姓名：曾泇睷　学号：1820221053　课程：离散数学（第五章）

5-2：(1) $\forall x \exists y (F(x) \wedge G(y))$

$\Leftrightarrow \forall x F(x) \wedge \exists y G(y)$

$\Leftrightarrow (F(a) \wedge F(b) \wedge F(c)) \wedge (G(a) \vee G(b) \vee G(c))$

(2) $\forall x \forall y (F(x) \vee G(y))$

$\Leftrightarrow \forall x F(x) \vee \forall y G(y)$

$\Leftrightarrow (F(a) \wedge F(b) \wedge F(c)) \vee (G(a) \wedge G(b) \wedge G(c))$

(3) $\forall x F(x) \to \forall y G(y)$

$\Leftrightarrow (F(a) \wedge F(b) \wedge F(c)) \to (G(a) \wedge G(b) \wedge G(c))$

(4) $\forall x (F(x,y) \to \cancel{G(x,y)}\ \exists y G(y))$

$\Leftrightarrow \exists x F(x,y) \to \exists y G(y)$

$\Leftrightarrow (F(a,y) \vee F(b,y) \vee F(c,y)) \to (G(a) \vee G(b) \vee G(c))$

5-12：(1) $\forall x F(x) \to \forall y G(x,y)$

$\Leftrightarrow \forall z F(z) \to \forall y G(x,y)$　改名规则

$\Leftrightarrow \forall x \forall y (F(x) \to G(x,y))$

(2) $\forall x (F(x,y) \to \exists y G(x,y,z))$

$\Leftrightarrow \forall x (F(x,y) \to \exists t G(x,t,z))$

$\Leftrightarrow \forall x \exists t (F(x,y) \to G(x,t,z))$

(3) $\forall x F(x,y) \leftrightarrow \exists x G(x,y)$

$\Leftrightarrow (\forall x F(x,y) \to \exists x G(x,y)) \wedge (\exists x G(x,y) \to \forall x F(x,y))$

$\Leftrightarrow (\forall x_1 F(x_1,y) \to \exists x_2 G(x_2,y)) \wedge (\exists x_3 G(x_3,y), \forall x_4 F(x_4,y))$

$\Leftrightarrow \exists x_1 \exists x_2 \forall x_3 \forall x_4 ((F(x_1,y) \to G(x_2,y)) \wedge (G(x_3,y) \to F(x_4,y))$

(4) $\forall x_1 (F(x_1) \to G(x_1,x_2)) \to (\exists x_2 H(x_2) \to \exists x_3 L(x_2,x_3))$

$\Leftrightarrow \forall x_1 (F(x_1) \to G(x_1,x_2)) \to (\exists x_4 H(x_4) \to \exists x_3 L(x_2,x_3))$

$\Leftrightarrow \forall x_1 (F(x_1) \to G(x_1,x_2)) \to \forall x_4 \exists x_3 (H(x_4) \to L(x_2,x_3))$

$\Leftrightarrow \exists x_1 \forall x_4 \exists x_3 ((F(x_1) \to G(x_1,x_2)) \to (H(x_4) \to L(x_2,x_3))$

(5) $\exists x_1 F(x_1, x_2) \to (F(x_1) \to \neg \exists x_2 G(x_1, x_2))$

$\Leftrightarrow \exists y_1 F(y_1, x_2) \to (F(x_1) \to \forall y_2 \neg G(x_1, y_2))$

$\Leftrightarrow \exists y_1 F(y_1, x_2) \to \forall y_2 (F(x_1) \to \neg G(x_1, y_2))$

$\Leftrightarrow \forall y_1 \forall y_2 (F(y_1, x_2) \to (F(x_1) \to \neg G(x_1, y_2))$

5-13： $F(x)$ ：x是汽车 ，$G(x)$：x是火车 ，$H(x)$：x是飞机 ，$L(x,y)$：x比y快

(1) $\exists x \exists y \cancel{\exists x}(F(x) \wedge \cancel{\exists y} G(y) \wedge L(x,y))$

(2) $\exists x \forall y (G(x) \wedge (F(y) \to L(x,y)))$

(3) $\exists x \exists y (F(x) \wedge G(y) \wedge L(x,y))$

(4) $\forall x \forall y (H(x) \to (F(y) \to L(x,y)))$

5-24：设 $F(x)$：x喜欢步行，$G(x)$：x喜欢自行车，$H(x)$：x喜欢乘汽车

前提：$\forall x(F(x) \to \neg G(x))$，$\forall x(G(x) \vee H(x))$，$\exists x \neg H(x)$

结论：$\exists x \neg F(x)$

①	$\forall x(F(x) \to \neg G(x))$	前提引入
②	$F(y) \to \neg G(y)$	① ∀-
③	$\forall x(G(x) \vee H(x))$	前提引入
④	$G(y) \vee H(y)$	③ ∀-
⑤	$\neg G(y) \to H(y)$	④ 置换
⑥	$F(y) \to H(y)$	②⑤ 假言三段论
⑦	$\neg H(y) \to \neg F(y)$	⑥ 置换
⑧	$\neg H(y) \to \exists x \neg F(x)$	⑦ ∃+
⑨	$\exists y \neg H(y) \to \exists x \neg F(x)$	⑧ ∃-
⑩	$\exists x \neg H(x)$	前提引入
⑪	$\exists x \neg F(x)$	⑨⑩ 假言推理

5-25：设 $F(x)$：x是刻苦钻研的，$G(x)$：x是科学工作者

$H(x)$：x是聪明的，$L(x)$：x在事业中获得成功，a：王大海

前提：$\forall x(G(x) \to F(x))$，$\forall x((F(x) \land H(x)) \to L(x))$，$G(a) \land H(a)$

结论：$L(a)$

① $G(a)$ 前提引入

② $\forall x(G(x) \to F(x))$ 前提引入

③ $G(a) \to F(a)$ ② $\forall-$

④ $F(a)$ ①③ 假言推理

⑤ $H(a)$ 前提引入

⑥ $F(a) \land H(a)$ ④⑤ 合取

⑦ $\forall x((F(x) \land H(x)) \to L(x))$ 前提引入

⑧ $(F(a) \land H(a)) \to L(a)$ ⑦ $\forall-$

⑨ $L(a)$ ⑥⑧ 假言推理