Complex Numbers

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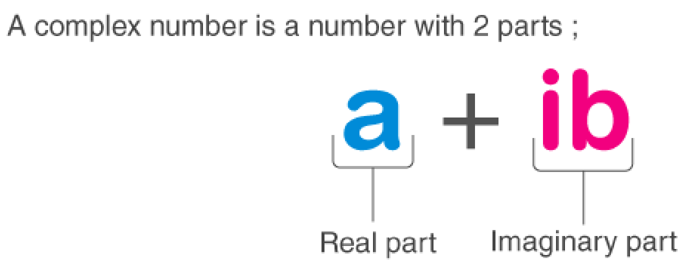
I will begin by introducing the concept of imaginary numbers and imaginary units

Complex numbers are the numbers that are expressed in the form of a+ib where, a,b are real numbers and 'i' is an imaginary number called "iota". The value of i = (√-1). For example, 2+3i is a complex number, where 2 is a real number (Re) and 3i is an imaginary number (Im).



I will apply this activity

Exploring the Complex Plane

- Provide graph paper or a coordinate plane template.

- Have students plot various complex numbers on the plane and identify their real and imaginary parts.

- Ask them to calculate the modulus and argument of each complex number and label them accordingly.

After that, you will go back with them what they studied the year before and ask them: What is the real number?

What Are Real Numbers?

Any number which is present in a number system such as positive, negative, zero, integer, rational, irrational, fractions, etc. are real numbers. It is represented as (Re). For example: 12, -45, 0, 1/7, 2.8, √5, etc., are all real numbers.

What Are Imaginary Numbers?

The numbers which are not real are imaginary numbers. When we square an imaginary number, it gives a negative result. It is represented as (Im). Example: √-2, √-7, √-11 are all imaginary numbers.

The complex numbers were introduced to solve the equation $x^2+1 = 0$. The roots of the equation are of form $x = \pm\sqrt{-1}$ and no real roots exist. Thus, with the introduction of complex numbers, we have Imaginary roots.

We denote √-1 with the symbol 'i', which denotes Iota (Imaginary number).

I start introducing them to the Notation that we will use in the lesson by having the students guess them and apply this activity

Complex Number Scavenger Hunt

- Hide cards around the classroom or school, each containing a complex number problem.
- Students work in teams to find the cards and solve the problems.
- Include a variety of problems covering basic operations, modulus, argument, and complex conjugates.

NOTATION

An equation of the form z= a+ib, where a and b are real numbers, is defined to be a complex number. The real part is denoted by Re z = a and the imaginary part is denoted by Im z = ib.

$$\boxed{Z = a + i\,b}$$

Combination of both the real number and imaginary number is a complex number.

I will give some simple examples to clarify

EXAMPLES OF COMPLEX NUMBERS:

- $1 + j$
- $-1^3 – 3i$
- $0.89 + 1.2\,i$
- $\sqrt{5} + \sqrt{2}i$

An imaginary number is usually represented by ‘i’ or ‘j’, which is equal to √-1. Therefore, the square of the imaginary number gives a negative value.

Since, $i = \sqrt{-1}$, so, $i^2 = -1$

- The main application of these numbers is to represent periodic motions such as water waves, alternating current, light waves, etc., which rely on sine or cosine waves, etc.

Is 0 a complex Number?

As we know, 0 is a real number. And real numbers are part of complex numbers. Therefore, 0 is also a complex number and can be represented as 0+0i.

Present an Argand diagram, which is a graphical representation of complex numbers in which the real part is plotted on the horizontal axis and the imaginary part on the vertical axis, and give students an example to solve on their own.
Then I will apply this activity

Visualizing Operations

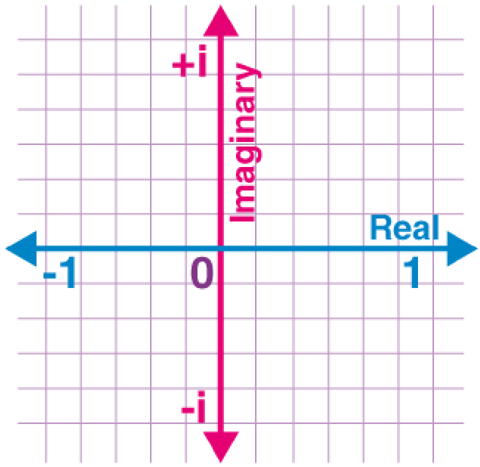
- Create flashcards with pairs of complex numbers.

- Ask students to perform addition, subtraction, multiplication, and division of the complex numbers on the flashcards.

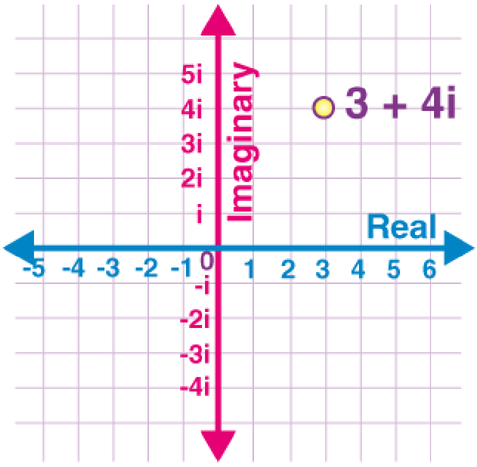
- Have them visualize the operations by drawing the complex numbers on the plane and using geometric methods to solve.

GRAPHICAL REPRESENTATION

In the graph below, check the representation of complex numbers along the axes. Here we can see, the x-axis represents real part and y represents the imaginary part.



Let us see an example here. If we have to plot a graph of complex number 3 + 4i, then:



The complex number 3+4i

ABSOLUTE VALUE

The absolute value of a real number is the number itself. The absolute value of x is represented by modulus, i.e. $|x|$. Hence, the modulus of any value always gives a positive value, such that;
$|3| = 3$
$|-3| = 3$

Now, in case of complex numbers, finding the modulus has a different method. Suppose, $z = x+iy$ is a complex number. Then, mod of z, will be:
$|z| = \sqrt{(x^2+y^2)}$

This expression is obtained when we apply the Pythagorean theorem in a complex plane. Hence, mod of complex number, z is extended from 0 to z and mod of real numbers x and y is extended from 0 to x and 0 to y respectively. Now these values form a right triangle, where 0 is the vertex of the acute angle. Now, applying Pythagoras theorem,
$|z|^2 = |x|^2+|y|^2$
$|z|^2 = x^2 + y^2$
$|z| = \sqrt{(x^2+y^2)}$

Teaching the basic operations of addition, subtraction, multiplication and division of complex numbers.

ALGEBRAIC OPERATIONS ON COMPLEX NUMBERS

There can be four types of algebraic operation on complex numbers which are mentioned below. Visit the linked article to know more about these algebraic operations along with solved examples. The four operations on the complex numbers include:

- Addition
- Subtraction
- Multiplication
- Division

ROOTS OF COMPLEX NUMBERS

When we solve a quadratic equation in the form of $ax^2 + bx + c = 0$, the roots of the equations can be determined in three forms;

- Two Distinct Real Roots
- Similar Root
- No Real roots (Complex Roots)

COMPLEX NUMBER FORMULAS

While performing the arithmetic operations of complex numbers such as addition and subtraction, combine similar terms. It means that combine the real number with the real number and imaginary number with the imaginary number.

ADDITION

$(a + ib) + (c + id) = (a + c) + i(b + d)$

SUBTRACTION

$(a + ib) – (c + id) = (a – c) + i(b – d)$

MULTIPLICATION

When two complex numbers are multiplied by each other, the multiplication process should be similar to the multiplication of two binomials. It means that the FOIL method (Distributive multiplication process) is used.

$(a + ib). (c + id) = (ac – bd) + i(ad + bc)$

DIVISION

The division of two complex numbers can be performed by multiplying the numerator and denominator by its conjugate value of the denominator, and then applying the FOIL Method.

$(a + ib) / (c + id) = (ac+bd)/ (c^2 + d^2) + i(bc – ad) / (c^2 + d^2)$

In this case I will give him som questions in a form of activity

Interactive Online Quizzes

- Utilize online platforms or apps that offer interactive quizzes on complex numbers.

- Include multiple-choice questions, matching exercises, and drag-and-drop activities to assess understanding of concepts.

- Provide immediate feedback to help students learn from their mistakes.

POWER OF IOTA (I)

Depending upon the power of "i", it can take the following values;

$i^{4k+1} = i.i^{4k+2} = -1\ i^{4k+3} = -i.i^{4k} = 1$

Where k can have an integral value (positive or negative).

Similarly, we can find for the negative power of i, which are as follows;

$i^{-1} = 1 / i$

Multiplying and dividing the above term with i, we have;

$i^{-1} = 1 / i \times i/i \times i^{-1} = i / i^2 = i / -1 = -i / -1 = -i$

Note: $\sqrt{-1} \times \sqrt{-1} = \sqrt{(-1 \times -1)} = \sqrt{1} = 1$ contradicts to the fact that $i^2 = -1$.

Therefore, for an imaginary number, $\sqrt{a} \times \sqrt{b}$ is not equal to $\sqrt{ab}$.

IDENTITIES

Let us see some of the identities.

1. $(z_1 + z_2)^2 = (z_1)^2 + (z_2)^2 + 2\ z_1 \times z_2$
2. $(z_1 – z_2)^2 = (z_1)^2 + (z_2)^2 – 2\ z_1 \times z_2$
3. $(z_1)^2 – (z_2)^2 = (z_1 + z_2)(z_1 – z_2)$
4. $(z_1 + z_2)^3 = (z_1)^3 + 3(z_1)^2\ z_2 + 3(z_2)^2\ z_1 + (z_2)^3$
5. $(z_1 – z_2)^3 = (z_1)^3 – 3(z_1)^2\ z_2 + 3(z_2)^2\ z_1 – (z_2)^3$

Properties

The properties of complex numbers are listed below:

- The addition of two conjugate complex numbers will result in a real number
- The multiplication of two conjugate complex number will also result in a real number
- If x and y are the real numbers and $x+yi = 0$, then $x = 0$ and $y = 0$
- If p, q, r, and s are the real numbers and $p+qi = r+si$, then $p = r$, and $q=s$
- The complex number obeys the commutative law of addition and multiplication.

  $z_1+z_2 = z_2+z_1$

  $z_1 . z_2 = z_2 . z_1$
- The complex number obeys the associative law of addition and multiplication.

  $(z_1+z_2) +z_3 = z_1 + (z_2+z_3)$

  $(z_1.z_2).z_3 = z_1.(z_2.z_3)$
- The complex number obeys the distributive law

  $z_1.(z_2+z_3) = z_1.z_2 + z_1.z_3$
- If the sum of two complex number is real, and also the product of two complex number is also real, then these complex numbers are conjugate to each other.
- For any two complex numbers, say z_1 and z_2, then $|z_1+z_2| \leq |z_1|+|z_2|$
- The result of the multiplication of two complex numbers and its conjugate value should result in a complex number and it should be a positive value.

I will give them some examples, and the rest of them will be Homework

SOLVED PROBLEMS

a) $16i + 10i(3-i)$

b) $(7i)\ (5i)$

c) $11i + 13i – 2i$

Solution:

a) $16i + 10i(3-i)$

$= 16i + 10i(3) + 10i\ (-i)$

$= 16i +30i – 10\ i^2$

$= 46\ i – 10\ (-1)$

$= 46i + 10$

b) and c) homework

finally, I will apply this activity

Group Discussions and Debates

- Divide the class into groups and assign each group a complex number-related topic, such as the significance of the imaginary unit or the geometric interpretation of complex numbers.

- Have groups discuss their assigned topics and prepare arguments to present to the class.

- Facilitate a debate where groups defend their positions and respond to counterarguments.