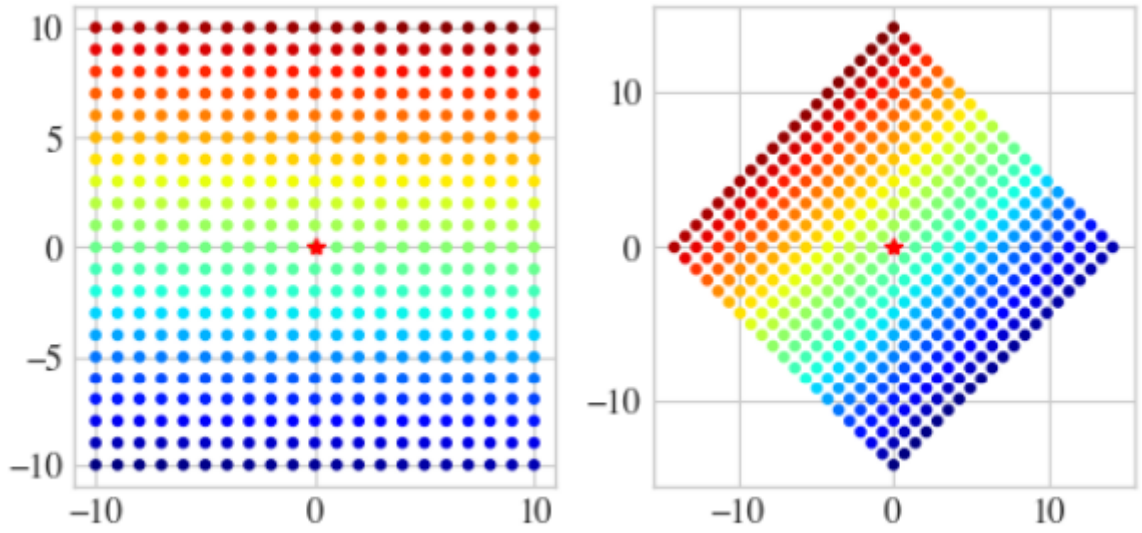
Transformations

Isometries/Orthogonal matrix: Rotations and Translations

If we want to rotate, we can add one single scalar. We can do that by using this transformation matrix, and justby using a single number we can rotate.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

This preserves angles and distances.



Why?

Of course you remember that each column of the transform where the unit vectors are gonna land.

Then, with $\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$, will make $\hat{j}$ land on $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ when $\theta = 0$. Then it will continue rotating left with higher angles.

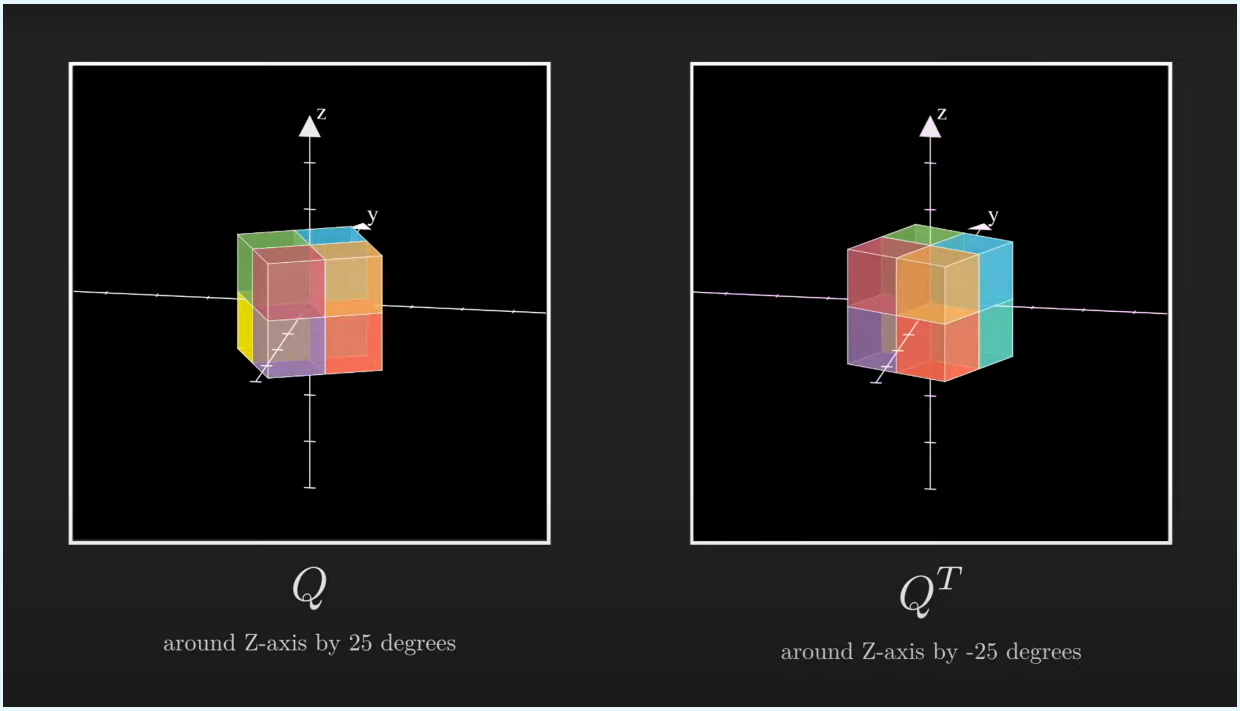
For $\hat{i}$ it's the same thing, the only difference is that it is at a different position in the unit circle.

In fact, when $\theta = 0$, $\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, which is exactly the basic $\hat{i}$.

Info

If you transpose an orthogonal matrix(rotation), you get its inverse matrix, which is the exact rotation but backwards.

So with AA^T, nothing changes.



Similarity: Scale, Flip, Rotation and Translations

Now we add a scaling factor to the two axes.

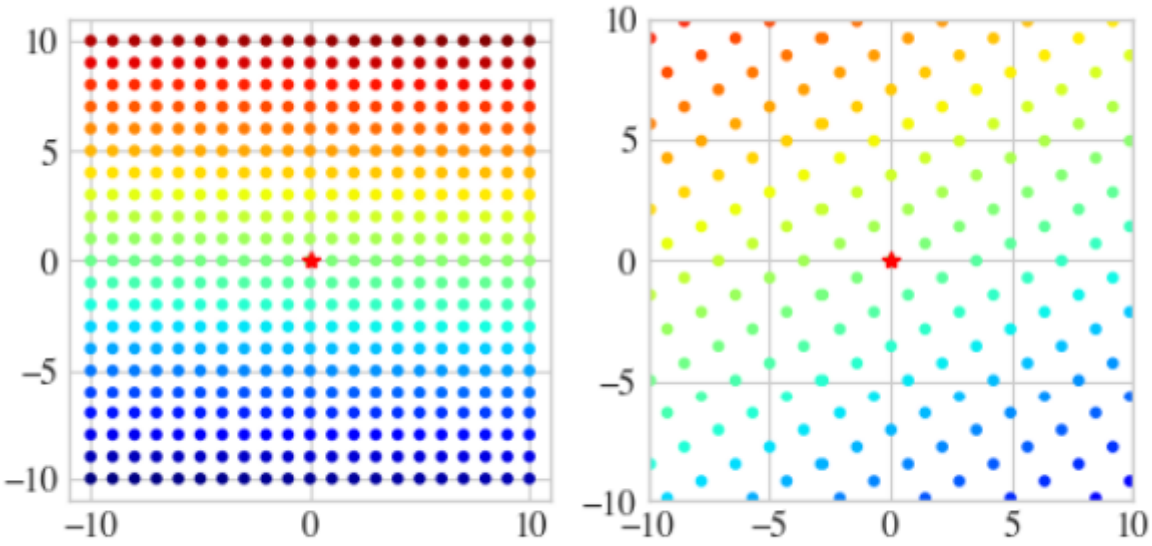
$$\begin{bmatrix} S_x\cos\theta & -\sin\theta \\ \sin\theta & S_y\cos\theta \end{bmatrix}$$

Preserve angles and ratio between distances.

Why the S coefficients?

Because as you remember, the unit vectors have two components, x and y.
The main component of $\hat{i}$ is x and the main component of $\hat{j}$ is y.

Here we are just scaling the main components of the two vectors.



Affine:

We add C_x and C_y to the axes that were previously 0.

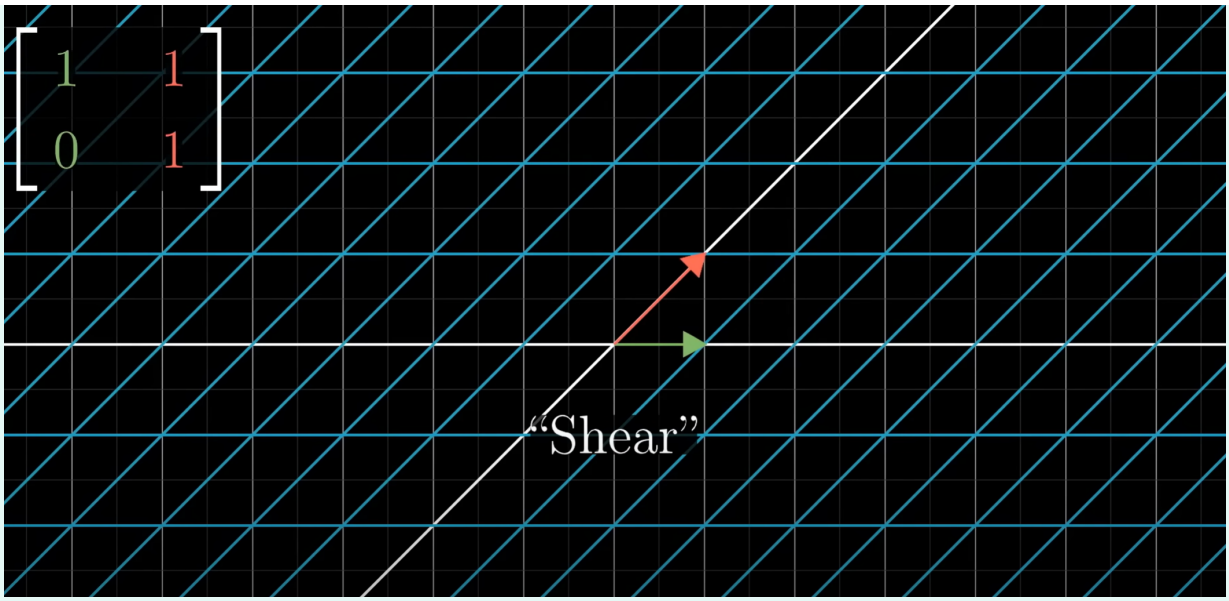
$$\begin{bmatrix} s_x \cos\theta & -c_x \sin\theta \\ c_y \sin\theta & s_y \cos\theta \end{bmatrix}$$

Preserve preserve parallelism but not angles.

Why the C coefficients?

There are two ways to shear:

- By rotating one vector and keeping the other the same:



- Rotate both the vectors and extend only one of them.

Since in this case we have only 1 angle, we are using the second way. The shear can only happen if we rotate the vectors, since if the secondary component is 0 in both of the basis vectors, unless it is rotated.

So, here, the C coefficients serve the purpose of scaling the secondary component of the unit vectors, which is rotated.