Programming Assignment - 1

AI1110 - Probability And Random Variables

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Instructions

1. The submission deadline on 9 February 25 is firm, i.e. no extensions will be permitted.
2. Submitting late will cost you 25% penalty per day.
3. Only `Jupyter notebooks` (`.ipynb`) should be submitted.
4. The code should be neatly documented. This includes proper variable names and functions. Comments should be used effectively.
5. Avoid usage of external libraries apart from `numpy`, `matplotlib`, `pandas`. Usage of other libraries may lead to a penalty.
6. Plagiarism in any form will result in an F-grade.

Questions

1. **Buffon's Matches**

   Refer to the following *video* for Buffon's matches problem.

   Buffon's matches problem is one of those problems that look at first sight an incredibly difficult one to solve, but on closer inspection turn out t have a beautifully simple solution. In this assignment we are going to estimate the value of π using a simulation of this problem.

   Write a function `buffons_matches(L, d)` where:

   - L is the length of the matches and d is the distance between the parallel lines.
   - Simulate the throwing of matchsticks on to the ruled sheet using `matplotlib` and also display the number of matches thrown and those that have crossed a line (use different color schemes to differentiate between the 2) along with current π estimate.

   Refer to the following *video* to see a sample simulation.

   Ensure your solution satisfies the following:

   - Your solution should work for any L, d **(even if they do not satisfy the Buffon's matches criteria)** i.e., $L \geq d$.

2. **The Birthday Problem**

   The birthday problem is a famous problem in probability theory that explores the likelihood of two or more people in a group sharing the same birthday. The paradox lies in how counterintuitive the probability is as the group size increases. Your task is to demonstrate how this probability varies with the number of people.

   Write a python code to:

   - Determine the minimum number of people required for the probability of at least two sharing the same birthday to be at least 70%.
   - Write a function **birthday_probability(n, k)** which takes the number of people n, the minimum number of people sharing the same birthday k as input and returns it's probability.
   - Keeping $k = 2$, vary the total number of people from 2 to 60. Plot a graph showing the relationship between the number of people and the corresponding probability.

   Also plot the graph of $1 - \exp(-\frac{n^2}{730})$. Notice the similarity between both the graphs, especially the shape. Is this just a coincidence? Can you think of the reason why the graph is so similar?