CIE IGCSE Maths: Core

Your notes

Types of Number

Contents

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Types of Number

You will come across vocabulary such as

- **Integers** and **natural** numbers
- **Rational** and **irrational** numbers
- **Multiples**
- **Factors**
- **Prime** numbers
- **Squares**, **cubes** and **roots**
- **Reciprocals**

Knowing what each of these terms mean is essential.

What are integers and natural numbers?

- **Integers** are **whole** numbers;
  - They can be **positive, negative** and **zero**
  - For example, -3, -2, -1, 0, 1, 2, 3 are all integers
- **Natural** numbers are the **positive integers**
  - They can be thought of as counting numbers
  - 1, 2, 3, 4, ... are the natural numbers
    - Notice that 0 is **not** included

What are rational and irrational numbers?

- A **rational** number is a number that can be written as a fraction in its simplest form
  - A rational number can be written in the form $\frac{a}{b}$
    - where a and b are both **integers,** and b is not zero
  - All terminating decimals are rational numbers
    - e.g. $0.32 = \frac{32}{100}$
  - All recurring decimals are rational numbers
    - e.g. $0.4444... = \frac{4}{9}$
    - Recurring digits are indicated by dots or bars
      e.g. $0.4\dot{5} = 0.4\overline{5} = 0.455\ 555\ 555\ ...$

e.g. $0.\dot{5}82\dot{4} = 0.\overline{5824} = 0.5824\ 5824\ 5824\ ...$

- An **irrational** number cannot be written as a fraction
  - All **non-terminating, non-recurring** decimals are irrational numbers
  - π (pi) is an irrational number
    - It has an endless amount of decimal places and there is **no pattern** or recurring digits
    - Any (simplified) fraction involving π, such as $\frac{\pi}{2}$, will also be irrational (as a in $\frac{a}{b}$ is not an integer)

Exam Tip

- In the calculator paper, use your calculator to your advantage!
  - To determine whether a number is rational or irrational, type it into your calculator and see if it can be displayed as a fraction (with integers)

Worked example

Explain why $0.\dot{3}49\dot{2}$ is a rational number.

The dots indicate the first and last of the recurring digits

$$0.\dot{3}49\dot{2} = 0.3492\ 3492\ 3492\ ...$$

$0.\dot{3}49\dot{2}$ is a rational number as it is a recurring decimal

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Multiples

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What are multiples?

- A **multiple** is a number which can be divided by another number, without leaving a remainder
  - For example, 12 is a multiple of 3
    - 12 divided by 3 is exactly 4
- Multiples can be considered as the numbers in a times table
- However multiples go beyond times tables and continue forever
  - For example, the multiples of 3 are 3, 6, 9, 12, 15, ..., 300, ..., 3000, ..., 34 567 896, ...
- Every non-zero number has an infinite number of multiples
- A **common multiple** is multiple that is shared by more than one number
  - For example, 12 is a common multiple of 4 and 6
- **Even** numbers (2, 4, 6, 8, 10, ...) are multiples of 2
- **Odd** numbers (1, 3, 5, 7, 9, ...) are **not** multiples of 2
- Multiples can be **algebraic**
  - For example, the multiples of k would be $k,\ 2k,\ 3k,\ 4k,\ 5k.\ ...$

How do I find the multiples of a number?

- Starting with a particular value, multiples can be listed by counting up in steps of that particular value
  - e.g. the multiples of 7 start with 7, then counting up in 7's will give 14, 21, 28, 35 and so on
- Multiples form a sequence
  - e.g. 7, 14, 21, 28, 35, ...
- Questions may ask you to state the multiples of a value between certain numbers
  - e.g. the multiples of 7 between 10 and 40 are 14, 21, 28 and 35

Worked example

a) List the first five multiples of 2.

$$2,\ 4,\ 6,\ 8,\ 10$$

b) List the multiples of 5 between 12 and 37.

$$15,\ 20,\ 25,\ 30,\ 35$$

Factors

What are factors?

- A **factor** of a given number is a value that divides the given number exactly, with **no remainder**
  - 6 is a factor of 18
    - because 18 divided by 6 is exactly 3
- Every integer greater than 1 has at least two factors
  - The integer itself, and 1
- A **common factor** is a factor that is shared by more than one number
  - For example, 3 is a common factor of both 21 and 18

How do I nd factors?

- Finding all the factors of a particular value can be done by finding factor pairs
- For example when finding the factors of 18
  - 1 and 18 will be the first factor pair
  - Divide by 2, 3, 4 and so on to test if they are factors
    - 18 ÷ 2 = 9, so 9 and 2 are factors
    - 18 ÷ 3 = 6, so 6 and 3 are factors
    - 18 ÷ 4 = 4.5, so 4 is not a factor
    - 18 ÷ 5 = 3.6, so 5 is not a factor
    - 18 ÷ 6 would be next, but we have already found that 6 was a factor
    - So we have now found all the factors of 18: 1, 2, 3, 6, 9

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How do I nd factors without a calculator?

- Use a **divisibility test**
  - Some tests easier to remember, and more useful, than others
- Once you know that the number has a particular factor, you can divide by that factor to find the factor pair
- Instead of a divisibility test, you could use a formal written method to divide by a value
  - If the result is an integer; you have found a factor

How do I test for divisibility by 2?

- A number is **divisible by 2** if the last digit is even (a multiple of 2)
  - 12**6**
    6 is even so 126 is divisible by 2
  - 13**5**
    5 is odd so 135 is not divisible by 5

How do I test for divisibility by 3?

- A number is **divisible by 3** if the sum of the digits is divisible by 3 (a multiple of 3)
  - 123
    $1 + 2 + 3 = 6$; 6 is a multiple of 3, so 123 is divisible by 3
  - 134
    $1 + 3 + 4 = 8$; 8 is not a multiple of 3, so 134 is not divisible by 3

How do I test for divisibility by 4 or 8?

- A number is **divisible by 4** if halving the number **twice** results in an integer
  - 128
    $128 \div 2 = 64$; $64 \div 2 = 32$; 32 is an integer so 128 is divisible by 4
  - 134
    $134 \div 2 = 67$; $67 \div 2 = 33.5$; 33.5 is not an integer so 134 is not divisible by 4
- A number is **divisible by 8** if it can be halved **3 times** and the result is an integer

How do I test for divisibility by 5 or 10?

- A number is **divisible by 5** if the last digit is a 0 or 5
  - 16**5**
    The last digit is 5; 165 is divisible by 5
  - 23**0**
    The last digit is 0; 230 is divisible by 5
  - 16**2**
    The last digit is 2; 162 is not divisible by 5
- A number is **divisible by 10** if the last digit is a 0

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What are some other divisibility tests?

- The following are harder divisibility tests
- You don't need to remember them, but they can speed up your working
- You could instead use a formal written method to carry out a division instead
- A useful fact is that if a value is divisible by two numbers, it is also divisible by the product of those two numbers
  - e.g. A number is **divisible by 6** if it is divisible by **both 2 and 3**
  - A number is **divisible by 12** if it is divisible by **both 4 and 3**
- A number is **divisible by 7** if you get a multiple of 7 when you double the last digit, and subtract it from the remaining part of the number
  - 245
    - Double 5 is 10
    - 24 - 10 = 14 which is a multiple of 7, so 245 is a multiple of 7
  - 906
    - Double 6 is 12
    - $90-12=78$ which is not a multiple of 7, so 906 is not a multiple of 7
- A number is **divisible by 9** if the sum of the digits is divisible by 9 (similar to the rule for 3)
- A number is **divisible by 11** if you get an answer of 0 or a multiple of 11 when you alternately add and subtract the digits
  - 1364
    - $+1-3+6-4=0$ so 1364 is a multiple of 11
  - 428
    - $+4-2+8=10$ so 428 is not a multiple of 11

Exam Tip

- On the calculator exam paper, use your calculator to test for divisibility
  - A factor pair will be found if the result of the calculation is an integer
- Being very familiar with times tables helps to reduce the need to use the divisibility tests

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Worked example

Find all the factors of 84.

The first factor pair will be 1 and the value itself

1	84

Test for divisibility by 2; the last digit of 84 is even; $84 \div 2 = 42$

2	42

Test for divisibility by 3, 4, 5, etc until either a value is reached that has already been found as a factor, or a factor pairs with itself

(Test for $\div 3$): $8 + 4 = 12$; 12 is a multiple of 3; 84 is a multiple of 3; $84 \div 3 = 28$

3	28

(Test for $\div 4$): $84 \div 2 = 42$; $42 \div 2 = 21$; 21 is an integer

4	21

(Test for $\div 5$): The last digit of 84 is neither 0 nor 5

(Test for $\div 6$): 84 is divisible by 2 and 3; $84 \div 6 = 14$

6	14

(Test for $\div 7$): List the multiples of 7 close to 84; 70, 77, **84**

7	12

(Test for $\div 8$): $84 \div 2 = 42$; $42 \div 2 = 21$; $21 \div 2 = 10.5$; 10.5 is not an integer

(Test for $\div 9$): List the multiples of 9 close to 84; 72, 81, 90

(Test for $\div 10$): The last digit of 84 is not 0

(Test for $\div 11$): List the multiples of 11 close to 84; 77, 88

The next number to test, 12, is already on the list, so the list is complete

Write the list of factors in order, being careful to not miss any out

The factors of 84 are 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42 and 84

This question could also be done by recalling times tables and using a formal written method for division, rather than divisibility tests

Prime Numbers

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What are prime numbers?

- A **prime** number is a number which has exactly two (distinct) **factors;** itself and 1
  - The first 10 prime numbers are
    - 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
    - You should remember at least the first ten prime numbers
- 1 is **not** a prime number, there are a few reasons for this such as
  - by definition, prime numbers are integers greater than or equal to 2
  - 1 only has one factor
- 2 is the only even prime number
- If a number has any factors other than itself and 1, it is not a prime number
    - For example, 27 is often mistaken for a prime number
      - but it is divisible by 3 and 9 (3 and 9 are a factor pair of 27)
      - so 27 is not a prime number

Worked example

Show that 51 is **not** a prime number.

If we can find a factor of 51 (that is not 1 or 51), this will prove it is not prime

51 is not even so is not divisible by 2
Next use the divisibility test for 3

$$5 + 1 = 6; \ \ 6 \text{ is divisible by 3; therefore 51 is divisible by 3}$$
$$51 \div 3 = 17$$

51 is not prime as it has more than two (distinct) factors

The factors of 51 are 1, 3, 17 and 51

Squares, Cubes & Roots

What are square numbers?

- A **square** number is the result of multiplying a number by itself
  - The first square number is $1 \times 1 = 1$, the second is $2 \times 2 = 4$ and so on
- The first 15 square numbers are: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225
  - Aim to remember at least the first fifteen square numbers
- In algebra, square numbers can be written using a power of 2
  - $a \times a = a^2$

What are cube numbers?

- A **cube** number is the result of multiplying a number by itself, twice
  - The first cube number is $1 \times 1 \times 1 = 1$, the second is $2 \times 2 \times 2 = 8$ and so on
- The first 5 cube numbers are 1, 8, 27, 64 and 125
  - Aim to remember at least the first five cube numbers
  - You should also remember $10^3 = 1000$
- In algebra, cube numbers can be written using a power of 3
  - $a \times a \times a = a^3$

What are square roots?

- The **square root** of a value, is the number that when multiplied by itself equals that value
  - For example, 4 is the square root of 16
  - It is the opposite of squaring
  - Square roots are indicated by the symbol $\sqrt{\ }$
    - e.g. The square root of 49 would be written as $\sqrt{49}$
  - Square roots can be positive and negative
    - e.g. The square roots of 25 are 5 and -5
  - If a negative square root is required then a - sign would be used
    - e.g. $\sqrt{25} = 5$ but $-\sqrt{25} = -5$
    - Sometimes both positive and negative square roots are of interest and would be indicated by $\pm\sqrt{25}$
- The square root of a non-square **integer** is also called a **surd**
  - e.g. $\sqrt{3}$ is a surd, as 3 is not a square number
  - surds are **irrational** numbers
    - where possible modern calculators will display **irrational** numbers as surds
  - $\sqrt{64}$ is **rational,** as it is equal to 8
    - 64 is a square number
  - However, $\sqrt{2}$ is **irrational**
    - 2 is not a square number
- You should aim to remember the square roots of the first 15 square numbers:
  - $\sqrt{1}, \sqrt{4}, \sqrt{9}, \sqrt{16}, \sqrt{25}, \sqrt{36}, \sqrt{49}, \sqrt{64}, \sqrt{81}, \sqrt{100}, \sqrt{121}, \sqrt{144}, \sqrt{169}, \sqrt{196}, \sqrt{225}$

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What are cube roots?

- The **cube root** of a value, is the number that when multiplied by itself twice equals that value
  - For example, 3 is the cube root of 27
  - It is the opposite of cubing
  - Cube roots are indicated by the symbol $\sqrt[3]{}$
    - e.g. The cube root of 64 would be written as $\sqrt[3]{64}$
  - You should remember the values of the following cube roots:
    - $\sqrt[3]{1}$, $\sqrt[3]{8}$, $\sqrt[3]{27}$, $\sqrt[3]{64}$, $\sqrt[3]{125}$, $\sqrt[3]{1000}$

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Worked example

Write down a number which is both a cube number and a square number, and hence express this number in two different ways using powers of 2 and 3.

Listing the first 12 square numbers

1, 4, 9, 16, 25, 36, 49, **64**, 81, 100, 121, 144

Listing the first 5 cube numbers

1, 8, 27, **64**, 125

64 appears in both lists, it is the 8th square number and 4th cube number

64 is both a square and cube number
$64 = 8^2$ and $64 = 4^3$

Reciprocals

What is a reciprocal?

- The **reciprocal** of a number is the result of **dividing 1 by that number**
  - Any number multiplied by its reciprocal will be equal to 1
- The reciprocal of 3 is $\frac{1}{3}$
  - The reciprocal of $\frac{1}{3}$ is 3
  - $3 \times \frac{1}{3} = \frac{1}{3} \times 3 = 1$
- The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$
  - The reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$
  - $\frac{2}{3} \times \frac{3}{2} = \frac{3}{2} \times \frac{2}{3} = 1$
- Algebraically the reciprocal of a is $\frac{1}{a}$
  - The reciprocal of $\frac{1}{a}$ is *a*
  - This can also be written using a power of -1
    - $\frac{1}{a} = a^{-1}$

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Worked example

Write down a fraction that completes this calculation: $\frac{3}{7} \times \frac{\cdots}{\cdots} = 1$

Recall that a number multiplied by its reciprocal is equal to 1

$$\frac{3}{7} \times \mathbf{\frac{7}{3}} = 1$$