Home Assignments
Digital Signal Processing

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12. FIR filters

a)

Given that:

$$H_b(\omega) = \frac{1}{4} + \frac{1}{2}\cos(\omega) \tag{1}$$

$$H_c(\omega) = \frac{1}{4} - \frac{1}{2}\cos(\omega) \tag{2}$$

$$\cos^2(x) = \frac{1 + \cos(x)}{2} \tag{3}$$

Follows:

$$
\begin{aligned}
H_{bc}(\omega) &= H_b(\omega)H_c(\omega) \\
&= \frac{1}{4} + \frac{1}{2}\cos(\omega) - \frac{1}{2}\cos(\omega) - \frac{1}{4}\cos^2(\omega) \\
&= \frac{1}{4} - \frac{1}{4}\cos^2(\omega) \\
&= \frac{1}{4} - \frac{1}{4}\frac{1 + \cos(2\omega)}{2} \\
&= \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos(2\omega) \\
&= \frac{1}{8} - \frac{1}{8}\cos(2\omega) \\
&= \frac{1}{8}\left(1 - \cos(2\omega)\right)
\end{aligned}
$$

b)

Given that:

$$H_{bc}(\omega) = \frac{1}{8} - \frac{1}{8}\cos(2\omega) \tag{4}$$

$$\cos(x) = \frac{e^x + e^{-x}}{2} \tag{5}$$

Follows:

$$
\begin{aligned}
\frac{1}{8} - \frac{1}{8}\cos(2\omega) &= \frac{1}{8} - \frac{1}{8}\frac{e^{2i\omega} + e^{-2i\omega}}{2} \\
&= -\frac{1}{16}e^{2i\omega} + \frac{1}{8}e^{0i\omega} - \frac{1}{16}e^{-2i\omega} \\
&\Rightarrow \left\{-\frac{1}{16},\ 0,\ \frac{1}{8},\ 0,\ -\frac{1}{16}\right\}
\end{aligned}
$$