Calculus 2nd unit — Written test
4th June 2025 — Sheet n. 00001

Instructions: the boxes (**T** / **F**) can be used to choose the answer: True (**T**) or False (**F**).
The box "**C**" can be used to correct mistakes, since it "flips" the given answer.
To choose a box, blacken it out **completely**: ■ (not ☒ or ☑).

Name: _____

Surname: _____

ID number: | | | | | | | |

	1A	1B	1C	1D	2A	2B	2C	2D	3A	3B	3C	3D	4A	4B	4C	4D
V	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
F	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
C	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

1) Say if the following statements are true or false (1.5 point for each right answer).

1A) We have

$$\int_{\frac{\pi}{2}}^{\pi} \frac{\cos(x)}{\sin(x)} dx = \ln(\sin(\pi))$$

1B) One has $\int_0^1 (7x^2 + 9x + 11) = \frac{7}{3} + \frac{9}{2} + 11$

1C) One has

$$\int_3^7 x \ln(2x) dx = \frac{1}{2}x^2 \left(\ln(2x^2) - \frac{1}{3} \right) \Big|_3^7$$

1D) One has $\int_{-7}^{7} e^{4x^2} \sin(2x) dx = 0$

2) Say if the following statements are true or false (1.5 point for each right answer).

2A) One has

$$\int_2^{\infty} \frac{\ln x}{x^2} < \infty$$

2B) One has

$$\int_0^1 \frac{e^x - 1}{x} dx < \infty$$

2C) One has

$$\int_0^{\infty} \frac{1}{1 + x^{1/2}} < \infty$$

2D) One has

$$\int_1^{\infty} x^4 \sin\left(\frac{1}{x}\right) dx < \infty$$

3) Say if the following statements are true or false (1.5 point for each right answer). Consider the series $\sum_{n=1}^{\infty} \frac{6n}{6n+1}(x-2)^n$

3A) The radius of convergence of the series is 1.
3B) The series is convergent at $x = 3$.
3C) The series is convergent at $x = 1$.
3D) The series is convergent at $x = 5/2$.

4) Say if the following statements are true or false (1.5 point for each right answer).

4A) The series $\sum_{n=1}^{\infty} \frac{n}{6n^3+8}$ is convergent.
4B) The series $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$ is divergent.
4C) The series $\sum_{n=1}^{\infty} \frac{n!}{\sqrt{(2n)!}}$ is divergent.
4D) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ is convergent.

Exercise 5			
☐☐	☐☐	☐☐	☐☐
a	b	c	d

Surname **Name** **ID number** **Sheet n. 00001**

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5) 2 points for each right answer = 8 points for the whole exercise.

a) Consider the differential equation

$$(\star) \qquad y''(t) + -5y'(t) + 6y(t) = \sin(4t)$$

Write the general integral of the associated homogeneous equation.

b) Find a particular solution of $(\star)$.

c) Write the general integral of the equation $(\star)$.

d) Solve the equation $y'(t) + y(t) = 1 - t$.
