Home Assignments

Digital Signal Processing

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12. Multipying frequency responses

a) Amplitude Plot

Given that:

$$H_b(\omega) = \frac{1}{4} + \frac{1}{2}\cos(\omega) \tag{1}$$
$$H_c(\omega) = \frac{1}{4} - \frac{1}{2}\cos(\omega) \tag{2}$$
$$\cos^2(x) = \frac{1+\cos(x)}{2} \tag{3}$$

Follows:

$$\begin{aligned}
H_{bc}(\omega) &= H_b(\omega)H_c(\omega) \\
&= \frac{1}{4} + \frac{1}{2}\cos(\omega) - \frac{1}{2}\cos(\omega) - \frac{1}{4}\cos^2(\omega) \\
&= \frac{1}{4} - \frac{1}{4}\cos^2(\omega) \\
&= \frac{1}{4} - \frac{1}{4}\frac{1+\cos(2\omega)}{2} \\
&= \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos(2\omega) \\
&= \frac{1}{8} - \frac{1}{8}\cos(2\omega) \\
&= \frac{1}{8}\left(1 - \cos(2\omega)\right)
\end{aligned}$$

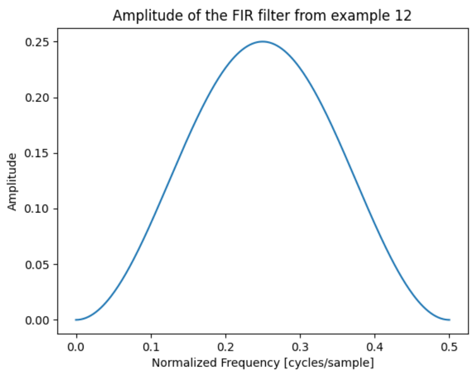

Figure 1: Amplitude for the given FIR filters.

b) Time Domain Coefficiants

Given that:

$$H_{bc}(\omega) = \frac{1}{8} - \frac{1}{8}\cos(2\omega) \tag{4}$$

$$\cos(x) = \frac{e^{x} + e^{-x}}{2} \tag{5}$$

Follows:

$$\begin{aligned}
\frac{1}{8} - \frac{1}{8}\cos(2\omega) &= \frac{1}{8} - \frac{1}{8}\frac{e^{2i\omega} + e^{-2i\omega}}{2} \\
&= -\frac{1}{16}e^{2i\omega} + \frac{1}{8}e^{0i\omega} - \frac{1}{16}e^{-2i\omega} \\
&\Rightarrow \left\{-\frac{1}{16},\ 0,\ \frac{1}{8},\ 0,\ -\frac{1}{16}\right\}
\end{aligned}$$

13. IRR filter

Given that:

$$\{a_{-1},\ a_0,\ a_1\} = \left\{-\frac{1}{2},\ 1,\ 0\right\} \tag{6}$$

$$\{b_{-1},\ b_0,\ b_1\} = \left\{\frac{1}{8},\ \frac{1}{4},\ \frac{1}{8}\right\} \tag{7}$$

Follows:

$$\begin{aligned}
H(\omega) &= \frac{Y(\omega)}{X(\omega)} \\
&= \frac{\frac{1}{8}e^{i\omega} + \frac{1}{4} + \frac{1}{8}e^{-i\omega}}{-\frac{1}{2}e^{i\omega} + 1} \\
&= \frac{\frac{1}{4}(1 + cos(\omega))}{1 - \frac{1}{2}e^{i\omega}} \\
&= \frac{1 + cos(\omega)}{4 - 2e^{i\omega}}
\end{aligned}$$

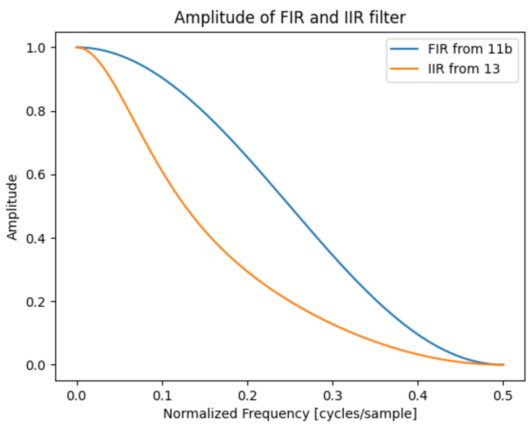

Figure 2: Amplitude comparison for the filters from example 11a and 13.