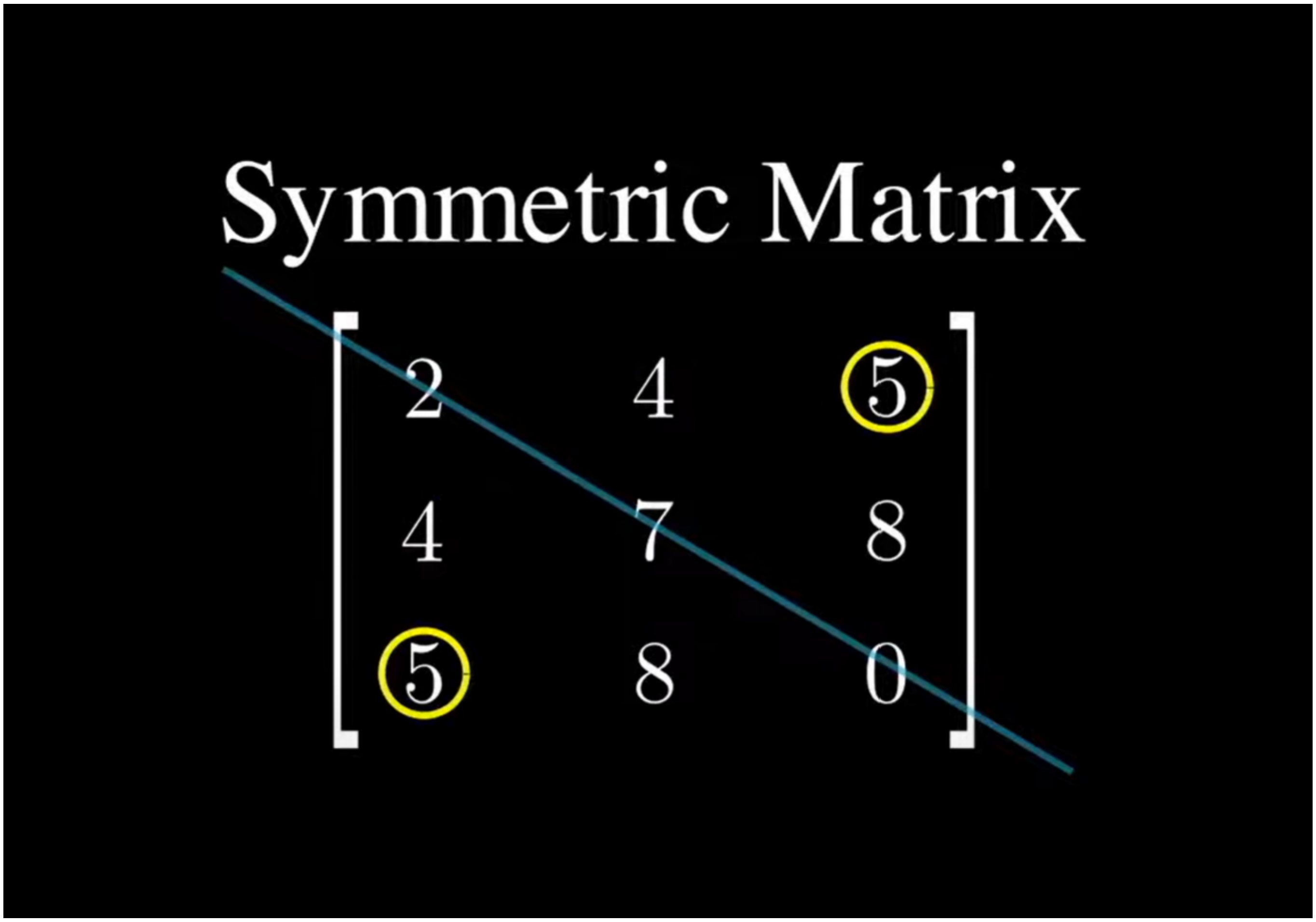
Symmetric matrix

A symmetric matrix is a matrix that has equal values on both sides of its diagonal.



It is describable by:

$$A_{ji} = A_{ij}$$

Why are they useful?

The eigenvectors are perpendicular to each other.

That means that if we pack the eigenvectors(normalized) into a matrix, we obtain an orthogonal matrix that describes a rotation of the basis vectors to the eigenvectors.

But the cool thing is:

If we transpose this orthogonal matrix, we get its inverse, so the counter to the original rotation.

Hint

Non-square matrices cannot be symmetrical.

Info

If the matrix is symmetrical, the eigenvectors are perpendicular.

Why?

Creating symmetry

Symmetric matrices are very powerful, so we artificially create symmetry to take advantage of it.

If we multiply any rectangular matrix to its transposed, we get a square matrix, that is also symmetrical.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}^T = \begin{bmatrix} 14 & 32 \\ 32 & 77 \end{bmatrix}$$

We can also do it both ways:

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}^T \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 17 & 22 & 27 \\ 22 & 29 & 36 \\ 27 & 36 & 45 \end{bmatrix}$$

So we just created two square symmetrical matrices from a singular rectangular matrix.

We all these two matrices:

$$S_L = \begin{bmatrix} 14 & 32 \\ 32 & 77 \end{bmatrix}, \quad S_R = \begin{bmatrix} 17 & 22 & 27 \\ 22 & 29 & 36 \\ 27 & 36 & 45 \end{bmatrix}$$