姓名：曾加健　　学号：18202210053　　课程：数值分析（第三章）

3.1：　高斯消元法

$$\begin{cases} 3x_1 + 2x_2 + 5x_3 = 6 \\ -x_1 + 4x_2 + 3x_3 = 5 \\ x_1 - x_2 + 3x_3 = 1 \end{cases}$$

$u_{11}=3$ $l_{11}=1$	$u_{12}=2$	$u_{13}=5$	$z_1=6$
$l_{21}=-\frac{1}{3}$	$u_{22}=4-(-\frac{1}{3}\times 2)=\frac{14}{3}$ $l_{22}=1$	$u_{23}=3-(-\frac{1}{3}\times 5)=\frac{14}{3}$	$z_2=5-(-\frac{1}{3}\times 6)=7$
$l_{31}=\frac{1}{3}$	$l_{32}=\frac{-1-(\frac{1}{3}\times 2)}{\frac{14}{3}}=-\frac{5}{14}$	$u_{33}=3-(\frac{1}{3}\times 5)-(-\frac{5}{14}\times\frac{14}{3})=3$	$z_3=1-(\frac{1}{3}\times 6)-(-\frac{5}{14}\times 7)=\frac{3}{2}$

$$\therefore \begin{cases} 3x_1 + 2x_2 + 5x_3 = 6 \\ \frac{14}{3}x_2 + \frac{14}{3}x_3 = 7 \\ 3x_3 = \frac{3}{2} \end{cases} \Rightarrow$$

解得 $x_3=\frac{1}{2}$

$x_2=\left(7-\frac{14}{3}(\frac{1}{2})\right)\times\frac{3}{14}=1$

$x_1=\left(6-2(1)-5(\frac{1}{2})\right)\times\frac{1}{3}=\frac{1}{2}$

3.2 $\begin{cases} 3x_1 - x_2 + 4x_3 = 7 \\ -x_1 + 2x_2 - 2x_3 = -1 \\ 2x_1 - 3x_2 - 2x_3 = 0 \end{cases}$，克劳特消元法

$u_{11} = 1$ $l_{11} = \frac{1}{3}$	$u_{12} = -\frac{1}{3}$	$u_{13} = \frac{4}{3}$	$z_1 = \frac{7}{3}$
$l_{21} = -1$	$u_{22} = 1$ $l_{22} = 2 - (-1)(-\frac{1}{3})$ $= \frac{5}{3}$	$u_{23} = \frac{-2-(-1\times\frac{4}{3})}{\frac{5}{3}}$ $= -\frac{2}{5}$	$z_2 = \frac{-1-(-1\times\frac{7}{3})}{\frac{5}{3}}$ $= \frac{4}{5}$
$l_{31} = 2$	$l_{32} = -3-(2\times-\frac{1}{3})$ $= -\frac{7}{3}$	$u_{33} = 1$ $l_{33} = -2-(2\times\frac{4}{3})-(-\frac{7}{3}\times\frac{-2}{5})$ $= -\frac{28}{5}$	$z_3 = \frac{0-(2\times\frac{7}{3})-(-\frac{7}{3}\times\frac{4}{5})}{-\frac{28}{5}}$ $= \frac{1}{2}$

$$\begin{cases} x_1 - \frac{1}{3}x_2 + \frac{4}{3}x_3 = \frac{7}{3} \\ x_2 - \frac{2}{5}x_3 = \frac{4}{5} \\ x_3 = \frac{1}{2} \end{cases} \Rightarrow \begin{cases} x_3 = \frac{1}{2} \\ x_2 = \frac{4}{5} + \frac{2}{5}(\frac{1}{2}) \\ \quad = 1 \\ x_1 = \frac{7}{3} + \frac{1}{3}(1) + -\frac{4}{3}(\frac{1}{2}) \\ \quad = 2 \end{cases}$$

3.3 $\begin{cases} 3x_1 - x_2 + 2x_3 = 7 \\ -x_1 + 2x_2 - 2x_3 = -1 \\ 2x_1 - 2x_2 + 4x_3 = 10 \end{cases}$ ，平方根法

$l_{11} = \sqrt{3} = u_{11}$	$u_{12} = -\frac{1}{\sqrt{3}}$	$u_{13} = \frac{2}{\sqrt{3}}$	$z_1 = \frac{7}{\sqrt{3}}$
$l_{21} = -\frac{1}{\sqrt{3}}$	$l_{22} = \left(2-\left(-\frac{1}{\sqrt{3}}\right)^2\right)^{\frac{1}{2}}$ $= 1.2910$ $= u_{22}$	$u_{23} = \frac{-2-\left(\frac{-1}{\sqrt{3}}\times\frac{2}{\sqrt{3}}\right)}{1.2910}$ $= -1.0328$	$z_2 = \frac{-1-\left(-\frac{1}{\sqrt{3}}\times\frac{7}{\sqrt{3}}\right)}{1.2910}$ $= 1.0328$
$l_{31} = \frac{2}{\sqrt{3}}$	$l_{32} = \frac{-3-\left(\frac{2}{\sqrt{3}}\right)\left(-\frac{1}{\sqrt{3}}\right)}{1.2910}$ $= 1.0328$	$l_{33} = \left(-2-\left(\frac{2}{\sqrt{3}}\times\frac{2}{\sqrt{3}}\right)-1.0328^2\right)^{\frac{1}{2}}$ $= 1.2649$ $= u_{33}$	$z_3 = \frac{0-\frac{2}{\sqrt{3}}\times\frac{7}{\sqrt{3}}-1.0328^2}{1.2649}$ $= -2.8461$

$$\begin{cases} \sqrt{3}x_1 - \frac{1}{\sqrt{3}}x_2 + \frac{2}{\sqrt{3}}x_3 = \frac{7}{\sqrt{3}} \\ 1.2910x_2 - 1.0328x_3 = 1.0328 \\ 1.2649x_3 = -2.8461 \end{cases}$$

$$\Rightarrow \begin{cases} x_3 = -2.25 \\ x_2 = \frac{1.0328 + 1.0328 \times (-2.25)}{1.2910} \\ \quad = -1 \\ x_1 = \left(\frac{7}{\sqrt{3}} + \frac{1}{\sqrt{3}}(-1) - \frac{2}{\sqrt{3}}(-2.25)\right) \times \frac{1}{\sqrt{3}} \\ \quad = 3.5 \end{cases}$$

3.4：
$$\begin{cases} 3x_1 - x_2 + 4x_3 = 3 \\ -x_1 + 2x_2 - 2x_3 = 2 \\ 2x_1 - 3x_2 - 2x_3 = -5 \end{cases}$$

列主元素法

第一列取 3 作为主元素

$$\begin{cases} 3x_1 - x_2 + 4x_3 = 3 & —① \\ -x_1 + 2x_2 - 2x_3 = 2 & —② \\ 2x_1 - 3x_2 - 2x_3 = -5 & —③ \end{cases}$$

$l_{21} = -\frac{1}{3}$ ， $l_{31} = \frac{2}{3}$

② $- l_{21} \times$ ①， ③ $- l_{31} \times$ ①

得
$$\begin{cases} 1.6667x_2 - 0.6667x_3 = 3 \\ -2.3333x_2 - 4.6667x_3 = -7 \end{cases}$$

第二列取 -2.3333 作主元素

交换得

$$\begin{cases} -2.3333x_2 - 4.6667x_3 = -7 & —④ \\ 1.6667x_2 - 0.6667x_3 = 3 & —⑤ \end{cases}$$

$l_{22} = 0.7143$

⑤ $- l_{22} \times$ ④

得 $2.1667x_3 = -2$

$x_3 = 0.5$

$$\begin{cases} x_3 = 0.5 \\ x_2 = \dfrac{3 + 0.6667(0.5)}{1.6667} \\ x_1 = \dfrac{3 + 2 - 4(0.5)}{3} \end{cases}$$

$= 2$

$= 1$

全主元素法

取 4 作为主元素

$$\begin{cases} 4x_3 + 3x_1 - x_2 = 3 & —⑥ \\ -2x_3 - x_1 + 2x_2 = 2 & —⑦ \\ -2x_3 + 2x_1 - 3x_2 = -5 & —⑧ \end{cases}$$

$l_{21} = -\frac{1}{2}$ ， $l_{31} = -\frac{1}{2}$

⑦ $- l_{21} \times$ ⑥， ⑧ $- l_{31} \times$ ⑥

得
$$\begin{cases} 0.5x_1 + 1.5x_2 = 3.5 \\ 3.5x_1 - 3.5x_2 = -3.5 \end{cases}$$

取 3.5 作为主元素

交换得

$$\begin{cases} 3.5x_1 - 3.5x_2 = -3.5 & —⑨ \\ 0.5x_1 + 1.5x_2 = 3.5 & —⑩ \end{cases}$$

$l_{21} = 0.1429$

⑩ $- l_{21} \times$ ⑨

得 $2x_2 = 4$

$x_2 = 2$

回代， $x_1 = \dfrac{3.5 - 1.5(2)}{0.5}$

$= 1$

$x_3 = \dfrac{3 - 3(1) + 2}{4}$

$= 0.5$

3.5:

$$\begin{cases} 2x_1 - x_2 = 0 \\ -x_1 + 2x_2 - x_3 = 1 \\ -x_2 + 2x_3 - x_4 = 0 \\ -x_3 - 2x_4 = 2.5 \end{cases} \qquad \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & -2 & 2.5 \end{bmatrix}$$

，追赶法

$u_{11} = 1$ $l_{11} = \frac{1}{2}$	$u_{12} = -0.5$	0	0	$z_1 = 0$
$l_{21} = -1$	$u_{22} = 1$ $l_{22} = 2-(-1\times-0.5))$ $= 1.5$	$u_{23} = -\frac{1}{1.5}$ $= -0.6667$	0	$z_2 = \frac{1}{1.5}$ $= 0.6667$
0	$l_{32} = -1$	$u_{33} = 1$ $l_{33} = 2-(-1)(-0.6667)$ $= 1.3333$	$u_{34} = -\frac{1}{1.3333}$ $= -0.75$	$z_3 = \frac{0-(-1)(0.6667)}{1.3333}$ $= 0.5$
0	0	$l_{43} = -1$	$u_{44} = 1$ $l_{44} = -2-(-1)(-0.75)$ $= -2.75$	$z_4 = \frac{2.5-(-1\times0.5)}{-2.75}$ $= -1.0909$

$$\begin{cases} x_1 - 0.5x_2 = 0 \\ x_2 - 0.6667x_3 = 0.6667 \\ x_3 - 0.75x_4 = 0.5 \\ x_4 = -1.0909 \end{cases} \Rightarrow$$

$x_4 = -1.0909$

$x_3 = 0.5 + 3(-1.0909)$
$= -0.3182$

$x_2 = 0.6667 + 0.6667(-0.3182)$
$= 0.4546$

$x_1 = 0.5(0.4546)$
$= 0.2273$