

Solution

B_i represent Belief after i th action and observation.

So according to conditions given in question :-

Roll number = 2019101080

$$\therefore x = 0.99$$

$$y = 1$$

Now let $P(O = \text{Red} \mid S = \text{Red}) = 0.98 = y_1$

and $P(O = \text{Green} \mid S = \text{Green}) = 0.8 = y_2$

$$B_{S_0} = \left[\frac{1}{3}, 0, \frac{1}{3}, 0, 0, \frac{1}{3} \right]$$

So If B_i is of form $[P(R), P(G)]$ then this means that he may be in a red state with probability $P(R)$ and in green with probability $P(G)$.

$$\text{So } B_0 = [1, 0]$$

(i) Action = Right and observation = Green.

$$P(S_1) = \frac{1}{3} (1-x) (1-y_1) = 0.000167$$

$$P(S_2) = \frac{1}{3} x x y_2 = 0.264$$

$$P(S_3) = \frac{1}{3} (1-x) (1-y_1) = 0.000167$$

$$P(S_4) = \frac{1}{3} x y_2 = 0.264$$

$$P(S_5) = 0$$

$$P(S_6) = \frac{1}{3} (1-y_1) = 0.016667$$

$$\text{Sum} = \sum P(S_i) = 0.545001$$

After normalizing :-

$$B_1 = BS_1 = \begin{bmatrix} 0.000306, 0.484403, 0.000306, \\ 0.484403, 0, 0.030582 \end{bmatrix}$$

$$B_1 = [0.031194, 0.968806]$$

(ii) Action = left and observation = red

$$\begin{aligned} P(s_1) &= BS_1(s_1) \cdot y_1 + BS_1(s_2) \cdot x \cdot y_1 \\ &= 0.455872 \end{aligned}$$

$$\begin{aligned} P(s_2) &= BS_1(s_2) \cdot (1-x) \cdot (1-y_2) + BS_1(s_3) \cdot x \cdot (1-y_2) \\ &= 0.001029 \end{aligned}$$

$$\begin{aligned} P(s_3) &= y_1 \times (BS_1(s_3) \cdot (1-x) + BS_1(s_4) \cdot x) \\ &= 0.455584 \end{aligned}$$

$$\begin{aligned} P(s_4) &= (1-y_2) (BS_1(s_4) \cdot (1-x) + BS_1(s_5) \cdot x) \\ &= 0.000969 \end{aligned}$$

$$\begin{aligned} P(s_5) &= (1-y_2) (BS_1(s_5) \cdot (1-x) + BS_1(s_6) \cdot x) \\ &= 0.006055 \end{aligned}$$

$$\begin{aligned} P(s_6) &= y_1 \times BS_1(s_6) \cdot (1-x) \\ &= 0.0002901 \end{aligned}$$

$$\text{Sum} = 0.9198$$

After normalizing :-

$$BS_2 = \begin{bmatrix} 0.495621, 0.001119, 0.495308, \\ 0.001053, 0.006582, 0.000316 \end{bmatrix}$$

$$B_2 = [0.991245, 0.008754]$$

(iii) Action = left and observation = Green.

$$P(S_1) = BS_2(S_1) \cdot (1-y_1) + BS_2(S_2)x \cdot (1-y_1) \\ = 0.024836$$

$$P(S_2) = (BS_2(S_2)(1-x) + BS_2(S_3)x) \cdot y_2 \\ = 0.392293$$

$$P(S_3) = (BS_2(S_3)(1-x) + BS_2(S_4)x) \cdot (1-y_1) \\ = 0.00029 \quad 0.0003$$

$$P(S_4) = (BS_2(S_4)(1-x) + BS_2(S_5)x) \cdot y_2 \\ = 0.005221$$

$$P(S_5) = (BS_2(S_5)(1-x) + BS_2(S_6)x) \cdot y_2 \\ = 0.000303$$

$$P(S_6) = BS_2(S_6)(1-x) \cdot (1-y_1) \\ = 0.000000$$

$$\text{Sum} = 0.422953$$

$$BS_2 = [0.024836, 0.392293, 0.0003, \\ 0.005221, 0.000303, 0.422]$$

$$BS_3 = [0.058720, 0.927510, 0.000709, \\ 0.012344, 0.000716, 0.000000]$$

$$B_3 = [0.059429, 0.94057]$$