Home Assignments

Digital Signal Processing

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14. Laplace Transform

a)

$$f(t) = \begin{cases} 0 & t < 0 \\ e^{2t} - e^{-3t} & t \geq 0 \end{cases} \tag{1}$$

$$\begin{aligned} L\{f(t)\} &= \int_0^\infty \left(e^{2t} - e^{-3t}\right) e^{-st} dt \\ &= \int_0^\infty \left(e^{2t-st} - e^{-3t-st}\right) dt \\ &= \left[\frac{e^{(2-s)t}}{2-s} - \frac{e^{(-3-s)t}}{-3-s}\right]_0^\infty \\ &= \frac{e^{(2-s)\infty}}{2-s} - \frac{e^{(-3-s)\infty}}{-3-s} - \frac{e^{(2-s)0}}{2-s} + \frac{e^{(-3-s)0}}{-3-s} \\ &= -\frac{1}{2-s} + \frac{1}{-3-s} \end{aligned}$$

Converges if $s > 2$ and $s > -3$. Instable because ROC does not cross zero.

b)

$$f(t) = \begin{cases} -e^{2t} & t < 0 \\ -e^{-3t} & t \geq 0 \end{cases} \tag{2}$$

$$\begin{aligned} L\{f(t)\} &= \int_{-\infty}^0 -e^{2t} e^{-st} dt + \int_0^\infty -e^{-3t} e^{-st} dt \\ &= \int_{-\infty}^0 -e^{(2-s)t} dt - \int_0^\infty e^{(-3-s)t} dt \\ &= \left[-\frac{e^{(2-s)t}}{2-s}\right]_{-\infty}^0 - \left[\frac{e^{(-3-s)t}}{-3-s}\right]_0^\infty \\ &= -\frac{1}{2-s} + \frac{1}{-3-s} \end{aligned}$$

Converges if $s < 2$ and $s > -3$. Stable because ROC crosses zero.

c)

$$f(t) = \begin{cases} -e^{2t} + e^{-3t} & t < 0 \\ 0 & t \geq 0 \end{cases} \tag{3}$$

$$\begin{aligned} L\{f(t)\} &= \int_{-\infty}^0 \left(-e^{2t} + e^{-3t}\right) e^{-st} dt \\ &= \int_{-\infty}^0 -e^{(2-s)t} + e^{(-3-s)t} dt \\ &= \left[-\frac{e^{(2-s)t}}{2-s} + \frac{e^{(-3-s)t}}{-3-s}\right]_{-\infty}^0 \\ &= -\frac{1}{2-s} + \frac{1}{-3-s} \end{aligned}$$

Converges if $s < 2$ and $s < -3$. Instable because ROC does not cross zero.