姓名：曾泇腱　学号：1820221053　课程：数值分析（第二章）

2.3：$x^3-x^2-1=0$，$x_0=1.5$，$\varepsilon=10^{-4}$

$y_{n+1}=\varphi(x_n)$

$z_{n+1}=\varphi(y_{n+1})$，改写 $x=\sqrt[3]{x^2+1}$

$$x_{n+1}=\frac{x_n z_{n+1}-y_{n+1}^2}{x_n-2y_{n+1}+z_{n+1}}$$

$$\begin{cases} x_0=1.5 \\ y_1=1.4812 \\ z_1=1.4727 \end{cases}$$

$$\begin{cases} x_1=1.4657 \\ y_2=1.4656 \\ z_2=1.4656 \end{cases}$$

$$\begin{cases} x_2=1.4656 \\ y_3=1.4656 \\ z_3=1.4656 \end{cases}$$

$\therefore x=1.4656$

2.4：$f(x)=x^3-x^2-x-1=0$，$\varepsilon=10^{-4}$

$f'(x)=3x^2-2x-1$

在 $x\in[1,2]$

牛顿迭代法

$$x_{n+1}=x_n-\frac{x_n^3-x_n^2-x_n-1}{3x_n^2-2x_n-1}=\frac{2x_n^3-x_n^2+1}{3x_n^2-2x_n-1}$$

$x_0=2$

$x_1=1.85714$

$x_2=1.83954$

$x_3=1.83929$

$x_4=1.83929$

$\therefore x^*=1.8393$

弦截法

$$x_{n+1}=x_n-\frac{f(x_n)}{f(x_n)-f(x_{n-1})}(x_n-x_{n-1})$$

$$=x_n-\frac{x_n^3-x_n^2-x_n-1}{x_n^3-x_n^2-x_n-1-(x_{n-1}^3-x_{n-1}^2-x_n-1)}(x_n-x_{n-1})$$

取 $x_0=1.5$，$x_1=2$

$x_2=1.78947$

$x_3=1.83310$

$x_4=1.83955$

$x_5=1.83929$

$x_6=1.83929$

$\therefore x^*=1.8393$

2.7 : $\begin{cases} x+2y-3=0 \\ 2x^2+y^2-5=0 \end{cases}$, 设 $f_1(x,y) = x+2y-3$, $f_2(x,y) = 2x^2+y^2-5$, $x^{(0)} = -1$, $y^{(0)} = 2$

$\therefore \frac{\partial f_1}{\partial x} = 1$, $\frac{\partial f_1}{\partial y} = 2$, $\frac{\partial f_2}{\partial x} = 4x$, $\frac{\partial f_2}{\partial y} = 2y$

$\frac{\partial f_2(x^{(0)}, y^{(0)})}{\partial x} = -4$, $\frac{\partial f_2(x^{(0)}, y^{(0)})}{\partial y} = 4$, $f_1(x^{(0)}, y^{(0)}) = 0$, $f_2(x^{(0)}, y^{(0)}) = 1$

得 $\begin{cases} \Delta x_1^{(0)} + 2\Delta x_2^{(0)} = 0 \\ -4\Delta x_1^{(0)} + 4\Delta x_2^{(0)} = -1 \end{cases}$, 解得 $\Delta x_1^{(0)} = 0.1667$, $\Delta x_2^{(0)} = -0.0833$

$x^{(1)} = x^{(0)} + \Delta x_1^{(0)} = -0.8333$, $y^{(1)} = y^{(0)} + \Delta x_2^{(0)} = 1.9167$

$\frac{\partial f_2(x^{(1)}, y^{(1)})}{\partial x} = -3.3332$, $\frac{\partial f_2(x^{(1)}, y^{(1)})}{\partial y} = 3.8334$, $f_1(x^{(1)}, y^{(1)}) = 0.0001$, $f_2(x^{(1)}, y^{(1)}) = 0.0625$

得 $\begin{cases} \Delta x_1^{(1)} + 2\Delta x_2^{(1)} = -0.0001 \\ -3.3332\Delta x_1^{(1)} + 3.8334\Delta x_2^{(1)} = -0.0625 \end{cases}$, 解得 $\Delta x_1^{(1)} = 0.0119$, $\Delta x_2^{(1)} = -0.0060$

$x^{(2)} = x^{(1)} + \Delta x_1^{(1)} = -0.8214$, $y^{(2)} = y^{(1)} + \Delta x_2^{(1)} = 1.9107$

$\frac{\partial f_2(x^{(2)}, y^{(2)})}{\partial x} = -3.2856$, $\frac{\partial f_2(x^{(2)}, y^{(2)})}{\partial y} = 3.8214$, $f_1(x^{(2)}, y^{(2)}) = 0$, $f_2(x^{(2)}, y^{(2)}) = 0.0002$

得 $\begin{cases} \Delta x_1^{(2)} + 2\Delta x_2^{(2)} = 0 \\ -3.2856\Delta x_1^{(2)} + 3.2814\Delta x_2^{(2)} = -0.0002 \end{cases}$, 解得 $\Delta x_1^{(2)} \approx 0$, $\Delta x_2^{(2)} \approx 0$

$x^{(3)} = -0.8214 = -0.821$

$y^{(3)} = 1.9107 = 1.911$ （近似至3位小数）

2.11 : $f(x) = x^2-2=0$, $x_0 = 0.5$, $x_{n+1} = \frac{1}{2}(x_0 + \frac{2}{x_0})$, $x_1 = 2.25$

$\because |f(x_1)| > |f(x_0)|$, 不满足下山条件 , 取 $\lambda = \frac{1}{2^i}$, $i = 1,2,3\cdots$

取 $i=1$, $y_1 = (1-\frac{1}{2^i})x_0 + \frac{1}{2^i}x_1 = (1-\frac{1}{2}) \times 0.5 + \frac{1}{2} \times 2.25$
$= 1.375$

$|f(1.375)| < |f(0.5)|$ 满足下山条件

$\therefore$ 取 $\lambda = \frac{1}{2}$ 即可