

Image Quantization Analysis

Graph Construction

- Since we have a dense graph it is not feasible to construct the full weighted graph for images.
- We instead calculate the required weight in the making of MST.

Minimum Spanning Tree Code

- We chose Prim's algorithm in getting the MST since we don't have a constructed graph and also we have a dense graph.

To get the MST we must have a list of the **distinct colors**, we get it by the following function:

`GetDistinctColorsList(ImageMatrix)`

- Takes $O(L*W)$, where L is the length and W is the width of the image.

```
private static List<int> GetDistinctColorsList(ReadOnlySpan<byte> ImageMatrix) //O(L*W)
{
    HashSet<int> distinctColors = new HashSet<int>();

    foreach (ReadOnlySpan<byte> pixel in ImageMatrix) // O(L*W) if C# handles resizing well
        distinctColors.Add(BitConverter.ToInt32(pixel, 0));

    List<int> colorsList = distinctColors.ToList();
    return colorsList;
}
```

We get the MST using Prim's algorithm by the following function:

`Prim(distinctColorsList)`

- Takes $O(D^2)$ where D is number of distinct colors in the image.

```
public static int[] Prim(List<int> distinctColors) //O(D^2)
{
    int v = distinctColors.Count;
    int[] parent = new int[v];
    int[] key = new int[v];
    bool[] mstSet = new bool[v];
    //initialize all keys as infinite
    for (int i = 0; i < v; i++) //O(D)
    {
        key[i] = int.MaxValue;
        mstSet[i] = false;
    }
    key[0] = 0;
    parent[0] = -1;
    for (int i = 0; i < v - 1; i++) //O(D^2)
    {
        int u = GetMinimumKey(key, mstSet); //O(D)
```

```

        mstSet[u] = true;
        //relax all edges connected to u
        for (int v = 0; v < V; v++)          //O(D)
        {
            int distance = ColorQuantization.Getweight( //O(1)
                distinctColors[u],
                distinctColors[v]
            );
            if (distance != 0
                && mstSet[v] == false
                && distance < key[v]
            )
            {
                parent[v] = u;

                key[v] = distance;
            }
        }
    }
    return parent;
}

```

The function `GetMinimumKey()` is used in `Prim()` :

- Takes **O(D)**, *where D is the number of distinct colors in the image.*

```

private static int GetMinimumKey(int[] key, bool[] mstSet) //O(D)
{
    int min = int.MaxValue;
    int min_index = -1;
    for (int v = 0; v < key.Length; v++)          //O(D)
    {
        if (mstSet[v] == false && key[v] < min)
        {
            min = key[v];
            min_index = v;
        }
    }
    return min_index;
}

```

Palette Generation Code

We use the function `GetColorPalette` to get and generate the palette, this function take a list of clusters `List<List<RGBPixel>> clusters` and return a list that each index `i` is an index of a cluster and the value of that index is the representative color for the whole cluster `i`.

`GetColorPalette(clusters)`

- Takes **O(D)**, where D is the number of distinct colors in the image.

```

public static List<RGBPixel> GetColorPalette(List<List<RGBPixel>> clusters)
//O(D)
{
    // for every member of cluster sum all values and get the mean for the sum
    List<RGBPixel> colorPallete = new List<RGBPixel>();
}

```

```

for (int clusterIndex = 0; clusterIndex < clusters.Count; clusterIndex++)
//this will loop over all nodes in all clusters,
//since number of nodes is D then this whole loop takes
//O(D)
{
    int sumRed = 0, sumGreen = 0, sumBlue = 0;
    int numberOfColorsInCluster = clusters[clusterIndex].Count;

    foreach (RGBPixel pixel in clusters[clusterIndex])
    {
        sumRed += pixel.red;
        sumBlue += pixel.blue;
        sumGreen += pixel.green;
    }

    sumRed = (int)Math.Ceiling((double)sumRed / numberOfColorsInCluster);
    sumGreen = (int)Math.Ceiling((double)sumGreen /
numberOfColorsInCluster);
    sumBlue = (int)Math.Ceiling((double)sumBlue / numberOfColorsInCluster);

    byte red = Convert.ToByte(sumRed);
    byte green = Convert.ToByte(sumGreen);
    byte blue = Convert.ToByte(sumBlue);

    RGBPixel representitaveColor = new RGBPixel(red, green, blue);
    colorPallete.Add(representitaveColor);
}
return colorPallete;
}

```