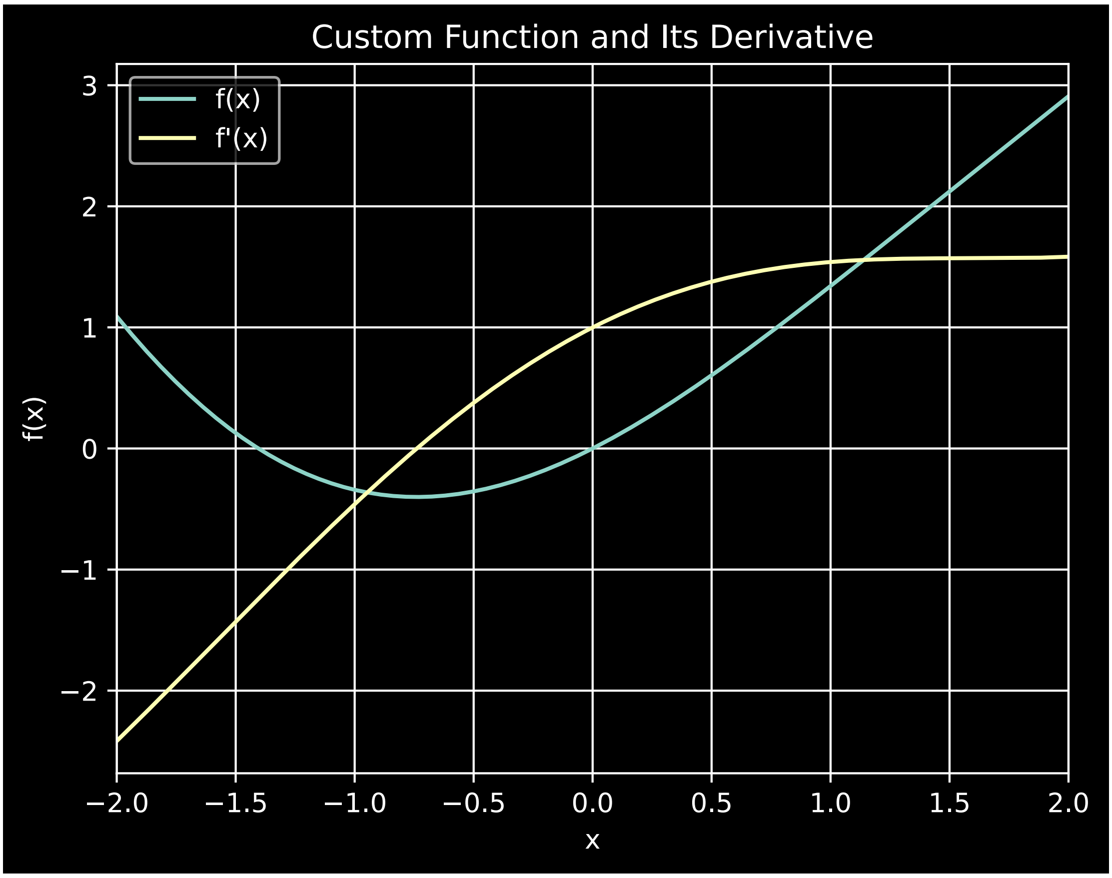
In [1]:
```
# import libraries
import numpy as np
import torch
import torch.nn as nn
import matplotlib.pyplot as plt
from IPython import display
display.set_matplotlib_formats('svg')


plt.style.use('dark_background')
```

```
/tmp/ipykernel_4691/3561430413.py:7: DeprecationWarning: `set_matplotlib_f
ormats` is deprecated since IPython 7.23, directly use `matplotlib_inlin
e.backend_inline.set_matplotlib_formats()`
  display.set_matplotlib_formats('svg')
```

In [2]:
```
# Define a range for x
x_values = np.linspace(-2, 2, 2001)

# Define a custom function and its derivative
def customFunction(x):
    return np.sin(x) + 0.5 * x ** 2

def customDerivative(x):
    return np.cos(x) + x

# Plot the function and its derivative
plt.plot(x_values, customFunction(x_values), x_values, customDerivative(x
plt.xlim(x_values[[0, -1]])
plt.grid()
plt.xlabel('x')
plt.ylabel('f(x)')
plt.legend(['f(x)', 'f\'(x)'])
plt.title('Custom Function and Its Derivative')
plt.show()
```



In [37]:

```
# Random starting point
initial_point = np.random.choice(x_values, 1)
print(initial_point)

# Learning rate and training epochs
learning_rate = 0.01
training_epochs = 20

# Perform gradient descent to find a local minimum
for epoch in range(training_epochs):
    gradient = customDerivative(initial_point)
    initial_point = initial_point - learning_rate * gradient

initial_point
```

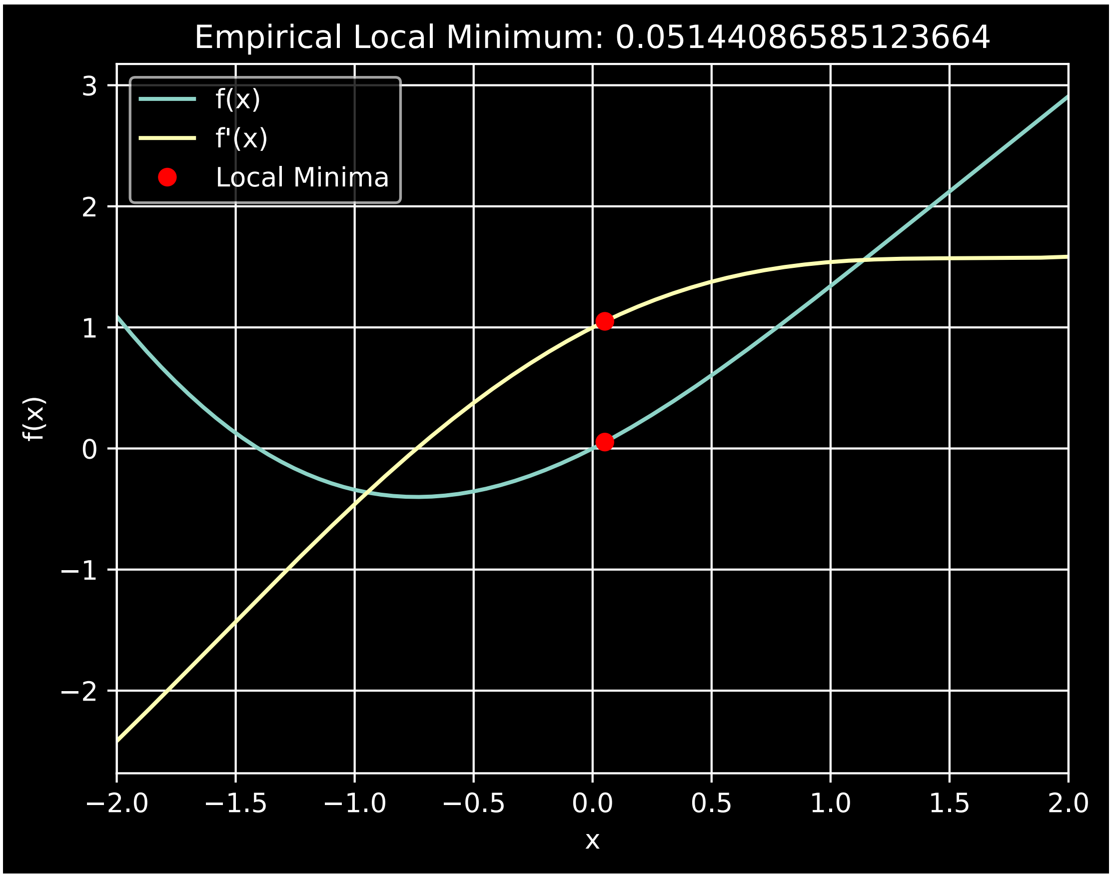
```
[0.282]
```

Out[37]:

```
array([0.05144087])
```

In [38]:

```
# Plot the results
plt.plot(x_values, customFunction(x_values), x_values, customDerivative(x
plt.plot(initial_point, customDerivative(initial_point), 'ro')
plt.plot(initial_point, customFunction(initial_point), 'ro')

plt.xlim(x_values[[0, -1]])
plt.grid()
plt.xlabel('x')
plt.ylabel('f(x)')
plt.legend(['f(x)', 'f\'(x)', 'Local Minima'])
plt.title('Empirical Local Minimum: %s' % initial_point[0])
plt.show()
```



In [26]:

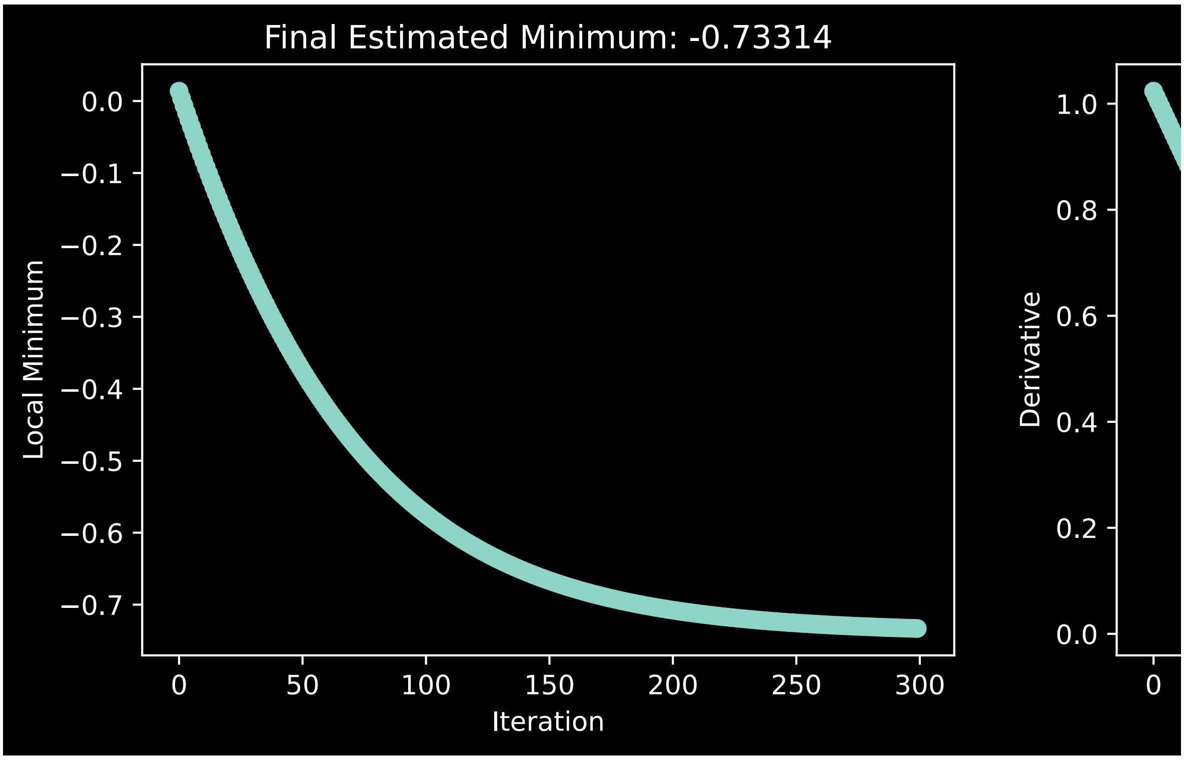
```
# Random starting point
initial_point = np.random.choice(x_values, 1)

# Learning rate and training epochs
learning_rate = 0.01
training_epochs = 300

# Run through training and store all the results
model_params = np.zeros((training_epochs, 2))
for epoch in range(training_epochs):
    gradient = customDerivative(initial_point)
    initial_point = initial_point - learning_rate * gradient
    model_params[epoch, 0] = initial_point
    model_params[epoch, 1] = gradient
```

```
/tmp/ipykernel_4691/1005108408.py:13: DeprecationWarning: Conversion of an
array with ndim > 0 to a scalar is deprecated, and will error in future. E
nsure you extract a single element from your array before performing this
operation. (Deprecated NumPy 1.25.)
  model_params[epoch, 0] = initial_point
/tmp/ipykernel_4691/1005108408.py:14: DeprecationWarning: Conversion of an
array with ndim > 0 to a scalar is deprecated, and will error in future. E
nsure you extract a single element from your array before performing this
operation. (Deprecated NumPy 1.25.)
  model_params[epoch, 1] = gradient
```

In [27]:

```
# Plot the gradient descent progress
fig, ax = plt.subplots(1, 2, figsize=(12, 4))

for i in range(2):
    ax[i].plot(model_params[:, i], 'o-')
    ax[i].set_xlabel('Iteration')
    ax[i].set_title(f'Final Estimated Minimum: {initial_point[0]:.5f}')
```

```
ax[0].set_ylabel('Local Minimum')
ax[1].set_ylabel('Derivative')

plt.show()
```

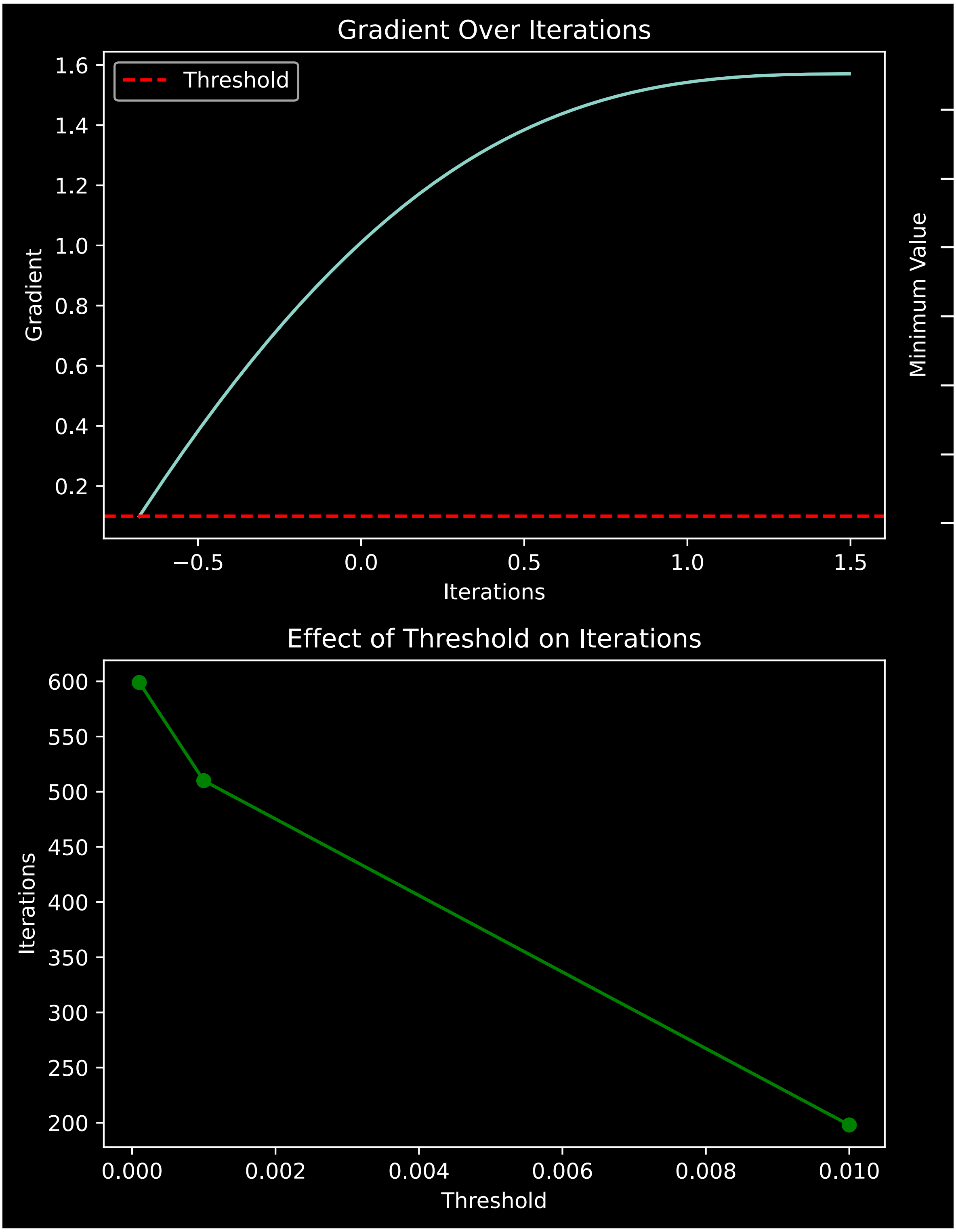


In [36]:

```
import numpy as np
import torch
import torch.nn as nn
import matplotlib.pyplot as plt
from IPython import display
display.set_matplotlib_formats('svg')

# Define a range for x
x_values = np.linspace(-2, 2, 2001)

# Define a custom function and its derivative
def customFunction(x):
    return np.sin(x) + 0.5 * x ** 2

def customDerivative(x):
    return np.cos(x) + x

def findLocalMinimumWithThreshold(learning_rate, threshold):
    # Random starting point
    initial_point = np.random.choice(x_values, 1)

    # Initialize variables
    gradient = customDerivative(initial_point)
    iteration = 0
    x_history = []
    gradient_history = []

    # Run through training until the derivative magnitude is smaller than
    while np.abs(gradient) > threshold:
        gradient = customDerivative(initial_point)
        initial_point = initial_point - learning_rate * gradient
```

```
        iteration += 1
        x_history.append(initial_point[0])
        gradient_history.append(gradient)

    return initial_point, iteration, x_history, gradient_history

# 1) Modify the code to end training when the derivative is smaller than
learning_rate = 0.01
threshold = 0.1
found_minima, iterations, x_history, gradient_history = findLocalMinimumW
print("Found Minimum:", found_minima[0])
print("Iterations:", iterations)

# 2) Explore the accuracy of the result with different threshold values
thresholds_to_test = [0.01, 0.001, 0.0001]
results = []

for threshold in thresholds_to_test:
    found_minima, iterations, _, _ = findLocalMinimumWithThreshold(learni
    results.append((threshold, found_minima[0], iterations))

# 3) Potential problems when the stopping criterion is based on the deriv
# - The choice of threshold can impact the accuracy and convergence speed
# - Negative derivatives may cause unexpected behavior. Adding safeguards
# - The derivative may become very small near local minima, leading to ea

# Plot the results
fig, axes = plt.subplots(2, 2, figsize=(12, 8))

# Plot for Question 1
axes[0, 0].plot(x_history, gradient_history)
axes[0, 0].set_xlabel('Iterations')
axes[0, 0].set_ylabel('Gradient')
axes[0, 0].set_title('Gradient Over Iterations')
axes[0, 0].axhline(y=0.1, color='r', linestyle='--', label='Threshold')
axes[0, 0].legend()

# Plot for Question 2
threshold_values = [result[0] for result in results]
minimum_values = [result[1] for result in results]
iterations_values = [result[2] for result in results]

axes[0, 1].plot(threshold_values, minimum_values, marker='o', linestyle='
axes[0, 1].set_xlabel('Threshold')
axes[0, 1].set_ylabel('Minimum Value')
axes[0, 1].set_title('Effect of Threshold on Minimum Value')

axes[1, 0].plot(threshold_values, iterations_values, marker='o', linestyl
axes[1, 0].set_xlabel('Threshold')
axes[1, 0].set_ylabel('Iterations')
axes[1, 0].set_title('Effect of Threshold on Iterations')

# Plot for Question 3 (No specific plot needed for this question)

plt.tight_layout()
plt.show()
```

```
/tmp/ipykernel_4691/4163496515.py:6: DeprecationWarning: `set_matplotlib_f
ormats` is deprecated since IPython 7.23, directly use `matplotlib_inlin
e.backend_inline.set_matplotlib_formats()`
  display.set_matplotlib_formats('svg')
Found Minimum: -0.679944839537824
Iterations: 269
```



Summary of Observations

- Lowering the number of epoch in the first example shows the importance of picking a good initial value. A low number of epochs and a random initial value will sometimes result in never reaching the minimum. In the case of thresholding, the randomness in the initial value also greatly impacts the number of iterations required to reach that threshold for each run

- Similarly, having too low of a learning rate will never let the gradient converge to the minimum, however too high of an learning rate might cause the gradient to shoot over the true minimum
- Having a well defined threshold to early stop the training can greatly help reduce the computational cost of the algorithm while also helping to prevent overfitting. Setting a threshold in the third example helped to reduce the overall number of iterations while not having a great impact into the calculated function minimum