Poprawka terenowa/topograficzna

1.

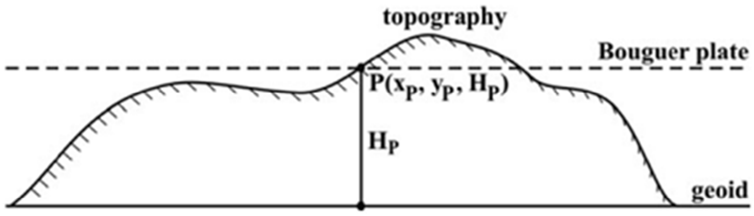

Fig. 8.1 The topography and the Bouguer plate

$$B(x_p, y_p, H_P) = G \iint\limits_E \int\limits_0^{H_P} \frac{\rho(x,y,z)(H_P - z)}{\left[(x_P - x)^2 + (y_P - y)^2 + (H_P - z)^2\right]^{3/2}}\, dx dy dz, \quad (8.4)$$

$$c(x_p, y_p, H_P) = G \iint\limits_E \int\limits_H^{H_P} \frac{\rho(x,y,z)(H_P - z)}{\left[(x_P - x)^2 + (y_P - y)^2 + (H_P - z)^2\right]^{3/2}}\, dx dy dz. \quad (8.5)$$

If the radius of the previously mentioned area is infinite, the Bouguer reduction in the case of a simple horizontal plate can be determined as:

$$B = 2\pi G \rho H_P. \quad (8.6)$$

Using the approximated value of G that has been already given in Chap. 1 (Sect. 1.2, in Part II of this book) and assuming a constant density $\rho = 2670\,\mathrm{kgm}^{-3}$ ($2.67\,\mathrm{gcm}^{-3}$), the simple Bouguer reduction reads

$$B = 0.1119 H_P, \quad (8.7)$$

that gives the reduction in mGal when H is given in meters.

The terrain correction formula (8.5) is a refinement of the simple case of the Bouguer plate, since it accounts for the surpluses and deficits of the actual Earth's topography from the aforementioned horizontal Bouguer plate (Fig. 8.2). Based on the kernel function of the terrain correction formula expressed by $(\Delta h / l^3)$, an area E of 100×100 km may be considered big enough to get a reasonable accuracy in the computation of the terrain correction at a point P lying at the center of this area (Peng 1994).

The 2D linearly approximated formula for the terrain correction at a point P on a plane reference surface E is derived from (8.5) integrating with respect to z and in this case the triple integral of the terrain correction reads as follows (Sideris 1985):

$$c(x_p, y_p) = G \iint\limits_E \frac{-\rho(x,y)}{\left(l_0^2 + z^2\right)^{1/2}} \Big|_0^{\Delta H}\, dx dy$$

$$= \iint\limits_E \frac{\rho(x,y)}{l_0} \left[1 - \left[1 + \left(\frac{\Delta H}{l_0}\right)^2\right]^{-1/2}\right] dx dy, \quad (8.8)$$

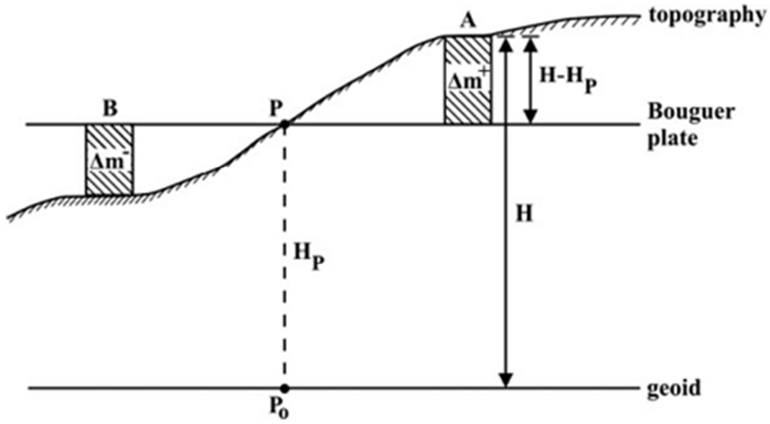

Fig. 8.2 The geometry of the planar Bouguer reduction and the terrain correction

where $\Delta H = H(x_P, y_P) - H(x, y)$ and $l_0^2 = (x_p - x)^2 + (y_p - y)^2$. For $(\frac{\Delta H}{l_0})^2 \leq 1$, the term $[1 + (\frac{\Delta H}{l_0})^2]^{-1/2}$ can be expanded into a series as follows:

$$\left[1 + \left(\frac{\Delta H}{l_0}\right)^2\right]^{-1/2} = 1 - \frac{1}{2}\left(\frac{\Delta H}{l_0}\right)^2 + \frac{1.3}{2.4}\left(\frac{\Delta H}{l_0}\right)^4 - \frac{1.3.5}{2.4.6}\left(\frac{\Delta H}{l_0}\right)^6 + \ldots . \tag{8.9}$$

Keeping the terms up to third order we get the following approximation of the terrain correction integral:

$$\begin{aligned} c(x_p, y_p) = \frac{1}{2} G \iint_E \frac{\rho(x,y)(H_P - H)^2}{\left[(x_P - x)^2 + (y_P - y)^2\right]^{3/2}} dx dy \\ - \frac{3}{8} G \iint_E \frac{\rho(x,y)(H_P - H)^4}{\left[(x_P - x)^2 + (y_P - y)^2\right]^{5/2}} dx dy \\ + \frac{5}{16} G \iint_E \frac{\rho(x,y)(H_P - H)^6}{\left[(x_P - x)^2 + (y_P - y)^2\right]^{7/2}} dx dy \end{aligned} \tag{8.10}$$

If $\frac{\Delta H}{l_0} \ll 1$ a good approximation is obtained by $[1 + (\frac{\Delta H}{l_0})^2]^{-1/2} \approx 1 - \frac{1}{2}(\frac{\Delta H}{l_0})^2$ and by substituting it into (8.8) we finally get:

$$c(x_P, y_P) = \frac{1}{2} G \iint_E \frac{\rho(x,y)\left[H(x_P, y_P) - H(x,y)\right]^2}{\left[(x_P - x)^2 + (y_P - y)^2\right]^{3/2}} dx dy. \tag{8.11}$$

This last equation represents the so-called *linear approximation* of the terrain correction. Similar approximation formulas for terrain correction as before can be

2.

3.2.2 Two-dimensional integration

To more rigorously account for the [potentially] rectangular shape of $\Delta\varphi$, $\Delta\lambda$ grid cells, mean kernel values are computed by means of the 2D-AI, shown in Sect. 2.4. For the 2D-AI of mean Stokes kernels $\overline{S}(u)$, the planar approximation $K_S(x,y)$ of Stokes's kernel $S(\psi)$ is required. With the $2/\psi$ approximation [Eq. (19)] and planar approximation of the spherical distance $\psi \approx \sqrt{x^2+y^2}$, the planar Stokes kernel $K_S(x,y)$ reads:

$$K_S(x,y) \approx \frac{2}{\sqrt{x^2+y^2}} \quad . \tag{22}$$

where (x,y) are the planar coordinates of the evaluation point Q relative to the computation point P, as introduced in Eq. (10). For a cell bounded by $v=(x_1,y_1,x_2,y_2)$, the mean kernel $\overline{K}_S(v)$ in planar approximation is computed using Eq. (13), to give:

$$\begin{aligned}\overline{K}_S(v) &= \frac{1}{\Delta x\Delta y}\int_{x=x_1}^{x_2}\int_{y=y_1}^{y_2} K_S(x,y)dxdy = \frac{1}{\Delta x\Delta y}\left\| F_S(x,y)\Big|_{y_1}^{y_2}\right|_{x_1}^{x_2} \\ &= \frac{1}{\Delta x\Delta y}[F_S(x_1,y_1)-F_S(x_2,y_1)-F_S(x_1,y_2)+F_S(x_2,y_2)]\end{aligned} \tag{23}$$

where $\Delta x\Delta y$ is the surface area of the cell and $F_S(x,y)$ the antiderivative of the planar kernel $K_S(x,y)$. The antiderivative $F_S(x,y)$ is given by (de Min 1994):

$$F_S(x,y) = 2[x\ln(y+\sqrt{x^2+y^2})+y\ln(x+\sqrt{x^2+y^2})] \ . \tag{24}$$

With the weighting factor $W = \overline{K}_S(v)/K_S(x,y)$ [Eq. (16)], point Stokes kernels $S(\psi)$ are converted to mean Stokes kernels $\overline{S}(u)$ by:

$$\overline{S}(u) = WS(\psi) \tag{25}$$

where the vector $u=(\lambda_1,\varphi_1,\lambda_2,\varphi_2)$ contains the coordinates of the cell corners. This transformation is possible because the first term in Stokes's kernel (Eq. 18) is dominant in the inner zone. Importantly, there is no need to analytically integrate the (complete) Stokes function to obtain analytical mean kernels.

elevation surface can be any digital elevation model, e.g., DTM2006.0 (Pavlis et al. 2012). Under a planar approximation, the RTM gravities, δg_{RTM}, evaluated at point P on the (topographic) surface is given by the integral of form in the triad coordinate system (x,y,z)

$$\delta g_{RTM}(P) = k\rho \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{z_1=h_{ref}}^{z_2=h} \frac{z - h_P}{r^3} dzdydx\,,\ r = \sqrt{(x - x_P)^2 + (y - y_P)^2 + (z - h_P)^2} \tag{1}$$

where k is the Newton's gravitational constant, ρ is an average density of the topographic mass (=2.67 g/cm^3), and h are the topographic heights from, given by, for instance, SRTM. If the mean elevation surface is a sufficiently long wavelength, then we can approximate Eq. (1) as

$$\delta g_{RTM}(P) \approx 2\pi k\rho(h_P - h_{ref}) - k\rho \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{z_1=h_P}^{z_2=h} \frac{z - h_P}{r^3} dzdydx \tag{2}$$

It should be noted that the first term of Eq. (2) refers to two Bouguer plates at h_P and h_{ref}, and the second one refers to terrain correction. In practice, $\delta g_{RTM}(P)$ is numerically computed by summation of N rectangular prisms within some radius around the evaluation point (P). As shown in Figure 1, the single prism is defined by the coordinates x_1, x_2, y_1, y_2, z_1, and z_2 in the left-handed coordinate system. Therefore, the second term of Eq. (2) can be written in the closed analytical forms of flat-top prisms as follows Sansó and Rummel (1997):

$$\delta g_{RTM}(P) \approx 2\pi k\rho(h_P - h_{ref}) - \sum_{i=1}^{N}(k\rho \,|||\, x\ln(y + r) + y\ln(x + r) - z\arctan\frac{xy}{zr} \Big|_{x_1}^{x_2}\Big|_{y_1}^{y_2}\Big|_{h_P}^{h})_i \tag{3}$$

In fact, the summation term of Eq. (3) is terrain corrections. Our aim is to assess gravity requirements with respect to spatial resolutions. We assume that, in the void areas, EGM2008 contributes long- and medium-wavelength information of the earth's gravity field. For all wavelength contents, the topography-implied gravity anomalies in those areas can be approximated by EGM2008 gravity anomalies, Δg_M, and δg_{RTM} as follows:

$$\Delta g \approx \Delta g_M - \delta g_{RTM} - 2\pi k\rho h_{ref} \tag{4}$$

In Eq. (4), refined Bouguer gravity anomalies [complete Bouguer gravity anomalies plus terrain corrections (Heiskanen and Moritz, 1967, p. 131)] are immediately obvious if we consider Δg_M as free-air gravity anomalies. For numerical computations with an average topographic mass (crust) density of 2670 kg/m^3, we calculated δg_{RTM} using 30 arcsecond SRTM data [derived from three-arcsecond SRTM data--averages over 30 arcsecond × 30arcsecond blocks] and stored it in database—the term h_{ref} were generated using DTM2006.0. The simulated amomalies in Eq. (4), then, were computed.

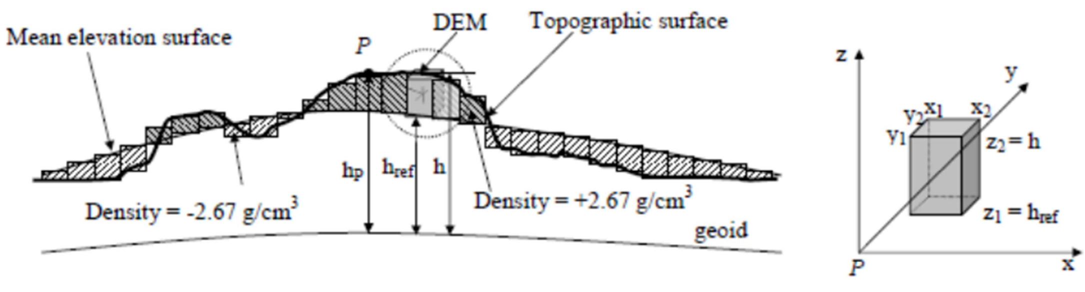

Figure 1: The residual terrain model

4.

TC estimates the effect of topography with respect to the Bouguer plate in a conventional manner. It integrates the gravitational effect produced by each prism constructed from the DEM (FORSBERG, 1984).

For a prism bounded by $\Delta x = xp - x$, $\Delta y = yp - y$, $\Delta z = hp - z$, the gravitational effect of terrain at a point P is given by equation (1):

$$TCR = G\rho \;|||\; x\,\mathbf{log}(y+r) + y\,\mathbf{log}(x+r) - z\,\mathbf{arctan}\frac{xy}{zr}\Big|_{xp}^{x}\Big|_{yp}^{y}\Big|_{zp}^{z} \qquad (1)$$

where ρ is the density, G is the gravitational constant and r is the spherical geocentric radius.

(xp, yp, hp) and (x, y, z) represent the Cartesian coordinates of the computation point of height hp and the corresponding coordinates of mass elements respectively.

The total TCR is given by the sum of the terrain effect from all prisms.

5.

$$\delta g^{T} = G\rho \int_{x=-x_0}^{x=x_0}\int_{y=-y_0}^{y=y_0}\int_{z=H_P}^{z=H(x,y)} \frac{z - H_P}{\left[\left(x_Q - x_P\right)^2 + \left(y_Q - y_P\right)^2 + \left(z_Q - H_P\right)^2\right]^{3/2}}\, dx_Q dy_Q dz_Q \qquad (2$$

gdzie:

- x_P, y_P – współrzędne płaskie stacji grawimetrycznej,
- H_P – wysokość stacji grawimetrycznej,
- x_Q, y_Q – współrzędne płaskie bieżącego elementu topografii (środka graniastosłupa),
- z_Q – wysokość bieżącego elementu topografii (graniastosłupa),
- dx_Q, dy_Q – kroki siatki modelu terenu, odpowiednio w kierunku północnym i wschodnim,
- dz_Q – różnica między wysokością stacji grawimetrycznej i wysokością bieżącego graniastosłupa tworzącego numeryczny model terenu,
- x_0, y_0 – współrzędne płaskie ograniczające obszar całkowania topografii terenu,
- ρ – gęstość mas tworzących topografię.

6.

4.1.1.1 Effect of the topographic and isostatic masses

The determination of the potential as well as its first and second derivatives by using the integration method based on rectangular prisms has been studied by many authors (e.g., Mader 1951; Nagy 1966; Forsberg 1984). Formulae for the calculation of the derivatives of the potential up to the third order are given by Mader (1951). Here we give only a short review of the gravitational potential, represented by rectangular prisms, as well as of its various derivatives. The gravitational potential of a rectangular prism of homogeneous mass-density ρ is described by Newton's integral (e.g., Heiskanen and Moritz 1967):

$$V_t\big|_{r=r_P} = G \iiint\limits_{v} \rho(Q) \frac{1}{l(\mathbf{r},\mathbf{r}_Q)}\bigg|_{r=r_P} dv_Q, \tag{4.1}$$

where $\rho(Q)$ is the density of the topographic masses or the density contrast of the topographic masses and the water masses of the oceans. Because of lack of more detailed knowledge, the density of the topographic masses is assumed to be constant (see, e.g., Vaníček and Kleusberg 1987; Featherstone 1992; Forsberg and Sideris 1993; Abd-Elmotaal 1999; Smith and Milbert 1999; Featherstone et al. 2001). However, the real density distribution can differ from this value by 10% or more (see, e.g., Martinec 1993; Tziavos el al. 1996; Pagiatakis and Armenakis 1998; Kuhn 2000a, b; Huang et al. 2001).

The point P in Eq.(4.1) is the computation point with the Cartesian coordinates x_P, y_P, z_P and Q (the center of the prism) represents the source point with the coordinates x', y' and z' referring to the same Cartesian coordinate system (Fig. 4.1). The quantity $l(\mathbf{r},\mathbf{r}_Q)$ is the distance between the computation point and the source point. It is given by the well-known formula,

$$l(\mathbf{r},\mathbf{r}_Q)\big|_{r=r_P} = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}\bigg|_{r=r_P}. \tag{4.2}$$

Insertion of Eq. (4.2) into Eq. (4.1) gives

$$V_t\big|_{r=r_P} = G \int\limits_{x1}^{x2}\int\limits_{y1}^{y2}\int\limits_{z1}^{z2} \rho(Q) \frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}\Bigg|_{r=r_P} dx'dy'dz'. \tag{4.3}$$

If the density of the prism for simplicity is assumed to be homogeneous and constant (ρ_{cr}), the integral of

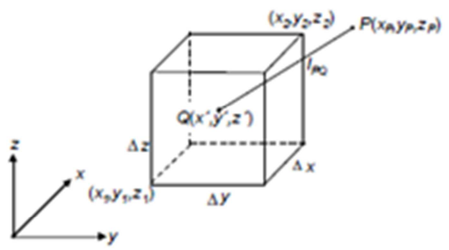

Fig. 4.1: Notation used for the definition of a prism

the Eq. (4.3) takes the form (Nagy 1966; Nagy et al. 2000):

$$\begin{aligned} V =& G\rho_{cr}\Bigg|\Bigg|\Bigg|(x-x')(y-y')\ln((z-z')+l(\mathbf{r},\mathbf{r}_Q))+(x-x')(z-z')\ln((y-y')+l(\mathbf{r},\mathbf{r}_Q))+ \\ &+(y-y')(z-z')\ln((x-x')+l(\mathbf{r},\mathbf{r}_Q))-\frac{(x-x')^2}{2}\arctan\frac{(y-y')(z-z')}{(x-x')l(\mathbf{r},\mathbf{r}_Q)}- \\ &-\frac{(y-y')^2}{2}\arctan\frac{(x-x')(z-z')}{(y-y')l(\mathbf{r},\mathbf{r}_Q)}-\frac{(z-z')^2}{2}\arctan\frac{(x-x')(y-y')}{(z-z')l(\mathbf{r},\mathbf{r}_Q)}\Bigg|_{r=r_P}\Bigg|_{x_1}^{x_2}\Bigg|_{y_1}^{y_2}\Bigg|_{z_1}^{z_2}. \end{aligned} \tag{4.4}$$

The first derivatives of the potential with respected to z are given by (Nagy 1966):

$$\frac{\partial V}{\partial z} = -G\rho_{cr}\int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2}\frac{(z-z')}{l(\mathbf{r},\mathbf{r}_Q)^{3/2}}\Bigg|_{r=r_P}dx'dy'dz', \tag{4.5}$$

$$\begin{aligned} \frac{\partial V}{\partial z} =& G\rho_{cr}\Bigg|\Bigg|\Bigg|(x-x')\ln((y-y')+l(\mathbf{r},\mathbf{r}_Q))+(y-y')\ln((x-x')+l(\mathbf{r},\mathbf{r}_Q)) \\ &-(z-z')\tan[(x-x')(y-y')/((z-z')l(\mathbf{r},\mathbf{r}_Q))]\Bigg|_{x_1}^{x_2}\Bigg|_{y_1}^{y_2}\Bigg|_{z_1}^{z_2}. \end{aligned} \tag{4.6}$$

The other two partial derivatives of the potential with respect to x, y can be obtained from Eq. (4.6) by cyclic permutation

$$\begin{aligned} \frac{\partial V}{\partial x} =& G\rho_{cr}\Bigg|\Bigg|\Bigg|(y-y')\ln((z-z')+l(\mathbf{r},\mathbf{r}_Q))+(z-z')\ln((y-y')+l(\mathbf{r},\mathbf{r}_Q)))- \\ &-(x-x')\tan[(y-y')(z-z')/((x-x')l(\mathbf{r},\mathbf{r}_Q))]\Bigg|_{x_1}^{x_2}\Bigg|_{y_1}^{y_2}\Bigg|_{z_1}^{z_2}, \end{aligned} \tag{4.7}$$

$$\begin{aligned} \frac{\partial V}{\partial y} =& G\rho_{cr}\Bigg|\Bigg|\Bigg|(z-z')\ln((x-x')+l(\mathbf{r},\mathbf{r}_Q))+(x-x')\ln((y-y')+l(\mathbf{r},\mathbf{r}_Q)))- \\ &-(y-y')\tan[(x-x')(z-z')/((y-y')l(\mathbf{r},\mathbf{r}_Q))]\Bigg|_{x_1}^{x_2}\Bigg|_{y_1}^{y_2}\Bigg|_{z_1}^{z_2}. \end{aligned} \tag{4.8}$$

The second derivatives of the potential are given by (ibid.)

$$\frac{\partial^2 V}{\partial x^2} = G\rho_{cr}\Bigg|\Bigg|\Bigg|-\arctan\frac{(y-y')(z-z')}{(x-x')l(\mathbf{r},\mathbf{r}_Q)}\Bigg|_{x_1}^{x_2}\Bigg|_{y_1}^{y_2}\Bigg|_{z_1}^{z_2}, \tag{4.9}$$

$$\frac{\partial^2 V}{\partial y^2} = G\rho_{cr}\Bigg|\Bigg|\Bigg|-\arctan\frac{(x-x')(z-z')}{(y-y')l(\mathbf{r},\mathbf{r}_Q)}\Bigg|_{x_1}^{x_2}\Bigg|_{y_1}^{y_2}\Bigg|_{z_1}^{z_2}, \tag{4.10}$$

$$\frac{\partial^2 V}{\partial z^2} = G\rho_{cr}\Bigg|\Bigg|\Bigg|-\arctan\frac{(x-x')(y-y')}{(z-z')l(\mathbf{r},\mathbf{r}_Q)}\Bigg|_{x_1}^{x_2}\Bigg|_{y_1}^{y_2}\Bigg|_{z_1}^{z_2}, \tag{4.11}$$

$$\frac{\partial^2 V}{\partial x\partial y} = G\rho_{cr}\Bigg|\Bigg|\Bigg|\ln((z-z')+l(\mathbf{r},\mathbf{r}_Q))\Bigg|_{x_1}^{x_2}\Bigg|_{y_1}^{y_2}\Bigg|_{z_1}^{z_2}, \tag{4.12}$$

$$\frac{\partial^2 V}{\partial x\partial z} = G\rho_{cr}\Bigg|\Bigg|\Bigg|\ln((y-y')+l(\mathbf{r},\mathbf{r}_Q))\Bigg|_{x_1}^{x_2}\Bigg|_{y_1}^{y_2}\Bigg|_{z_1}^{z_2}, \tag{4.13}$$

$$\frac{\partial^2 V}{\partial y\partial z} = G\rho_{cr}\Bigg|\Bigg|\Bigg|\ln((x-x')+l(\mathbf{r},\mathbf{r}_Q))\Bigg|_{x_1}^{x_2}\Bigg|_{y_1}^{y_2}\Bigg|_{z_1}^{z_2}. \tag{4.14}$$

From Eqs. (4.4) to (4.14), it is easy to realize that these formulae show singularities for special locations of P (e.g., if P coincides with a prism's corner or is situated on an edge of the prism); the respective limit values have been derived by Nagy et al. (2000, 2002). The above relations can be used for calculating the potential and its first and second derivatives of the topographic masses on or above the surface of the Earth. Since the prism matches the concept of the DTM, the TC software (Forsberg 1984) for prismatic topographic elements situated in an intermediate zone can be used after modifying it for calculating different topographic-isostatic models.

Due to the decrease of the gravitational effects with increasing distance, the complex formula (4.4) can be substituted by a simpler one for prisms which are located at a large distance from the computation point. This simpler formula can be derived based on a Taylor expansion of the integration kernel in Eq. (4.1). The Taylor expansion is fixed at the geometrical centre of the prism. The gravitational potential of the rectangular prism neglecting terms of order four and higher is given by MacMillan's (1930) formula (also see Anderson 1976; Forsberg 1984; Heck and Seitz 2006):

$$V = G\rho_{cr}\Delta x\Delta y\Delta z\left[\frac{1}{l_0}+\frac{3(x_0-x)^2-l_0^2}{24l_0^5}\Delta x^2+\frac{3(y_0-y)^2-l_0^2}{24l_0^5}\Delta y^2+\frac{3(z_0-z)^2-l_0^2}{24l_0^5}\Delta z^2+....\right], \tag{4.15}$$

where

$$\begin{aligned} x_o &= (x_1+x_2)/2, \\ y_o &= (y_1+y_2)/2, \\ z_o &= (z_1+z_2)/2. \end{aligned} \tag{4.16}$$

and l_0 denotes the Euclidean distance between the computation point P and the geometrical center (x_0, y_0, z_0) of the computation prism with $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$ and $\Delta z = z_2 - z_1$. The first and higher order derivatives of the potential can be simply found by differentiation of Eq. (4.15) with respect to x, y and z. Near the computation point (Eqs. 4.4 to 4.14), the flat-topped approximation of the topographic prism is assumed. This approximation produces some errors for the computation of the gravitational functions. A better approximation can be supplied by a prism topped by an inclined plane (Koch 1965) or by a bilinear surface (Smith 2000; Tsoulis et al. 2003). This procedure can be transformed to polyhedral bodies which can easily be determined by numerical methods (Talwani and Ewing 1960; Paul 1974; Petrović 1996).

7.

The terrain correction is computed by numerical integration. The figure shows the *prism method*, and how both prisms, I and II, lead to a positive correction, because prism I is computationally added and prism II removed when applying the terrain correction. One needs a digital terrain model, DTM, which must be, especially around the evaluation point, extremely dense: according to experience, 500 m is the maximum point density in a country like Finland, in the mountains one needs even 50 m. The systematic nature of the terrain correction makes a too sparse terrain model cause, possibly serious, *biases* in the insufficiently corrected gravity anomalies.

For computing the terrain correction with the prism method we use the following formula (assuming a constant density ρ, flat Earth) in rectangular map co-ordinates x, y:

$$TC(x,y) = \frac{1}{2} G\rho \int_{-D}^{+D} \int_{-D}^{+D} \left(h(x',y') - h(x,y)\right)^2 \ell^{-3} dx' dy',$$

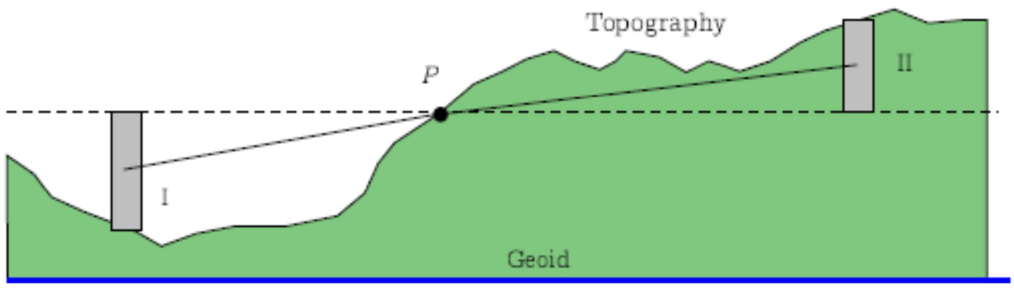

Figure 5.5 – Calculating the classical terrain correction by the prism method.

where

$$\ell = \sqrt{(x'-x)^2 + (y'-y)^2 + \left\{\frac{1}{2}\left(h(x',y') - h(x,y)\right)\right\}^2}$$

is the distance between the evaluation point $\begin{bmatrix} x & y & h(x,y) \end{bmatrix}^T$ and the centre point of the prism $\begin{bmatrix} x & y & \frac{1}{2}\left(h(x,y) + h(x',y')\right) \end{bmatrix}^T$. Of course this is only an approximation, but it works well enough in terrain where slopes generally do not exceed 45°. In the integral above, the limit D is typically tens or hundreds of kilometres. In the latter case, Earth curvature already starts having an effect, which the formula does not consider.