CSE313 Spring2016

Due: 5-31-2016

Midterm Examination

Problems:

1. [50 points] Consider the two numbers in radix decimal: A = 123 and B = -1023.

1.1. Explicitly convert each decimal number to binary (radix two) in 2s complement

(a) A = 123 is positive.

123 / 2 = 61 + 1

61 / 2 = 30 + 1

30 / 2 = 15 + 0

15 / 2 = 7 + 1

7 / 2 = 3 + 1

3 / 2 = 1 + 1

1 / 2 = 0 + 1

so binary of A in 2's complement is 01111011.

(b) B = - 1023.

According to Yale, Sanjay(2004, Page 26) *Example 2.1*. I followed steps from this example.

1. Let B be +1023 and find the binary fo +1023.

1023 / 2 = 511 + 1

511 / 2 = 255 + 1

255 / 2 = 127 + 1

127 / 2 = 63 + 1

63 / 2 = 31 + 1

31 / 2 = 15 + 1

15 / 2 = 7 + 1

7 / 2 = 3 + 1

3 / 2 = 1 + 1

1 / 2 = 0 + 1

So, the representation for +1023 is 01111111111

2. The complement of +1023 is 10000000000

3. Adding 1 to 1000000000 gives us 10000000001 and it's 2's complement binary.

1.2. Explicitly convert each decimal number to hexadecimal (radix sixteen) in 1s complement.

Accroding to Yale, Sanjay(2004, Page 42, ¶ 5) "If we had first broken the string at four-bit boundaries and then conveted each four-bit string to its equivalent hex digit.

(a) For A. We have 01111011 in 2's complement.

Convert it to 1's complement: 10000100.

Break it to four-bit boundaries from left to right: 0001 0000 0100.

Convert to hex for each part: 1 0 4.

So, the representnation of 1's complement hexadecimal number for A is x104.

(b) For B, we have 10000000001 in 2's complement.

Convert it to 1's complement: 01111111110.

Break it to four-bit Boundaries from left to right: 0011 1111 1110.

Convert it to hex for each part: 3 F E.

So, the representation of 1's complement hexadeciaml number for B is x3FE.

1.3. Using 2s complement integer arithmetic, explicitly find the result A + B in binary.

According to Yale, Sanjay(2004, Page 30) *Example 2.3.*

The decimal value A = 123 is represented as 01111011.

The decimal value B = -1023 is represented as 10000000001.

A+B is

```
     00001111011
 +   10000000001
 ---------------
     10001111100
```

So, A+B represented <u>10001111100</u>.

1.4. Using 2s complement integer arithmetic, explicitly find the result A - B in binary.

According to Yale, Sanjay(2004, Page 30) *Example 2.4.*

A - B we can change it to A + (-B)

-B is +1023 and the representation of it is 01111111111.

A + (-B) is

```
     00001111011
 +   01111111111
 ---------------
     10001111010
```

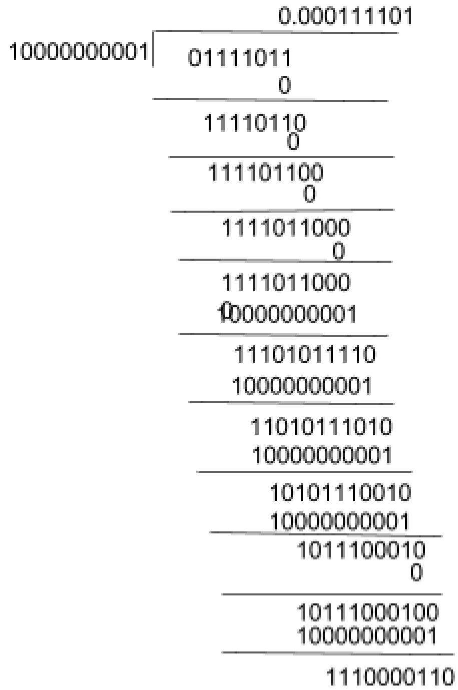
So, A - B respresented <u>10001111010</u>.

1.5. Using 2s complement integer arithmetic, explictly find the result A/B (long division) in binary.

The decimal value A = 123 is represented as 01111011.

The decimal value B = -1023 is represented as 10000000001.

Find A / B by long division:



1.6. Repeat the division in (1.5), but now show the result including any explicit binary fractions, and express the result with the explicit binary point on display

I don't understand this question very well. This is all I can do.

A / B =

0.00011110101110000101000111101011100001010001111010111000010100...

2. [50 points] Consider the real number X = -1024 (not an integer), and the real number Y = 2 (not an integer).

2.1. Encode X in IEEE 754 single precision floating point, showing explicitly your construction.

Consider the decimal number X = -1024.

(a) the binary value for 1024 is

1024 / 2 = 512 + 0

512 / 2 = 251 + 0

251 / 2 = 125 + 1

125 / 2 = 62 + 1

62 / 2 = 31 + 1

31 / 2 = 15 + 1

15 / 2 = 7 + 1

7 / 2 = 3 + 1

3 / 2 = 1 + 1

1 / 2 = 0 + 1

the binary value for 1024 is 1111111100

(b) the sign bit 1 reflect the fact -1024 is negative.

(c)Normalizing the binary value the result obtained is -1.1111111×2^9

(d)The exponent is unsigned number 132, represented as 10001000 in binary, because the real exponent is +9 (136 - 127 = +9).

(e) The fraction is 23 bits of precision, removing the leading 1. So, the fraction is

11111110000000000000000

(f) The floating point representation of -1024 is as follows:

sign	Exponent	Fraction
1	10001000	11111110000000000000000

so, the answer is <u>1 10001000 11111110000000000000000</u>

2.2. Encode Y in IEEE 754 single precision floating point, showing explicitly your construction.

Consider the decimal number Y = 2.

(a) the binary value for 2 is $(10)_2$

2 / 2 = 1 + 0

1 / 2 = 0 + 1

(b) the sign bit 0 reflect the fact 2 is positive.

(c) Normalizing the binary value the result obtained is 1.0×2^1

(d) The exponent is unsigned number 128, represented as 10000000 in binary, because the real exponent is +1 (128 - 127 = +1).

(e) The fraction is 23 bits of precision, removing the leading 1. So, the fraction is

00000000000000000000000

(f) The floating point representation of 2 is as follows:

sign	Exponent	Fraction
0	10000000	00000000000000000000000

so, the answer is <u>0 10000000 00000000000000000000000</u>

2.3. Explicitly show the steps required in IEEE 754 single precision floating point to form X + Y .

On the Website, *Floating Point Tutorial* (2012, ¶ 8) states the following steps to adding floating point.

	S1	E1	M1
X =	1	10001000	11111110000000000000000
	S2	E2	M2
Y =	0	10000000	00000000000000000000000

(a) Absolute value of X is greater than absolute value of Y.

(b) Result of initial exponent $E = E1 = (10001000)_2 = (136)_{10}$

(c) E1 - E2 = (100100 - 1000000) = 136 -128 = 6

(d) Shift the mantissa M2 by (E1-E2) so that the exponents are same for both numbers.

M2 = 1.00000000000000000000000

= 0.00000100000000000000000

(e) Sign is not equal. So, replace S with 1 and subtract M1 and shifted M2 for M.

```
      1.11111110000000000000000
  -   0.00000100000000000000000
-------------------------------
      1.11111010000000000000000

            S      E              M
X + Y =     1   10001000   11111010000000000000000
```

so, the repsentation for X + Y is <u>1 10001000 11111010000000000000000</u>

References

Floating Point Tutorial. Retrieved 2012, from the World Wide Web:
http://www.rfwireless-world.com/Tutorials/floating-point-tutorial.html

Yale P. and Sanjay P. (2004). *introduction to computing systems,* New York: The McGraw-Hill companies, Inc.