Covariance matrix

The covariance matrix is defined as a square matrix where the diagonal elements represent the variance and the off-diagonal elements represent the covariance.

$$\begin{bmatrix} var(x) & cov(x,y) \\ cov(x,y) & var(y) \end{bmatrix}$$

How to compute it

$$C_{i,j} = cov(D_i\,,\ D_j)$$

where D_i and D_j are the variables/dimensions(ex. x and y).
Visually:

$$C = \begin{bmatrix} cov(x,x) & cov(x,y) \\ cov(x,y) & cov(y,y) \end{bmatrix}$$

Since $cov(k,k) = var(k)$, the elements on the diagonal are all variances.

An alternative formula that takes a whole matrix as input:

$$C = \frac{XX^T}{1-n}$$

Example

Take this sample data, the variables are x and y:

Index	X	Y
0	-2.123062	-2.267402
1	-1.775958	0.070899
2	-1.582416	-3.072345
3	-0.492453	-0.920361

Let's actually compute it using the formula $C = \frac{XX^T}{1-n}$:

$$\begin{bmatrix} -2.12 & -2.26 \\ -1.77 & 0.07 \\ -1.58 & -3.07 \\ -0.49 & -0.92 \end{bmatrix} \begin{bmatrix} -2.12 & -1.77 & -1.58 & -0.49 \\ -2.26 & 0.07 & -3.07 & -0.92 \end{bmatrix}^T$$

$$\begin{bmatrix} -2.12 \begin{bmatrix} -2.12 \\ -1.77 \\ -1.58 \\ -0.49 \end{bmatrix} + -2.26 \begin{bmatrix} -2.26 \\ 0.07 \\ -3.07 \\ -0.92 \end{bmatrix}, & \end{bmatrix}$$