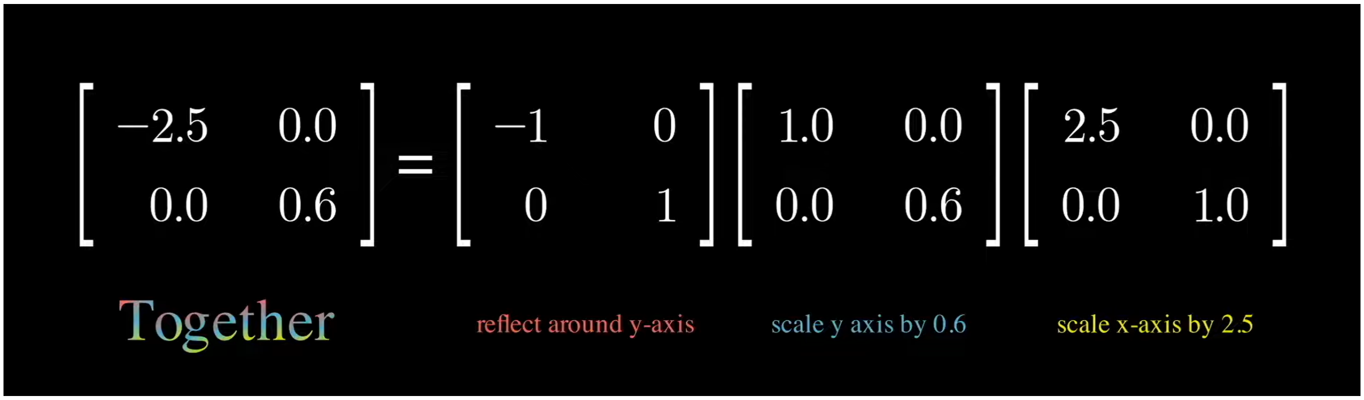
# Spectral decomposition

We know that we can actually combine multiple transformations into just one matrix.

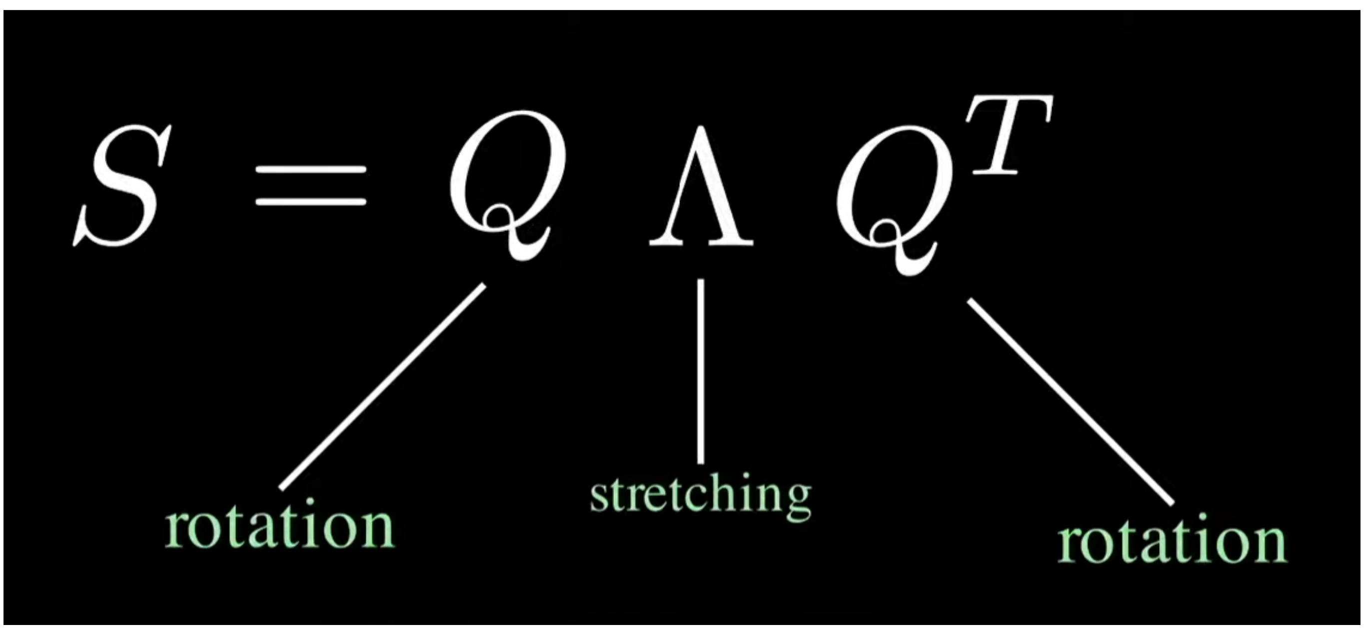
This matrix would applied would have the same result than if we applied the individual matrices one after the other.




And this is called composition of matrices, but did you know that we can also do the opposite process?

## Decomposition

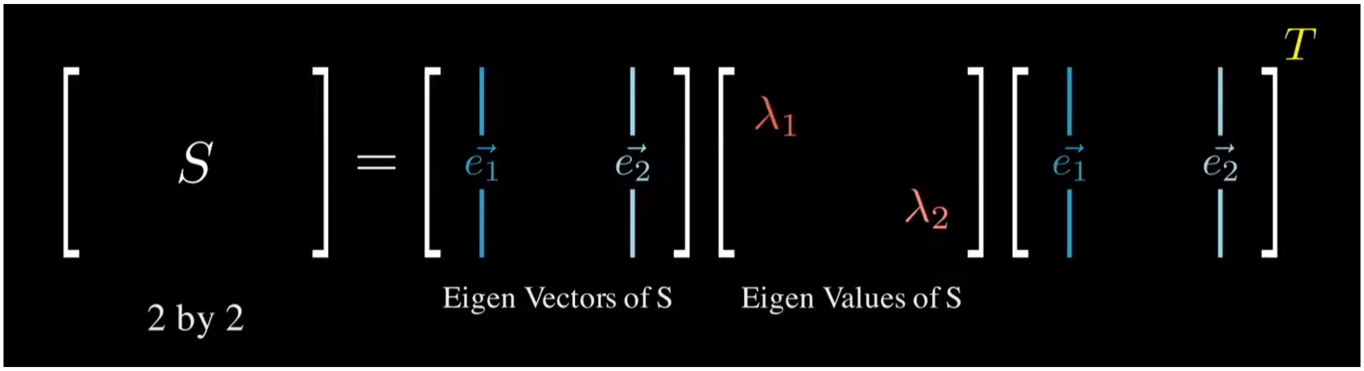
Whenever you have a symmetric matrix S, we can always and unconditionally decompose it in a sequence of three simple matrices:




What is happening is that:
$Q^T$) We rotate the plane so that the eigenvectors end up on the basis vectors.

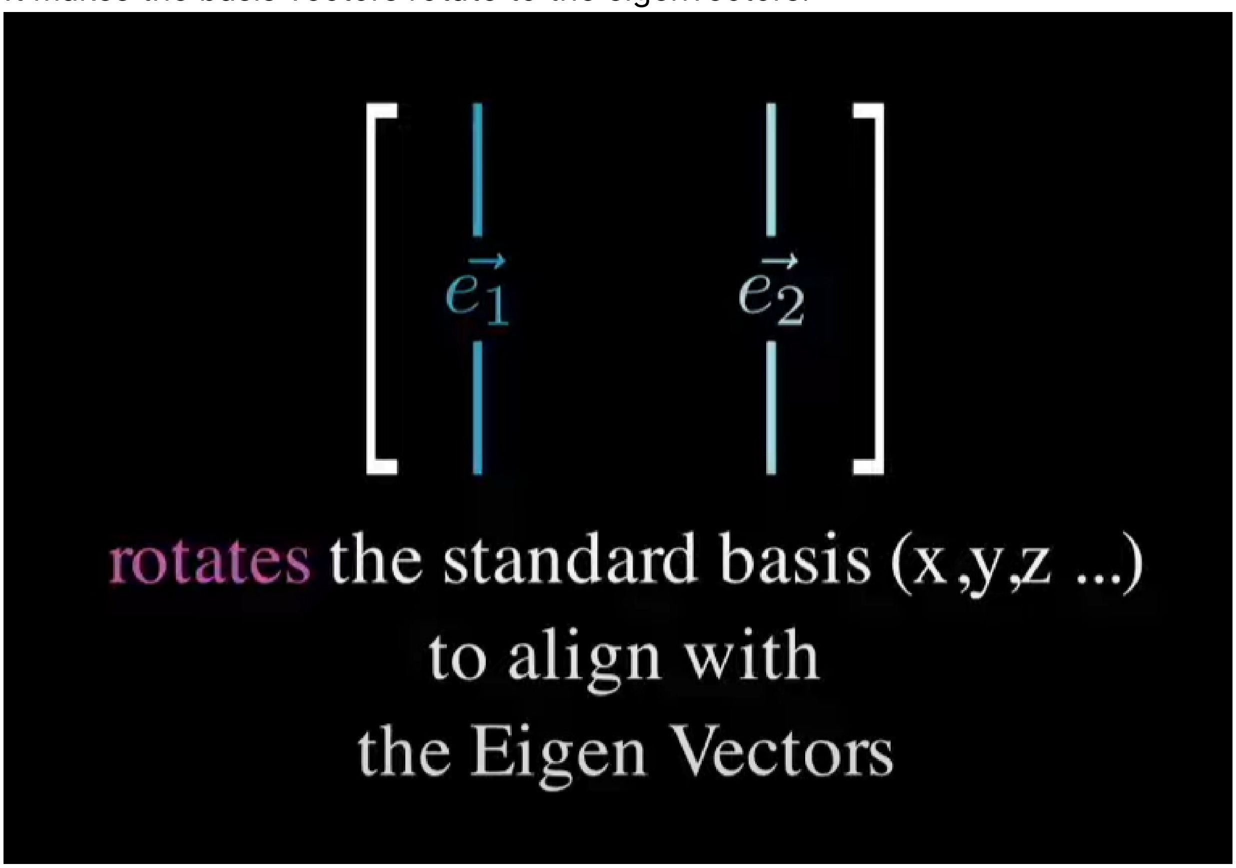
$A$) Once we have the eigenvectors there, we scale them.
$Q$) We rotate the plane so that the eigenvectors end up shere they were before. But now they are scaled.

## More detail:



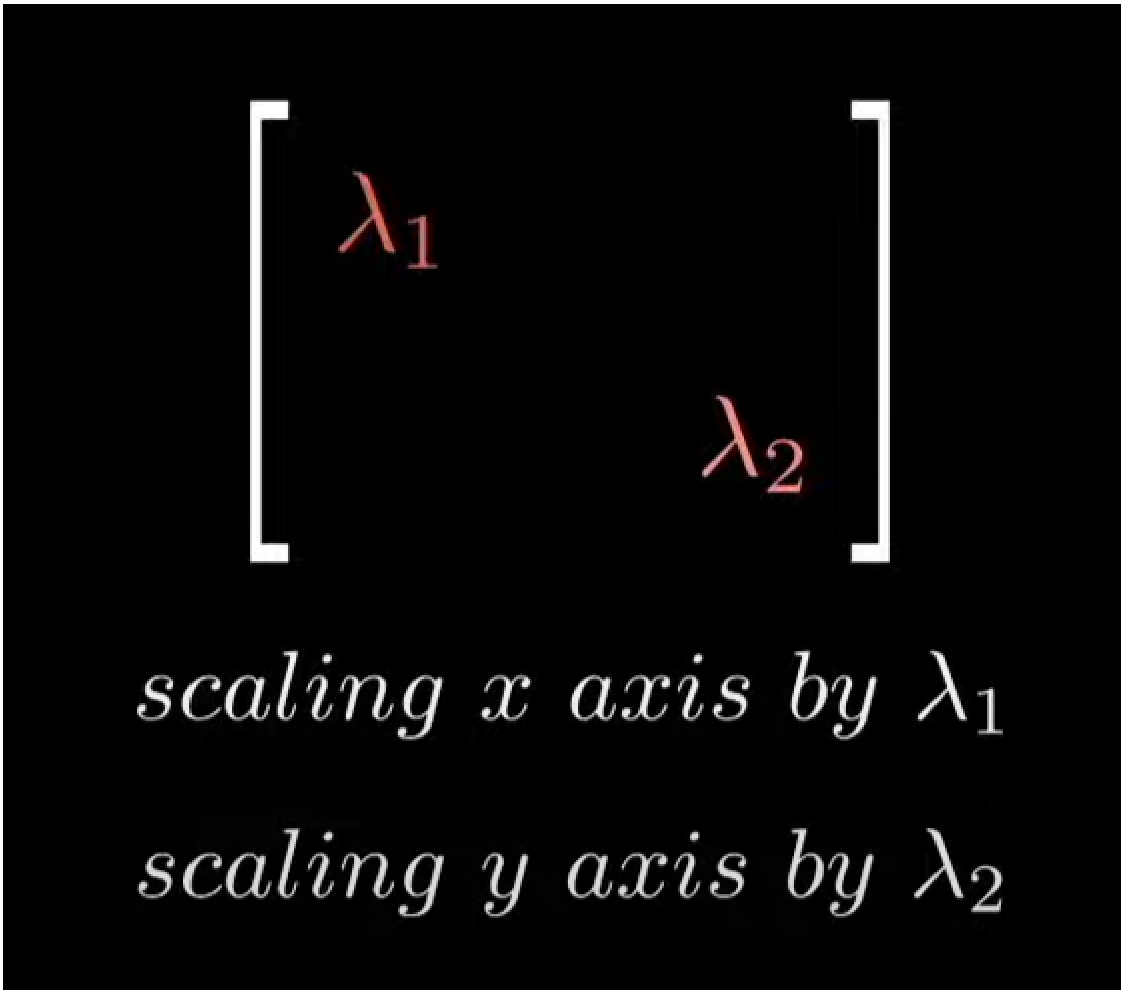

(The steps are not in order)

- $Q$ is an orthogonal matrix, which means that it's a rotation. Its columns are the eigenvectors of S.
  - It makes the basis vectors rotate to the eigenvectors.




- $Q^T$ is the inverse of Q, which means that it counters the rotation of Q perfectly.
- A is a diagonal matrix, which means that all the values except the ones on the diagonal, are zero.

- Its purpose is to scale the axes. We rotated everything so that we could scale the eigenvectors with this simple matrix.




> **Hint**
>
> You can only do this with symmetric matrices.