姓名 曾迦睫 学号 1820221053 课程 数值分析（第六章）

6.1 (1) $\int_0^1 \frac{x}{4+x^2}dx$, $M=8$, $h=\frac{1-0}{8}=\frac{1}{8}$

$x_i = a + ih = ih$

复化梯形公式

$$\int_0^1 \frac{x}{4+x^2}dx \simeq \frac{h}{2}\left[f(a)+f(b)+2\sum_{i=1}^{7} f(x_i)\right]$$

$$= \frac{1}{16}\left[f(0)+f(1)+2f(\tfrac{1}{8})+2f(\tfrac{1}{4})+2f(\tfrac{3}{8})+2f(\tfrac{1}{2})+2f(\tfrac{5}{8})+2f(\tfrac{3}{4})+2f(\tfrac{7}{8})\right]$$

$$= \frac{1}{16}\left[\frac{0}{4+0^2}+\frac{1}{4+1^2}+2\left(\frac{\frac{1}{8}}{4+(\frac{1}{8})^2}\right)+2\left(\frac{\frac{1}{4}}{4+(\frac{1}{4})^2}\right)+2\left(\frac{\frac{3}{8}}{4+(\frac{3}{8})^2}\right)+2\left(\frac{\frac{1}{2}}{4+(\frac{1}{2})^2}\right)+2\left(\frac{\frac{5}{8}}{4+(\frac{5}{8})^2}\right)+2\left(\frac{\frac{3}{4}}{4+(\frac{3}{4})^2}\right)+2\left(\frac{\frac{7}{8}}{4+(\frac{7}{8})^2}\right)\right]$$

$$= \frac{1}{16}\left[0+0.2+2(0.03113)+2(0.06154)+2(0.09057)+2(0.11765)+2(0.14235)+2(0.16438)+2(0.18361)\right]$$

$$= 0.11140$$

复化辛卜生公式

$$\int_0^1 \frac{x}{4+x^2}dx \simeq \frac{h}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+4f(x_5)+2f(x_6)+4f(x_7)+f(x_8)\right]$$

$$= \frac{1}{24}\left[f(0)+4f(\tfrac{1}{8})+2f(\tfrac{1}{4})+4f(\tfrac{3}{8})+2f(\tfrac{1}{2})+4f(\tfrac{5}{8})+2f(\tfrac{6}{8})+4f(\tfrac{7}{8})+f(1)\right]$$

$$= \frac{1}{24}\left[0+4(0.03113)+2(0.06154)+4(0.09057)+2(0.11765)+4(0.14235)+2(0.16438)+4(0.18361)+0.2\right]$$

$$= 0.11157$$

(3) $\int_0^9 \sqrt{x}\,dx$, $M = 4$, $h = \frac{9-0}{4} = \frac{9}{4}$, $x_1 = x_0 + 1h = 1$

复化梯形公式

$$\int_0^9 \sqrt{x}\,dx = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$= \frac{9}{8}\left[f(0) + 2f\left(\frac{9}{4}\right) + 2f\left(\frac{18}{4}\right) + 2f\left(\frac{27}{4}\right) + f(9)\right]$$

$$= \frac{9}{8}\left[0 + 2(1.5) + 2(2.12132) + 2(2.59808) + 3\right]$$

$$= 17.36865$$

复化辛卜生公式

$$\int_0^9 \sqrt{x}\,dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$$= \frac{9}{12}\left[f(0) + 4f\left(\frac{9}{4}\right) + 2f\left(\frac{18}{4}\right) + 4f\left(\frac{27}{4}\right) + f(9)\right]$$

$$= \frac{9}{12}\left[0 + 4(1.5) + 2(2.12132) + 4(2.59808) + 3\right]$$

$$= 17.72622$$

63. $\int_1^3 \frac{dy}{y}$, $f(y) = \frac{1}{y}$

(1) $T_1 = \frac{2}{2}[f(1) + f(3)] = 1.33333$

$T_2 = \frac{1}{2}T_1 + \frac{2}{2}f(2) = 1.16667$

$S_1 = \frac{4T_2 - T_1}{3} = 1.11111$

$T_4 = \frac{1}{2}T_2 + \frac{2}{4}[f(\frac{3}{2}) + f(\frac{5}{2})] = 1.11667$

$S_2 = \frac{4T_4 - T_2}{3} = 1.10000$

$C_1 = \frac{16S_2 - S_1}{15} = 1.09926$

$T_8 = \frac{1}{2}T_4 + \frac{2}{8}[f(\frac{5}{4}) + f(\frac{7}{4}) + f(\frac{9}{4}) + f(\frac{11}{4})] = 1.10321$

$S_4 = \frac{4T_8 - T_4}{3} = 1.09872$

$C_2 = \frac{16S_4 - S_2}{15} = 1.09863$

$R_1 = \frac{64C_2 - C_1}{63} = 1.09862$

$T_{16} = \frac{1}{2}T_8 + \frac{2}{16}[f(\frac{9}{8}) + f(\frac{11}{8}) + f(\frac{13}{8}) + f(\frac{15}{8}) + f(\frac{17}{8}) + f(\frac{19}{8}) + f(\frac{21}{8}) + f(\frac{23}{8})]$

$= 1.09977$

$S_8 = \frac{4T_{16} - T_8}{3} = 1.09862$

$C_4 = \frac{16S_8 - S_4}{15} = 1.09861$

$R_2 = \frac{64C_4 - C_2}{63} = 1.09861$

∴ 取 $z = 1.09861$

(2) $\int_1^3 \frac{dy}{y}$ ，设 $y = \frac{3+1}{2} + \frac{3-1}{2}t$ ， $dy = dt$

$= 2+t$

$$\int_{-1}^{1} \frac{1}{2+t}dt$$

当 $n=3$， $\int_1^3 \frac{dy}{y} \approx \int_{-1}^{1} \frac{1}{2+t}dt$

$$= 0.8888888889\left(\frac{1}{2+0}\right) + 0.5555555556\left(\frac{1}{2-0.774596692} + \frac{1}{2+0.774596692}\right)$$

$$= 1.09804$$

当 $n=5$， $\int_1^3 \frac{dy}{y} \approx \int_{-1}^{1} \frac{1}{2+t}dt$

$$= 0.5688888889\left(\frac{1}{2+0}\right) + 0.4786286705\left(\frac{1}{2-0.5384693101} + \frac{1}{2+0.5384693101}\right) + 0.2369268851\left(\frac{1}{2-0.9061798459} + \frac{1}{2+0.9061798459}\right)$$

$$= 1.10861$$

6 4 (2) $\int_0^2 \frac{1}{1+x}dx$ ，取 $M=10$ ，$h=\frac{2}{10}=\frac{1}{5}$

复化梯形公式

$$\int_0^2 \frac{dx}{1+x} \approx \frac{1}{10}\left[f(0)+2f(\tfrac{1}{5})+2f(\tfrac{2}{5})+2f(\tfrac{3}{5})+2f(\tfrac{4}{5})+2f(1)+2f(\tfrac{6}{5})+2f(\tfrac{7}{5})+2f(\tfrac{8}{5})+2f(\tfrac{9}{5})+f(2)\right]$$

$$=\frac{1}{10}\left[1+2(0.83333+0.71429+0.62500+0.55556+0.50000+0.45455+0.41667+0.38462+0.35714)+0.33333\right]$$

$$=1.1016$$

复化辛卜生公式

$$\int_0^2 \frac{dx}{1+x} \approx \frac{1}{15}\left[f(0)+4f(\tfrac{1}{5})+2f(\tfrac{2}{5})+4f(\tfrac{3}{5})+2f(\tfrac{4}{5})+4f(1)+2f(\tfrac{6}{5})+4f(\tfrac{7}{5})+2f(\tfrac{8}{5})+4f(\tfrac{9}{5})+f(2)\right]$$

$$=\frac{1}{15}\left[1+4(0.83333+0.62500+0.50000+0.41667+0.35714)+2(0.71429)+0.55556+0.45455+0.38462)+0.33333\right]$$

$$=1.0987$$

龙贝格公式

$$T_1 = \frac{2}{2}\left(f(0)+f(2)\right) = 1\ 33333$$

$$T_2 = \frac{1}{2}T_1 + \frac{2}{2}f(1) = 1\ 16667$$

$$S_1 = \frac{4T_2 - T_1}{3} = 1\ 11112$$

$$T_4 = \frac{1}{2}T_2 + \frac{2}{4}\left[f(\tfrac{1}{2}) + f(\tfrac{3}{2})\right] = 1\ 11667$$

$$S_2 = \frac{4T_4 - T_2}{3} = 1\ 10000$$

$$C_1 = \frac{16S_2 - S_1}{15} = 1\ 09926$$

$$T_8 = \frac{1}{2}T_4 + \frac{2}{8}\left[f(\tfrac{1}{4}) + f(\tfrac{3}{4}) + f(\tfrac{5}{4}) + f(\tfrac{7}{4})\right] = 1\ 10321$$

$$S_4 = \frac{4T_8 - T_4}{3} = 1\ 09872$$

$$C_2 = \frac{16S_4 - S_2}{15} = 1\ 09863$$

$$R_1 = \frac{64C_2 - C_1}{63} = 1\ 09862$$

取 $z = 1\ 0986$

高斯求积公式

$$\int_0^2 \frac{dx}{1+x}, \quad 设\ x = 1+t, \quad dx = dt$$

$$\int_0^2 \frac{dx}{1+x} \approx \int_{-1}^{1} \frac{dt}{2+t}$$

$$= 0\ 4786286705\left(\frac{1}{2-0\ 5384693101} + \frac{1}{2+0\ 5384693101}\right)$$

$$+ 0\ 2369268851\left(\frac{1}{2-0\ 9061798459} + \frac{1}{2+0\ 9061798459}\right)$$

$$+ 0\ 588888889\left(\frac{1}{2+0}\right)$$

$$= 1\ 1086$$

(3) $\int_0^1 \frac{dx}{1+x^2}$ ，设 $M=10$ ， $h=\frac{1}{10}$

复化梯形公式

$$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{20}\Big[f(0)+2f(\tfrac{1}{10})+2f(\tfrac{2}{10})+2f(\tfrac{3}{10})+2f(\tfrac{4}{10})+2f(\tfrac{5}{10})+2f(\tfrac{6}{10})+2f(\tfrac{7}{10})+2f(\tfrac{8}{10})+2f(\tfrac{9}{10})+f(1)\Big]$$

$$= \frac{1}{20}\Big[1+0\ 5+2(0\ 99010+0\ 96154+0\ 91743+0\ 86207+0\ 80000+0\ 73529+0\ 67114+0\ 60976+0\ 55249)\Big]$$

$$= 0\ 7850$$

复化辛卜生公式

$$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{30}\Big[f(0)+4f(\tfrac{1}{10})+2f(\tfrac{2}{10})+4f(\tfrac{3}{10})+2f(\tfrac{4}{10})+4f(\tfrac{5}{10})+2f(\tfrac{6}{10})+4f(\tfrac{7}{10})+2f(\tfrac{8}{10})+4f(\tfrac{9}{10})+f(1)\Big]$$

$$= \frac{1}{30}\Big[1+0\ 5+4(0\ 99010+0\ 91743+0\ 80000+0\ 67114+0\ 55249)+2(0\ 96154+0\ 86207+0\ 73529+0\ 60976)\Big]$$

$$= 0\ 7854$$

龙贝格公式

$$T_1 = \frac{1}{2}[f(0)+f(1)] = 0\ 75$$

$$T_2 = \frac{1}{2}T_1 + \frac{1}{2}f(0\ 5) = 0\ 775$$

$$S_1 = \frac{4T_2 - T_1}{3} = 0\ 78333$$

$$T_4 = \frac{1}{2}T_2 + \frac{1}{4}\left[f\left(\frac{1}{4}\right)+f\left(\frac{3}{4}\right)\right] = 0\ 78279$$

$$S_2 = \frac{4T_4 - T_2}{3} = 0\ 78539$$

$$C_1 = \frac{16S_2 - S_1}{15} = 0\ 78553$$

$$T_8 = \frac{1}{2}T_4 + \frac{1}{8}\left[f\left(\frac{1}{8}\right)+f\left(\frac{3}{8}\right)+f\left(\frac{5}{8}\right)+f\left(\frac{7}{8}\right)\right] = 0\ 78475$$

$$S_4 = \frac{4T_8 - T_4}{3} = 0\ 78540$$

$$C_2 = \frac{16S_4 - S_2}{15} = 0\ 78540$$

$$\cdot\quad z = 0\ 7854$$

高斯求积公式

$$\int_0^1 \frac{dx}{1+x^2}, \quad x = \frac{1}{2} + \frac{1}{2}t$$

$$dx = \frac{1}{2}dt$$

$$\int_0^1 \frac{dx}{1+x^2} \approx \int_{-1}^1 \frac{1}{1+(\frac{1}{2}+\frac{1}{2}t)^2} dt$$

$$= 0\ 888\ 888\ 888\ 9\left(\frac{1}{1+(\frac{1}{2}+\frac{1}{2}(0))^2}\right) +$$

$$0\ 555\ 555\ 555\ 6\left(\frac{1}{1+(\frac{1}{2}+\frac{1}{2}(-0.774596692))^2} + \frac{1}{1+(\frac{1}{2}+\frac{1}{2}(0\ 774596692))^2}\right)$$

$$= 0\ 7853$$