姓名：曾洳腱　　学号：1820221053　　课程：数值分析（第四章）

4.1：

$$\begin{cases} 20x_1 + 2x_2 + 3x_3 = 24 \\ x_1 + 8x_2 + x_3 = 12 \\ 2x_1 - 3x_2 + 15x_3 = 30 \end{cases}$$

简单迭代法：　　取 $x_1^{(0)} = 1$，$x_2^{(0)} = 1$，$x_3^{(0)} = 1$

迭代方程：

$$\begin{cases} x_1^{(k+1)} = 1.2 - 0.1x_2^{(k)} - 0.15x_3^{(k)} \\ x_2^{(k+1)} = 1.5 - 0.125x_1^{(k)} - 0.125x_3^{(k)} \\ x_3^{(k+1)} = 2.0 - 0.133x_1^{(k)} + 0.2x_3^{(k)} \end{cases}$$

当k=0，~~当k=0~~，即

$$x_1^{(1)} = 1.2 - 0.1(1) - 0.15(1) = 0.95$$
$$x_2^{(1)} = 1.5 - 0.125(1) - 0.125(1) = 1.25$$
$$x_3^{(1)} = 2.0 - 0.133(1) + 0.2(1) = 2.067$$

当k=1，

$$x_1^{(2)} = 1.2 - 0.1(1.25) - 0.15(2.067) = 0.765$$
$$x_2^{(2)} = 1.5 - 0.125(0.95) - 0.125(2.067) = 1.123$$
$$x_3^{(2)} = 2.0 - 0.133(0.95) + 0.2(1.25) = 2.124$$

当k=2，

$$x_1^{(3)} = 1.2 - 0.1(1.123) - 0.15(2.124) = 0.769$$
$$x_2^{(3)} = 1.5 - 0.125(0.765) - 0.125(2.124) = 1.139$$
$$x_3^{(3)} = 2.0 - 0.133(0.765) + 0.2(1.123) = 2.123$$

当k=3，

$$x_1^{(4)} = 1.2 - 0.1(1.139) - 0.15(2.123) = 0.768$$
$$x_2^{(4)} = 1.5 - 0.125(0.769) - 0.125(2.123) = 1.139$$
$$x_3^{(4)} = 2.0 - 0.133(0.769) + 0.2(1.139) = 2.126$$

当k=4，

$$x_1^{(5)} = 1.2 - 0.1(1.139) - 0.15(2.126) = 0.767$$
$$x_2^{(5)} = 1.5 - 0.125(0.768) - 0.125(2.126) = 1.138$$
$$x_3^{(5)} = 2.0 - 0.133(0.768) + 0.2(1.139) = 2.126$$

当k=5，

$$x_1^{(6)} = 1.2 - 0.1(1.138) - 0.15(2.126) = 0.767$$
$$x_2^{(6)} = 1.5 - 0.125(0.767) - 0.125(2.126) = 1.138$$
$$x_3^{(6)} = 2.0 - 0.133(0.767) + 0.2(1.138) = 2.126$$

$\therefore$

$$x_1 = 0.767$$
$$x_2 = 1.138$$
$$x_3 = 2.126$$

赛德尔迭代法

迭代方程
$$\begin{cases} x_1^{(k+1)} = 1.2 - 0.1\,x_2^{(k)} - 0.15\,x_3^{(k)} \\ x_2^{(k+1)} = 1.5 - 0.125\,x_1^{(k+1)} - 0.125\,x_3^{(k)} \\ x_3^{(k+1)} = 2.0 - 0.133\,x_1^{(k+1)} + 0.2\,x_2^{(k+1)} \end{cases}$$

当 $k=0$，
$$x_1^{(1)} = 1.2 - 0.1(1) - 0.15(1) = 0.95$$
$$x_2^{(1)} = 1.5 - 0.125(0.95) - 0.125(1) = 1.256$$
$$x_3^{(1)} = 2.0 - 0.133(0.95) + 0.2(1.256) = 2.125$$

当 $k=1$，
$$x_1^{(2)} = 1.2 - 0.1(1.256) - 0.15(2.125) = 0.756$$
$$x_2^{(2)} = 1.5 - 0.125(0.756) - 0.125(2.125) = 1.140$$
$$x_3^{(3)} = 2.0 - 0.133(0.756) + 0.2(1.140) = 2.127$$

当 $k=2$，
$$x_1^{(3)} = 1.2 - 0.1(1.140) - 0.15(2.127) = 0.767$$
$$x_2^{(3)} = 1.5 - 0.125(0.767) - 0.125(2.127) = 1.138$$
$$x_3^{(3)} = 2.0 - 0.133(0.767) + 0.2(1.138) = 2.126$$

当 $k=3$，
$$x_1^{(4)} = 1.2 - 0.1(1.138) - 0.15(2.126) = 0.767$$
$$x_2^{(4)} = 1.5 - 0.125(0.767) - 0.125(2.126) = 1.138$$
$$x_3^{(4)} = 2.0 - 0.133(0.767) + 0.2(1.138) = 2.126$$

$\therefore$
$$x_1 = 0.767$$
$$x_2 = 1.138$$
$$x_3 = 2.126$$

4.4： $\begin{cases} 5x_1 + 2x_2 + x_3 = -12 \\ -x_1 + 4x_2 + 2x_3 = 20 \\ 2x_1 - 3x_2 + 10x_3 = 3 \end{cases}$ ，$w = 0.9$ ，取 $x_1^{(0)}, x_2^{(0)}, x_3^{(0)} = 1$

迭代公式： $\cancel{x_1^{(k+1)} = x_1^{(k)} + \frac{w}{5}(-12-5}$

$$x_1^{(k+1)} = (1-w)x_1^{(k)} + \frac{w}{5}\left(-12 - 2x_2^{(k)} - x_3^{(k)}\right)$$

$$x_2^{(k+1)} = (1-w)x_2^{(k)} + \frac{w}{4}\left(20 + x_1^{(k+1)} - 2x_3^{(k)}\right)$$

$$x_3^{(k+1)} = (1-w)x_3^{(k)} + \frac{w}{10}\left(3 - 2x_1^{(k+1)} + 3x_2^{(k+1)}\right)$$

当 $k=0$，
$x_1^{(1)} = 0.1x_1^{(0)} + 0.18(-12 - 2x_2^{(0)} - x_3^{(0)}) = 0.1(1) + 0.18(-12 - 2(1) - (1)) = -2.6$
$x_2^{(1)} = 0.1x_2^{(0)} + 0.225(20 - x_1^{(1)} - 2x_3^{(0)}) = 0.1(1) + 0.225(20 + (-2.6) - 2(1)) = 3.565$
$x_3^{(1)} = 0.1x_3^{(0)} + 0.09(3 - 2x_1^{(1)} + 3x_2^{(1)}) = 0.1(1) + 0.09(3 - 2(-2.6) + 3(3.565)) = 1.80055$

当 $k=1$，
$x_1^{(2)} = 0.1(-2.6) + 0.18(-12 - 2(3.565) - (1.80055)) = -4.02750$
$x_2^{(2)} = 0.1(3.565) + 0.225(20 - (-4.02750) - 2(1.80055)) = 3.14007$
$x_3^{(2)} = 0.1(1.80055) + 0.09(3 - 2(-4.02750) + 3(3.14007)) = 2.02282$

当 $k=2$，
$x_1^{(3)} = 0.1(-4.02750) + 0.18(-12 - 2(3.14007) - (2.02282)) = -4.05728$
$x_2^{(3)} = 0.1(3.14007) + 0.225(20 - (-4.05728) - 2(2.02282)) = 2.99085$
$x_3^{(3)} = 0.1(2.02282) + 0.09(3 - 2(4.05728) + 3(2.99085)) = 2.01012$

当 $k=3$，
$x_1^{(4)} = 0.1(-4.05728) + 0.18(-12 - 2(2.99085) - (2.01012)) = -4.00426$
$x_2^{(4)} = 0.1(2.99085) + 0.225(20 - (-4.00426) - 2(2.01012)) = 2.99357$
$x_3^{(4)} = 0.1(2.01012) + 0.09(3 - 2(-4.00426) + 3(2.99357)) = 2.00004$

当 $k=4$，
$x_1^{(5)} = 0.1(-4.00426) + 0.18(-12 - 2(2.99357) - (2.00004)) = -3.99812$
$x_2^{(5)} = 0.1(2.99357) + 0.225(20 - (-3.99812) - 2(2.00004)) = 2.99976$
$x_3^{(5)} = 0.1(2.00004) + 0.09(3 - 2(-3.99812) + 3(2.99976)) = 1.99960$

当 $k=5$，
$x_1^{(6)} = 0.1(-3.99812) + 0.18(-12 - 2(2.99976) - (1.99960)) = -3.99965$
$x_2^{(6)} = 0.1(2.99976) + 0.225(20 - (-3.99965) - 2(1.99960)) = 3.00023$
$x_3^{(6)} = 0.1(1.99960) + 0.09(3 - 2(-3.99965) + 3(3.00023)) = 1.99996$

当 $k=6$，$x_1^{(7)} = -4.00004$，$x_2^{(7)} = 3.00003$，$x_3^{(7)} = 2.00001$

当 $k=7$，$x_1^{(8)} = -4.00002$，$x_2^{(8)} = 2.99999$，$x_3^{(8)} = 2.00000$

当 $k=8$，$x_1^{(9)} = -4.00000$，$x_2^{(9)} = 3.00000$，$x_3^{(9)} = 2.00000$

当 $k=9$，
$x_1^{(10)} = -4.00000$
$x_2^{(10)} = 3.00000$
$x_3^{(10)} = 2.00000$

$\because \|X^{(k+1)} - X^{(k)}\|_\infty < 10^{-5}$

$\therefore$ 迭代终止