Homework 4
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1. Suppose the "Birman-Schiper-Stephenson Protocol" is used to enforce "Causal Ordering of Messages" of a system that has four processes, P_1, P_2, P_3, and P_4. With the help of diagrams, explain clearly what the process would do in each of the following cases.

   a) The current vector time of Process P_3 was $\mathbf{C_3}$ when it recieved a message from P_2 along with vector time stamp $\mathbf{t_m}$, where

   $$\mathbf{C_3} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} \qquad \mathbf{t_m} = \begin{pmatrix} 1 \\ 3 \\ 3 \\ 1 \end{pmatrix}$$

   Should P_3 deliver the message immediately? Why? If not, what should it do?

   b) Process P_2 with current vector time $\mathbf{C_2}$ recieved a message from P_1 along with vector time stamp $\mathbf{t_m}$, where

   $$\mathbf{C_2} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \qquad \mathbf{t_m} = \begin{pmatrix} 2 \\ 2 \\ 3 \\ 5 \end{pmatrix}$$

   Should P_2 deliver the message immediately? Why? If not, what should it do?

2. Consider a cut $\mathbf{C}$:

   $$\mathbf{C} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

   where c_1, c_2, and c_3 are the cut events with vector clocks $\mathbf{C_1}, \mathbf{C_2}, \mathbf{C_3}$ respectively:

   $$\mathbf{C_1} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad \mathbf{C_2} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \qquad \mathbf{C_3} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

   Calculate $T_C = sup(C_1, C_2, C_3)$. Is $\mathbf{C}$ a consistemt cut? Why?

3. A banking system uses Chandy-Lamport global state recording protocol (Snapshot Algorithm) to record its global state; markers are sent along channels where FIFO is assumed. The system has three branches P_1, P_2, and P_3 and are connected by communication channels C_{ij}, where $i, j = 1, 2, 3$. Suppose LS_i denote the local state (the money the branch possesses at the time of recording) of branch P_i.

   P_1 initiated the recording process. Right before P_1 sent out the marker, a \$1 transaction was in transit on C_{12}, a \$2 transit on C_{21}, a \$3 transit on C_{23}, and a \$10 transit on C_{31} (assume

that the units are in million dollars) and branches P_1, P_2, and P_3 had \$40, \$50, and \$60 respectively (not including any money in transit). Assume that the branches do not send out any other money during the whole recording process and the markers from P_1 arrived at other banks earlier than other markers. With the help of diagrams, find out the state LS_i of P_i and channel states C_{ij} where i, j = 1, 2, 3. Tabulate your results in the following format:

State	Money
LS_1	..
LS_2	..
LS_3	..
C_{12}	..
C_{13}	..
C_{21}	..
C_{23}	..
C_{31}	..
C_{32}	..

Show your steps clearly.

4. In Lamport's algorithm for mutual exclusion, Process P_i enters CS when the following 2 conditions are satisfied:

   1) P_i's request is at the head of $request\mbox{ }queue_i$
   2) P_i has received a (REPLY) message from every other process time-stamped later than t_{s_i}

   Condition 1) can hold concurrently at several sites. Why then is 1) needed to guarantee mutual exclusion?
   Does the algorithm work if condition 2) is removed? Why? Give an example with illustrations (drawings) to support your argument.

5. In Lamport's algorithm of mutual exclusion, if a site S_i is executing the critical section, is it necessary that S_i's request need to be **always** at the top of the request-queue at another site S_j? Explain and give an example (with diagrams) to support your argument.

6. Can Byzantine agreement be always reached among four processors if two processors are faulty? With the help of diagrams, explain your answer.

7. Maekawa's Algorithm is used to achieve mutual exclusion for 13 sites. Suppose the sites are labeled 1, 2, ..., 13. Find the request sets R1, R2, ... , R13.

   Suppose sites 1, 6, 12 want to enter a critical section (CS) and they have sent requests in the order 1, 6, 12. The following sequence of events have occurred in the order listed:

   1. The requests of site 1 have arrived at site 2, and site 3. Its request to site 4 is on the way.
   2. The requests of site 6 have arrived at site 9, and site 12. Its request to site 2 is on the way.

3. The requests of site 12 have arrived at site 4, and site 7 and 8.

Draw a diagram to show which sites have been locked and locked by whom.

Suppose the transit requests have arrived at their destinations. At this point can any site enter the CS? Why? If not, how do the sites resolve the problem?

8. In a distribution system, there are 20 servers. Suppose the utilization of a server is 60%. Calculate the probability that the system has at least one task waiting and at least one server lying idle. Show your steps clearly.