

1 Optimization

1. Suppose there are mn people forming m rows with n columns. Let a denote the height of the tallest person taken from the shortest people in each column. Let b denote the height of the shortest person taken from the tallest people in each row. What is the relationship between a and b ?

2 Subgradients

1. (a) True or False: A subgradient of $f : \mathbb{R}^n \rightarrow \mathbb{R}$ at x is normal to a hyperplane that globally underestimates the graph of f .
(b) True or False: If $g \in \partial f(x)$ then $-g$ is a descent direction of f .
(c) True or False: For $f : \mathbb{R} \rightarrow \mathbb{R}$, if $1, -1 \in \partial f(x)$ then x is a global minimizer of f .
(d) True or False: Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ and let $g \in \partial f(x)$. Then $\alpha g \in \partial f(x)$ for all $\alpha \in [0, 1]$.
(e) True or False: If the sublevel sets of a function are convex, then the function is convex.
2. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by $f(x_1, x_2) = |x_1| + 2|x_2|$. Compute $\partial f(x_1, x_2)$ for each $x_1, x_2 \in \mathbb{R}^2$.