Question 1. A Diagonal Rod. A rod of rest length L moves with speed v along the horizontal direction. In its rest frame, the rod makes an angle θ with respect to the x' axis. Determine the length of the rod as measured by a stationary observer and the angle θ' the rod makes with the x axis.

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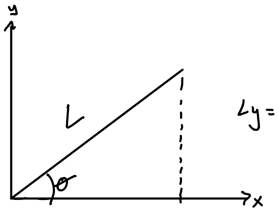


$L_y = L\sin\theta$

$L_x = L\cos\theta$

$\vec{v}$

→ Since the rod is moving along the x-direction, that means:

$$L'_y = L_y$$

but for L'_x, length is contracted by a factor of γ so:

$$L'_x = L_x/\gamma = L_x\sqrt{1-\beta^2}$$

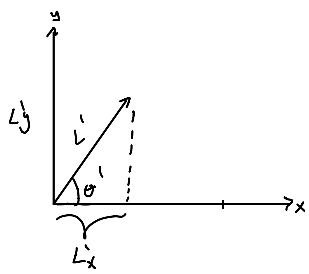
→ So, $L' = \sqrt{L'^2_x + L'^2_y}$ (Pythagorean theorem)

$$L' = \sqrt{(L\cos\theta)^2(1-\beta^2) + (L\sin\theta)^2}$$

$$L' = \sqrt{L^2\cos^2\theta(1-\beta^2) + L^2\sin^2\theta}$$

$$L' = \sqrt{L^2\cancel{\cos^2\theta} - L^2\cos^2\theta\beta^2 + L^2\cancel{\sin^2\theta}}$$

$$L' = \sqrt{L^2 - L^2\cos^2\beta^2}$$



now that we know L', we can say:

$$\tan\theta' = \frac{L'_y}{L'_x}$$

$$\tan\theta' = \frac{\cancel{L}\sin\theta}{\cancel{L}\cos\theta\sqrt{1-\beta^2}}$$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

$$\tan\theta' = \frac{\tan\theta}{\sqrt{1-\beta^2}}$$

$$\theta' = \tan^{-1}\left(\frac{\tan\theta}{\sqrt{1-\beta^2}}\right)$$

Question 2. A Common Approximation. Show that for particles moving at speed $c-\epsilon$, where $\epsilon \ll c$,

$$\gamma \approx \sqrt{\frac{c}{2\epsilon}}.$$

$$V = c - \epsilon$$

$$\gamma = \frac{1}{\sqrt{1-\frac{(c-\epsilon)^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{c^2-2c\epsilon+\epsilon^2}{c^2}}} = \frac{1}{\left(1-\frac{c^2-2c\epsilon+\epsilon^2}{c^2}\right)^{1/2}}$$

$$= \left(1-\left(\frac{c^2-2c\epsilon+\epsilon^2}{c^2}\right)\right)^{-1/2} = \left(\cancel{1}\cancel{-1} + \frac{2c\epsilon}{c^2} - \cancel{\frac{\epsilon^2}{c^2}}\right)^{-1/2}$$

$\epsilon <<< c$ so $\frac{\epsilon^2}{c^2}$ will cancel

$$= \frac{1}{\left(\frac{2c\epsilon}{c^2}\right)^{1/2}} = \boxed{\sqrt{\frac{c}{2\epsilon}}}.$$

Question 3. **A Simple Simultaneity Example.** A train 0.5 km long (as measured by an observer on the train) is traveling at a speed of 44 m/s. Two lightning bolts strike the ends of the train simultaneously as determined by an observer on the ground. What is the time separation as measured by an observer on the train?

$$\gamma = \frac{1}{\sqrt{1 - \frac{(44)^2}{(3.0 \times 10^8)^2}}}$$

$V = 44 \text{ m/s}$

$C = 3.0 \times 10^8 \text{ m/s}$

$L' = 500 \text{ m}$

$\gamma = 1.0000000018$

$L' = L/\gamma \quad : \quad L = L'/\gamma$

$$L = \frac{500}{1.0000000018} = 499.9999991 \text{ m} = .4999999991 \text{ km}$$

$$\Delta t = \frac{LV}{C^2}$$

$$(499.9999991) \cdot \frac{(44)}{(3.0 \times 10^8)^2}$$

$$= 2.44.. \times 10^{-13} \text{ s}$$