Explanation

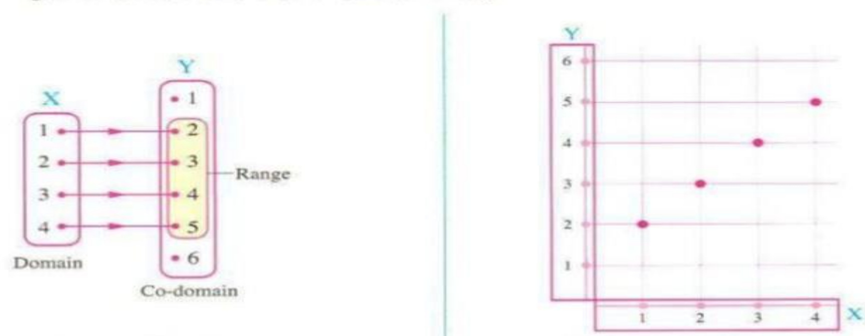
Real functions

In the beginning, we will start with an activity that is: Two people from each group present in the class come forward and stand in two rows, so that each person from a group does not have another person from the same group with him.

After forming the two rows, we use ropes so that each person from the first row is matched by a person in the second row, so that Each person in the first row can only hold one rope, but in the second row a person can hold one or more ropes. This is what happens in the ordered set of pairs. It is not possible for an element on the x-axis to be repeated more than once, but on the y-axis it can be repeated.

For example :

* If $X = \{1, 2, 3, 4\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and the function $f : X \longrightarrow Y$ where $f(X) = X + 1$, then the set of ordered pairs of the function $= \{(1, 2), (2, 3), (3, 4), (4, 5)\}$



Arrow diagram   Cartesian diagram

• Not each relation from X to Y is a function but all functions from X to Y are relations satisfy that:

- Each element in X appears once as a first projection in one of the ordered pairs of the relation.

Each element in X has only one arrow going out to an element of Y in the arrow diagram which represents the relation.

* The function $f : f(\mathcal{X}) = a_0 + a_1 \mathcal{X} + a_2 \mathcal{X}^2 + a_3 \mathcal{X}^3 + \cdots + a_n \mathcal{X}^n$ where :
  $a_0 , a_1 , a_2 , a_3 , \ldots , a_n$ are constants , $a_n \in \mathbb{R} - \{0\}$
  is called polynomial function of n^{th} degree and its domain and range are $\mathbb{R}$ if its not mention other than that.

* The function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(\mathcal{X}) = a \mathcal{X}^n$ where $a \in \mathbb{R}^*$, $n \in \mathbb{Z}^+$ is called power function , so at adding or subtracting power functions with constants , we get a polynomial function.

* Set of zeroes of polynomial function f is the set of values of $\mathcal{X}$ that make $f(\mathcal{X}) = 0$ and equals the set of $\mathcal{X}$-coordinates of the points of intersection of the curve of the function with $\mathcal{X}$-axis.

Activity 2

Drawing functions

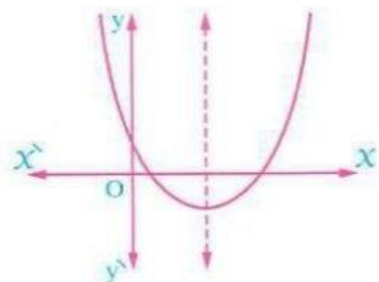
Using a phone or laptop, use programming or graphical tools to draw various real functions. You can design simple functions such as linear and quadratic functions, as well as more complex functions such as algebraic, translational, logarithmic, trigonometric, exponential, and more. You can analyze the characteristics and general behavior of the drawn functions, and each group will draw a different type of function and analyze what you understood from the drawing through the prior explanation. You will be using the site that works on the phone or laptop.

(https://www.desmos.com/calculator)

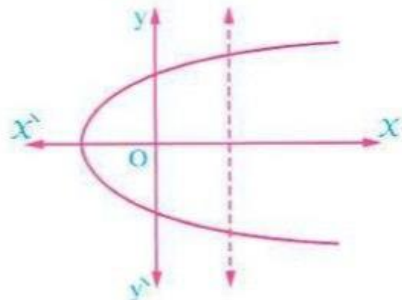
(1) Algebraically : The relation is a function if every value of the variable $x \in X$ is related with only one value of the variable $y \in Y$

(2) Graphically (The vertical line test) :

The relation is not a function if there exists at least one straight line parallel to y-axis and intersects the graph of the relation at more than one point.



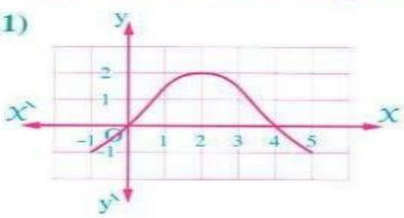
The graphical representation of the relation represents a Function from $X \longrightarrow Y$



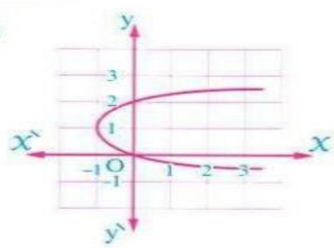
The graphical representation of the relation doesn't represent a Function from $X \longrightarrow Y$

Show which of the following graphs represents a function on $\mathbb{R}$, which doesn't represent a function giving reasons :

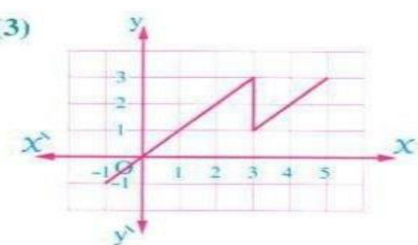
(1)



(2)



(3)



Solution

(1) Represents a function for each vertical line intersects the curve at one point at most.

(2) Does not represent a function for there are many vertical lines intersect the curve at two points.

(3) Does not represent a function for there is a vertical line passing through the point (3 , 0) and intersect the curve at a set of points.

Second Identifying the domain of the function from its rule

1 Polynomial function

The domain of the polynomial function is $\mathbb{R}$ unless it is defined on a subset of it.

For example : $f : f(X) = 3$ (Constant polynomial) , its domain $= \mathbb{R}$

, $f : f(X) = 2X + 1$ (First degree polynomial) , its domain $= \mathbb{R}$

, $f : f(X) = X^2 - 4X + 3$ (Second degree polynomial) , its domain $= \mathbb{R}$

2 Rational function

If f is a rational function where $f(X) = \frac{h(X)}{g(X)}$, h and g are two polynomials

, then the domain of the function $f = \mathbb{R}$ – the set of zeroes of the denominator.

State the domain of each of the real functions which are defined by th

(1) $f(X) = \sqrt{X + 2}$

(2) $f(X) = \sqrt{-2X + 3}$

Solution

(1) $\because$ The index of the root is an even number.

$\therefore$ The function is defined where $X + 2 \geq 0$

$\therefore X \geq -2$ $\qquad$ $\therefore$ The domain $= [-2, \infty[$

(2) $\because$ The index of the root is an even number.

$\therefore -2X + 3 \geq 0$ $\qquad$ $\therefore X \leq \frac{3}{2}$ $\qquad$ $\therefore$ The domain $=]-\infty, \frac{3}{2}]$

Determine the domain of each of the two functions defined by the following rules :

(1) $f(x) = \begin{cases} 2 - x & , \quad x < 0 \\ x - 2 & , \quad x > 0 \end{cases}$

(2) $f(x) = \begin{cases} x^2 & , \quad -2 \le x < 0 \\ x & , \quad 0 \le x \le 1 \\ \frac{1}{x} & , \quad x > 1 \end{cases}$

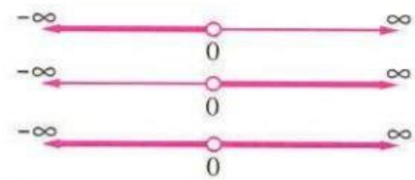
Solution

(1) The function f is defined on two intervals as the following :

Defined when $x \in]-\infty , 0[$

, defined when $x \in]0 , \infty[$

$\therefore$ Domain of $f =]-\infty , 0[\cup]0 , \infty[= \mathbb{R} - \{0\}$



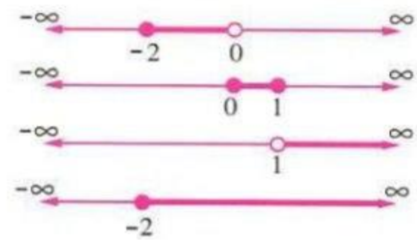
(2) The function f is defined on three intervals as the following :

Defined when $x \in [-2 , 0[$

, defined when $x \in [0 , 1]$

, defined when $x \in]1 , \infty[$

$\therefore$ Domain of $f = [-2 , 0[\cup [0 , 1] \cup]1 , \infty[= [-2 , \infty[$



Definition (2) ***(Decreasing function) :***

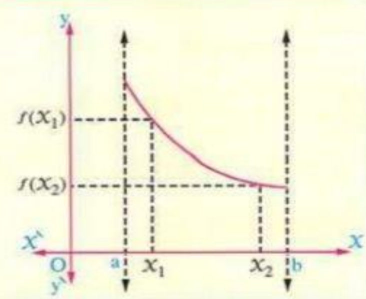
The function

f is said to be decreasing on

an interval $]a , b[$ if :

$x_2 > x_1 \Rightarrow f(x_2) < f(x_1)$ for every

$x_1 , x_2 \in]a , b[$



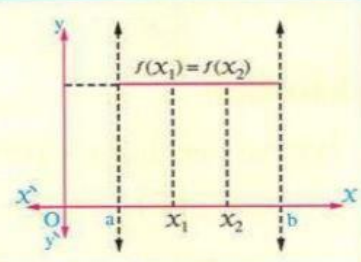
Definition (3) ***(Constant function) :***

The function

f is said to be constant on an interval $]a , b[$ if :

$x_2 > x_1 \Rightarrow f(x_2) = f(x_1)$ for every

$x_1 , x_2 \in]a , b[$



If f_1 , f_2 are two functions whose domains are D_1 and D_2 respectively , then :

(1) $(f_1 \pm f_2)(X) = f_1(X) \pm f_2(X)$ and the domain of $(f_1 \pm f_2)$ is $D_1 \cap D_2$

(2) $(f_1 \times f_2)(X) = f_1(X) \times f_2(X)$ and the domain of $(f_1 \times f_2)$ is $D_1 \cap D_2$

(3) $\left(\frac{f_1}{f_2}\right)(X) = \frac{f_1(X)}{f_2(X)}$ such that $f_2(X) \neq$ zero

, the domain of $\left(\frac{f_1}{f_2}\right)$ is $(D_1 \cap D_2) - Z(f_2)$ where $Z(f_2)$ is the set of zeroes of f_2

Noticing that in all the operations on the functions , the domain of the resulting function equals the intersection of the domains of the two functions except the zeroes of the divisor in the division operation.