5 2 8 3 13 4 1 7 16 18 17 19 6 15	11299399	No improvement is possible
Start from stored pattern	14 9 11 10 12 5 4 8 3 13 2 1 7 16 18 17 19 6 15	11300745	
Exchange 2 facilities	14 9 11 10 12 5 3 8 4 13 2 1 7 16 18 17 19 6 15	11281887	Possible improvement due to exchange
Exchange 2 facilities	14 9 11 10 12 5 3 8 4 13 2 1 7 16 18 17 19 6 15	11281887	No improvement is possible
Start from stored patterns and exchange facilities	No improvement possible after considering 50 patterns		
Terminate, Best Solution is	14 9 11 10 12 5 3 8 4 13 2 1 7 16 18 17 19 6 15	11281887	Final Solution

TABLE 2. RESULTS OF THE COMPARISON BETWEEN THE HEURISTIC AND SEVERAL OTHERS

Problem size	Hillier	Hillier-Connors	Craft	Biased Sampling	Neghabat	Part A	Part A + Part B	Best known cost
	Average Cost*					Cost		
5 × 5	27.6	29.4	28.2	27.6	25	26	26	25†
6 × 6	44.2	44.2	44.2	44.6	43	43	43	43†
7 × 7	78.8	78.4	79.6	77.2	74	78	75	74†
8 × 8	114.4	110.2	113.4	111.6	118	114	114	107†
12 × 12	317.4	310.2	296.2	304.7	334	295	295	289†
15 × 15	632.6	600.2	606.0	603.0	618	645	614	575
20 × 20	1400.4	1345	1339.0	1339.0	1385	1345	1299	1304
30 × 30	3267.2	3206.8	3189.6	3189.6	3260	3170	3125	3093

* Resulting from several random initial solutions.
† Optimum Solution.

improvement of 19.2% in the total cost (expressed in terms of passenger metres travelled per year) and took 136 sec. Furthermore, the derived solution proved compatible with some medical considerations that management was keen to satisfy in the new layout.

ACKNOWLEDGEMENTS

The author wishes to acknowledge his joint work on the Quadratic Assignment Problem with Professor M. S. Bazaraa. Results of that work will appear in a separate paper prepared jointly. The author also wishes to acknowledge the comments by two anonymous referees which helped improve the presentation of the paper.

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