

Home Assignments

Digital Signal Processing

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12. Multiplying frequency responses

a) Amplitude Plot

Given that:

$$H_b(\omega) = \frac{1}{4} + \frac{1}{2} \cos(\omega) \quad (1)$$

$$H_c(\omega) = \frac{1}{4} - \frac{1}{2} \cos(\omega) \quad (2)$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2} \quad (3)$$

Follows:

$$\begin{aligned} H_{bc}(\omega) &= H_b(\omega)H_c(\omega) \\ &= \frac{1}{4} + \frac{1}{2} \cos(\omega) - \frac{1}{2} \cos(\omega) - \frac{1}{4} \cos^2(\omega) \\ &= \frac{1}{4} - \frac{1}{4} \cos^2(\omega) \\ &= \frac{1}{4} - \frac{1}{4} \frac{1 + \cos(2\omega)}{2} \\ &= \frac{1}{4} - \frac{1}{8} - \frac{1}{8} \cos(2\omega) \\ &= \frac{1}{8} - \frac{1}{8} \cos(2\omega) \\ &= \frac{1}{8} (1 - \cos(2\omega)) \end{aligned}$$

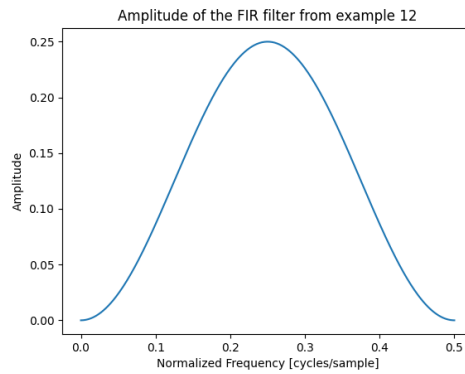


Figure 1: Amplitude for the given FIR filters.

b) Time Domain Coefficients

Given that:

$$H_{bc}(\omega) = \frac{1}{8} - \frac{1}{8} \cos(2\omega) \quad (4)$$

$$\cos(x) = \frac{e^x + e^{-x}}{2} \quad (5)$$

Follows:

$$\begin{aligned} \frac{1}{8} - \frac{1}{8} \cos(2\omega) &= \frac{1}{8} - \frac{1}{8} \frac{e^{2i\omega} + e^{-2i\omega}}{2} \\ &= -\frac{1}{16} e^{2i\omega} + \frac{1}{8} e^{0i\omega} - \frac{1}{16} e^{-2i\omega} \\ &\Rightarrow \left\{ -\frac{1}{16}, 0, \frac{1}{8}, 0, -\frac{1}{16} \right\} \end{aligned}$$

12. IIR filter

Given that:

$$\{a_{-1}, a_0, a_1\} = \left\{ -\frac{1}{2}, 1, 0 \right\} \quad (6)$$

$$\{b_{-1}, b_0, b_1\} = \left\{ \frac{1}{8}, \frac{1}{4}, \frac{1}{8} \right\} \quad (7)$$

Follows:

$$\begin{aligned} H(\omega) &= \frac{Y(\omega)}{X(\omega)} \\ &= \frac{\frac{1}{8}e^{i\omega} + \frac{1}{4} + \frac{1}{8}e^{-i\omega}}{-\frac{1}{2}e^{i\omega} + 1} \\ &= \frac{\frac{1}{4}(1 + \cos(\omega))}{1 - \frac{1}{2}e^{i\omega}} = \frac{1 + \cos(\omega)}{4 - 2e^{i\omega}} \end{aligned}$$

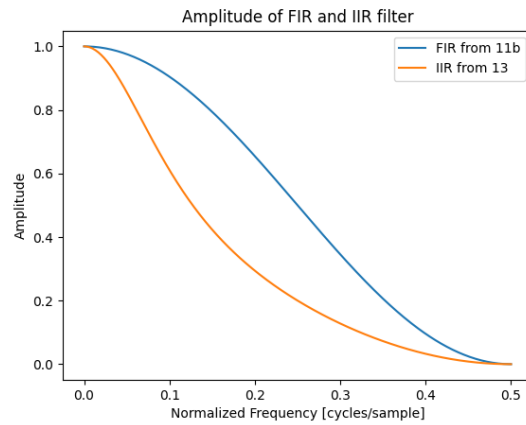


Figure 2: Amplitude comparison for the filters from example 11a and 13.