Exercise 4

Given: $\{t_i\}_{i=1}^{N}$ and $\{\sigma_{t_i}^2\}_{i=1}^{N}$ for some unknown Quantity T.

where each t_i differ from the 'true' value, T, by a Gaussian distribution offset, taken from a Gaussian with mean $= 0$, variance $= \sigma_{t_i}^2$

→ true according to us, but not really true. ↓ No one knows.

In the ~~probability~~ frequency distribution equation, our $f(x_i) = T$ (the 'true' value that the model ~~should~~ will predict)

$$\therefore P(t_i | T, \sigma_{t_i}) = \frac{1}{\sqrt{2\pi\sigma_{t_i}^2}} \exp\left\{-\frac{(T-t_i)^2}{2\sigma_{t_i}^2}\right\}$$

$$\therefore \mathcal{L} = \prod_{i=1}^{N} P(t_i | T, \sigma_{t_i})$$

$$\ln \mathcal{L} = \sum_{i=1}^{N} \left(\ln\left(\frac{1}{\sqrt{2\pi\sigma_{t_i}^2}}\right) + \left\{-\frac{(T-t_i)^2}{2\sigma_{t_i}^2}\right\} \right)$$

$$= K - \sum_{i=1}^{N} \frac{(T-t_i)^2}{2\sigma_{t_i}^2}$$

Set $\frac{d \ln \mathcal{L}}{dT} = 0$. [finding maxima.]

$$-\sum_{i=1}^{N} \frac{2(T-t_i)}{2\sigma_{t_i}^2} = 0$$

$$\Rightarrow \sum_{i=1}^{N} \frac{T}{\sigma_{t_i}^2} = \sum_{i=1}^{N} \frac{t_i}{\sigma_{t_i}^2}$$

$$\Rightarrow T \sum_{i=1}^{N} \frac{1}{\sigma_{t_i}^2} = \sum_{i=1}^{N} \frac{t_i}{\sigma_{t_i}^2}$$

$$\Rightarrow T = \sum_{i=1}^{N} \frac{t_i}{\sigma_{t_i}^2} \Big/ \sum_{i=1}^{N} \frac{1}{\sigma_{t_i}^2}$$