姓名：曾加健　学号：1820221053　课程：数值分析（第五章）

5.1：

x	1.45	1.36	1.14
y	3.14	4.15	5.65

，$x = 1.4$

$$l_0(x) = \frac{(x-1.36)(x-1.14)}{(1.45-1.36)(1.45-1.14)} = \frac{(x-1.36)(x-1.14)}{0.0279}$$

$$l_1(x) = \frac{(x-1.45)(x-1.14)}{(1.36-1.45)(1.36-1.14)} = \frac{(x-1.45)(x-1.14)}{-0.0198}$$

$$l_2(x) = \frac{(x-1.45)(x-1.36)}{(1.14-1.45)(1.14-1.36)} = \frac{(x-1.45)(x-1.36)}{0.0682}$$

$$L_2(x) = 3.14 \cdot \frac{(x-1.36)(x-1.14)}{0.0279} + 4.15 \cdot \frac{(x-1.45)(x-1.14)}{-0.0198} + 5.65 \cdot \frac{(x-1.45)(x-1.36)}{0.0682}$$

当$x = 1.4$时，$L_2(1.4) = 3.7295$

$\therefore f(1.4) = 3.7295 \approx 3.73$

5.2：差商表：

x_i	$f(x_i)$	一阶 $f(x_i, x_{i+1})$	二阶 $f(x_i, x_{i+1}, x_{i+2})$	三阶	四阶
93.0	11.38				
		0.44375			
96.2	12.80		0.00804		
		0.5		0	
100.00	14.70		0.00804		0
		0.56429		-0.00003	
104.2	17.07		0.00768		
		0.63111			
108.7	19.91				

~~$P_2(x) = f(x_{i0}) + (x-x_{i0}) f(x_{i0}, x_{i1}) + (x-x_{i0})(x-x_{i1}) f(x_{i0}, x_{i1}, x_{i2})$~~

当$x = 102$，$P_2(102) = 11.38 + (102-93)(0.44375) + (102-93)(102-96.2)(0.00804) + 0 + 0$

$= 15.79343$

$\therefore y(102) = 15.79343 \approx 15.79$

5.5 ： 差分表：

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
0.0	1.000 00				
		0.22140			
0.2	1.22140		0.04902		
		0.27042		0.01086	
0.4	1.49182		0.05988		0.00238
		0.33030		0.01324	
0.6	1.82212		0.07312		
		0.40342			
0.8	2.22554				

，$h = 0.2$

牛顿前插公式

$$P_n(x) = y_0 + (x-x_0)\frac{\Delta y_0}{1!h} + (x-x_0)(x-x_1)\frac{\Delta^2 y_0}{2!h^2} + \cdots + (x-x_0)(x-x_1)\cdots(x-x_{n-1})\frac{\Delta^n y_0}{n!h^n}$$

$$\therefore P_4(0.05) = 1.00000 + (0.05-0)\frac{0.22140}{1!(0.2)} + (0.05)(0.05-0.2)\frac{0.04902}{2!(0.2)^2} + (0.05)(0.05-0.2)(0.05-0.4)\frac{0.01086}{3!(0.2)^3}$$

$$+ (0.05)(0.05-0.2)(0.05-0.4)(0.05-0.6)\frac{0.00238}{4!(0.2)^4}$$

$$= 1.0513$$

$$\therefore y(0.05) = 1.0513$$

斯梯林插值公式 ，$t = \frac{0.42-0.4}{0.2} = 0.1$

$$y(0.42) \approx 1.49182 + \frac{0.1}{1!}\cdot\frac{0.27042+0.33030}{2} + \frac{0.1^2}{2!}\cdot 0.05988 + \frac{(0.1)(0.1^2-1^2)}{3!}\cdot\frac{0.01086+0.01324}{2}$$

$$+ \frac{0.1^2(0.1^2-1^2)}{4}\cdot 0.00238$$

$$= 1.5220$$

牛顿后插公式

$$y(0.75) \approx 2.2254 + (0.75-0.8)\frac{0.40342}{1!\,0.2} + (0.75-0.8)(0.75-0.6)\frac{0.07312}{2!\,0.2^2} + \cancel{0.75}$$

$$+ (0.75-0.8)(0.75-0.6)(0.75-0.4)\frac{0.01324}{3!\,0.2^3} + (0.75-0.8)(0.75-0.6)(0.75-0.4)(0.75-0.2)\frac{0.00238}{4!\,0.2^4}$$

$$= 2.1169$$

5.6 : (1)

x	y	一阶差商	二阶差商	三阶差商	四阶差商	五阶
-1	-1					
		0				
-1	-1		1			
		1		-2		
0	0		-1		1.5	
		0		1		-1.5
0	0		1		-1.5	
		1		-2		
1	1		-1			
		0				
1	1					

$\therefore P_5(x) = -1 + (x+1)(0) + (x+1)^2(1) + (x+1)^2(x-0)(-2) + (x+1)^2(x-0)^2(1.5) + (x+1)^2(x-0)^2(x-1)(-1.5)$

$= -1 + x^2 + 2x + 1 - 2x^3 - 4x^2 - 2x + 1.5x^4 + 3x^3 + 1.5x^2 + (x^5 + 2x^4 + x^3 - x^4 - 2x^3 - x^2)(-1.5)$

$= -1 + x^2 + 2x + 1 - 2x^3 - x^2 - 2x + 1.5x^4 + 3x^3 + 1.5x^2 - 1.5x^5 - 3x^4 - 1.5x^3 + 1.5x^4 + 3x^3 + 1.5x^2$

$= -1.5x^5 + 2.5x^3 + 3x^2$

(2)

x	y	一阶	二阶	三阶	四阶
0	0				
		0			
0	0		1		
		1		-1	
1	1		0		$\frac{1}{4}$
		1		$-\frac{1}{2}$	
1	1		-1		
		0			
2	1				

$P_4(x) = 0 + (x-0)(0) + (x-0)^2(1) + (x-0)^2(x-1)(-1) + (x-0)^2(x-1)^2\left(\frac{1}{4}\right)$

$= x^2 - x^3 + x^2 + \frac{1}{4}x^4 - \frac{1}{2}x^3 + \frac{1}{4}x^2$

$= \frac{1}{4}x^4 - \frac{3}{2}x^3 + \frac{9}{4}x^2$

5.8 :

$$X = \frac{(y-y_1)(y-y_2)(y-y_3)(y-y_4)(y-y_5)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)(y_0-y_4)(y_0-y_5)} X_0 + \frac{(y-y_0)(y-y_2)(y-y_3)(y-y_4)(y-y_5)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)(y_1-y_4)(y_1-y_5)} X_1 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_3)(y-y_4)(y-y_5)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)(y_2-y_4)(y_2-y_5)} X_2 + \frac{(y-y_0)(y-y_1)(y-y_2)(y-y_4)(y-y_5)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)(y_3-y_4)(y_3-y_5)} X_3 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)(y-y_3)(y-y_5)}{(y_4-y_0)(y_4-y_1)(y_4-y_2)(y_4-y_3)(y_4-y_5)} X_4 + \frac{(y-y_0)(y-y_1)(y-y_2)(y-y_3)(y-y_4)}{(y_5-y_0)(y_5-y_1)(y_5-y_2)(y_5-y_3)(y_5-y_4)} X_5$$

$$= 0.4769$$

5.9 :

$$x^{(0)} = x_0 + \frac{y - f(x_0)}{f[x_1, x_0]}$$

$y=0$

x_i	y_i	一阶差商	二阶差商
2.4	0.0025		
		-0.509	
2.5	-0.0484		0.125
		-0.484	
2.6	-0.0968		

$$= 2.4049$$

$$x^{(1)} = x_0 + \frac{y - f(x_0)}{f[x_1, x_0] + (x^{(0)} - x_0) f[x_0, x_1, x_2]}$$

$$= 2.4047 \approx 2.405$$

5-12：由于 $\sin x$ 和 $\cos x$ 是一个周期为 2π 的函数

设等距节点，$x_i = ih$，$0 \le i \le M$

$$h = \frac{2\pi}{M}$$

线性插值余项：$f(x) - L_1(x)$

$$= \frac{1}{2} f''(\xi_i)(x - x_i)(x - x_{i+1}),\quad \xi_i \in (x_i, x_{i+1})$$

$$\therefore |f(x) - L_1(x)| \le \frac{1}{8}h^2$$

$$\frac{1}{8}h^2 \le \frac{1}{2} \times 10^{-8}$$

$$h^2 \le 4 \times 10^{-8}$$

$$h \le 2 \times 10^{-4}$$

，取 8 位小数

5-16：$k = 0.5 \times 10^{-5}$

$$\varepsilon = \left(-\frac{1}{6} + 0.9048374\right)(0.5 \times 10^{-7}) + \left(\frac{2}{3} + 0.860708\right)(0.5 \times 10^{-7}) +$$
$$\left(\frac{2}{3} + 0.7788008\right)(2.5 \times 10^{-7}) + \left(-\frac{1}{6} + 0.7408182\right)(0.5 \times 10^{-7})$$
$$= 0.2 \times 10^{-6}$$

总误差，$\varepsilon = 0.5 \times 10^{-5} + 0.2 \times 10^{-6}$

$$= 0.5 \times 10^{-5}$$