# Exercise 5

Given: $\chi^2 = (Y-AX)^T C^{-1} (Y-AX)$

Consider $R = Y - AX$

$\therefore$ Jacobian $= -A = \begin{bmatrix} \frac{\partial R_1}{\partial x_1} & \frac{\partial R_1}{\partial x_2} & \cdots & \frac{\partial R_1}{\partial x_n} \\ \frac{\partial R_2}{\partial x_1} & \frac{\partial R_2}{\partial x_2} & \cdots & \frac{\partial R_2}{\partial x_n} \\ \vdots & & \ddots & \\ \frac{\partial R_n}{\partial x_1} & \frac{\partial R_n}{\partial x_2} & \cdots & \frac{\partial R_n}{\partial x_n} \end{bmatrix}$

$\chi^2 = R^T C^{-1} R$

Notice: $\chi^2$ is a scalar. $\Longrightarrow (\chi^2)^T = \chi^2$

~~This can be seen by taking $(\chi^2)^T$ too.~~

~~$(R_1^T C^{-1} R_2)^T = R_2^T C^{-1\,T} R_1^{T\,T}$~~

~~$= R_2^T (C^{-1})^T R_1$~~

~~But $C^{-1}$ is symm. $\because C$ is sym.~~

~~$\therefore (\chi^2)^T = \chi^2$~~

$$\frac{d\chi^2}{dx} = \frac{d}{dx}\left(R^T C^{-1} R\right) = \frac{d}{dx}\left( (\cdots) \begin{pmatrix} \cdot & & \\ & \ddots & \\ & & \cdot \end{pmatrix} \begin{pmatrix} \cdot \\ \vdots \\ \cdot \end{pmatrix} \right)$$

the product of $(R^T C^{-1} R)$ is a scalar, and the elements of $R^T$ and $R$ are the same.

Hence when taking derivative, we get two equal terms in product rule.

$$\frac{d\chi^2}{dx} = \cancel{\frac{d}{dx}}\; 2R^T C^{-1}(-A) = 0 \quad \text{[setting it 0, for extremum]}$$

Taking T on both the sides.

[C is symm.]

$A^T C^{-1} R = 0$

$A^T C^{-1} Y = A^T C^{-1} A X$

$X = [A^T C^{-1} A]^{-1} [A^T C^{-1} Y]$