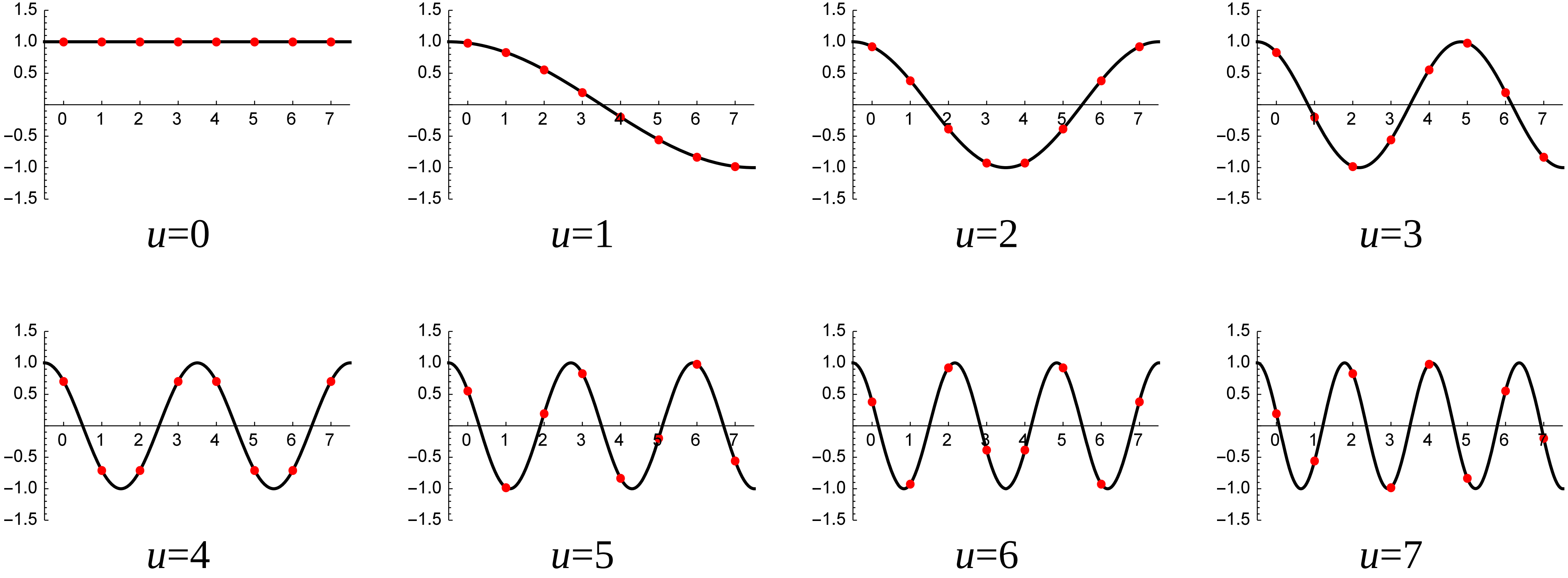
1D DCT & IDCT

$$C(u)=\alpha(u)\sum_{x=0}^{N-1} f(x)\cos\frac{(2x+1)u\pi}{2N}$$

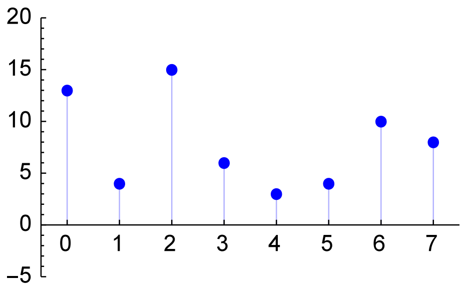
$$f(x)=\sum_{u=0}^{N-1}\alpha(u)C(u)\cos\frac{(2x+1)u\pi}{2N}$$

$$\alpha(u)=\begin{cases}\sqrt{1/N} & \text{for} \quad u=0\\ \sqrt{2/N} & \text{for} \quad u=1,2,\ldots,N-1\end{cases}$$

1D DCT



1D DCT



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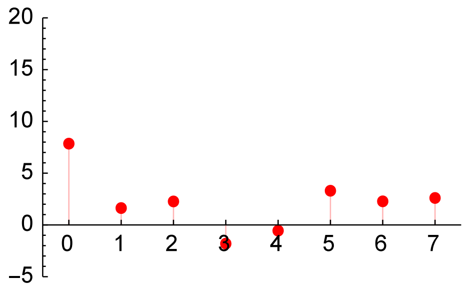
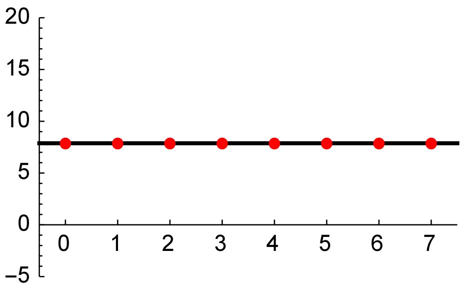


αC

1D DCT



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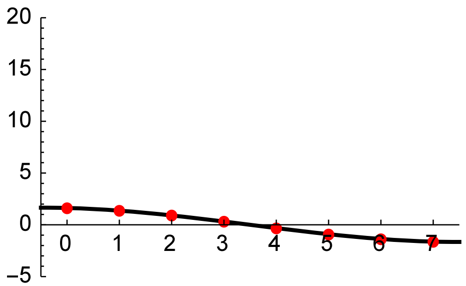

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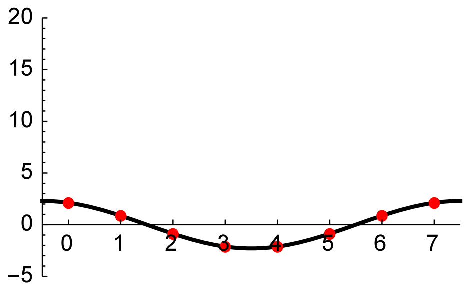

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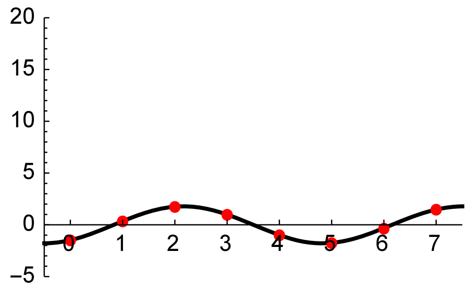

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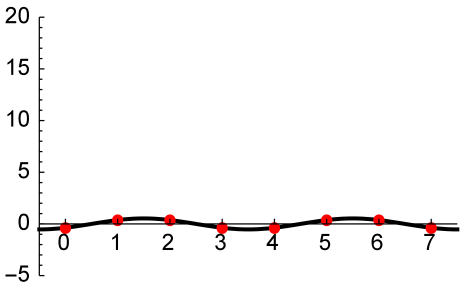

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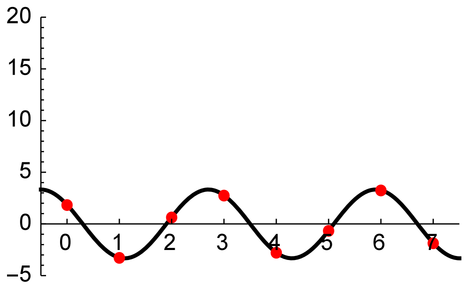

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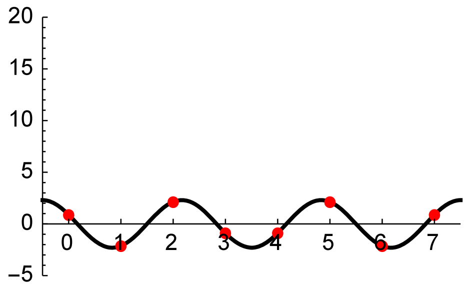

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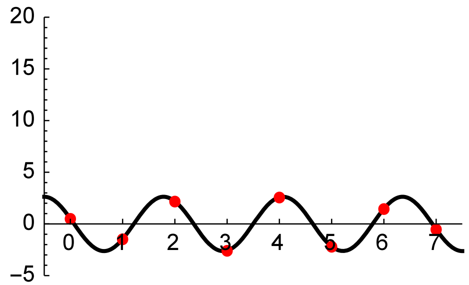

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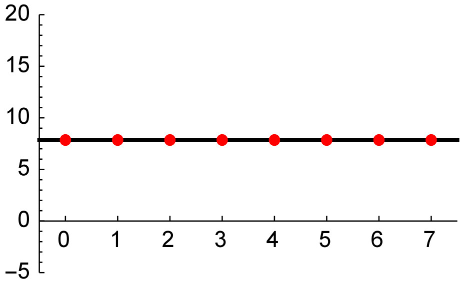
1D IDCT



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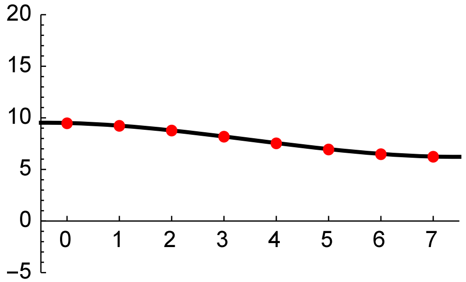

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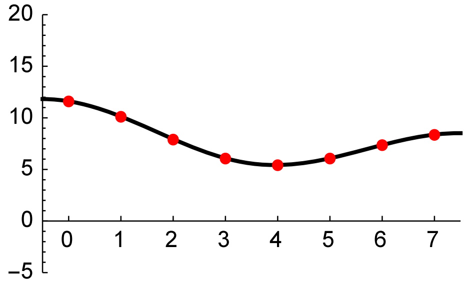

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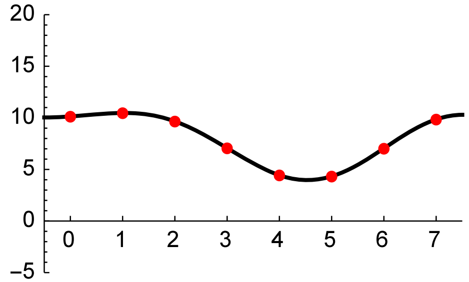

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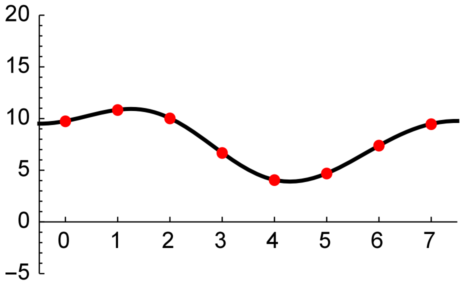

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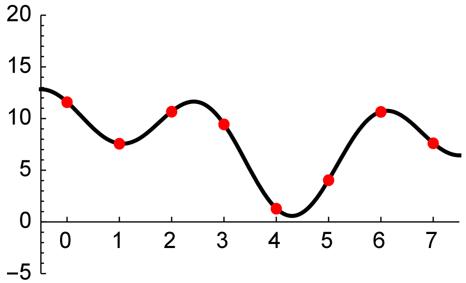

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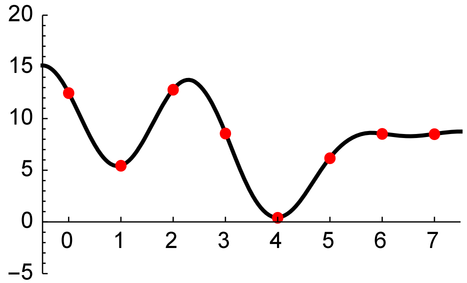

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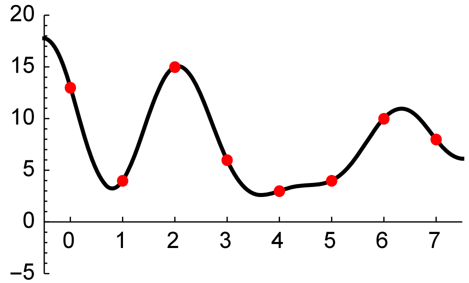

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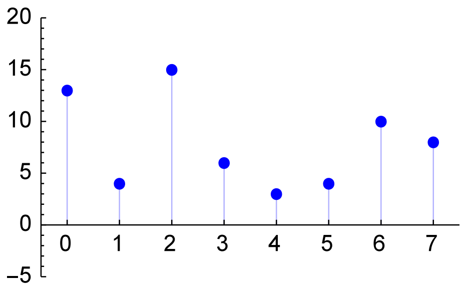
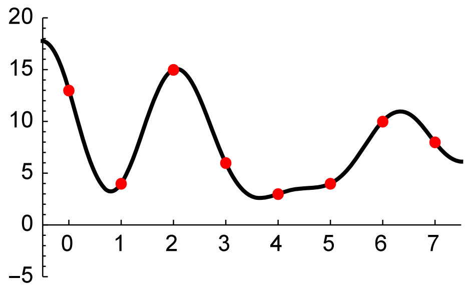
1D DCT



input



ricostruzione

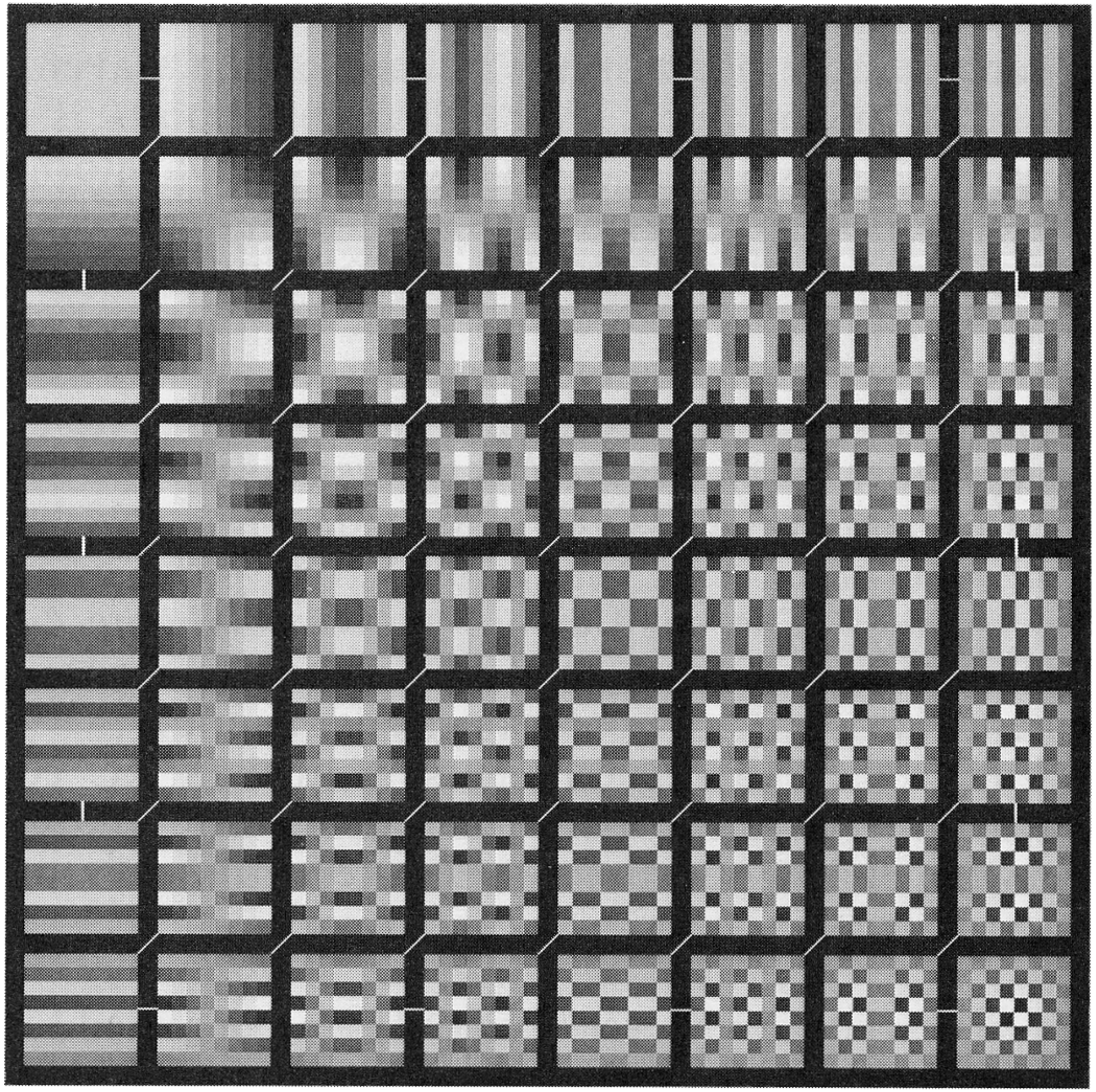
2D DCT & IDCT

$$C(u,v)=\alpha(u)\alpha(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\cos\frac{(2x+1)u\pi}{2N}\cos\frac{(2y+1)v\pi}{2N}$$

$$f(x,y)=\sum_{u=0}^{N-1}\sum_{v=0}^{N-1}\alpha(u)\alpha(v)C(u,v)\cos\frac{(2x+1)u\pi}{2N}\cos\frac{(2y+1)v\pi}{2N}$$

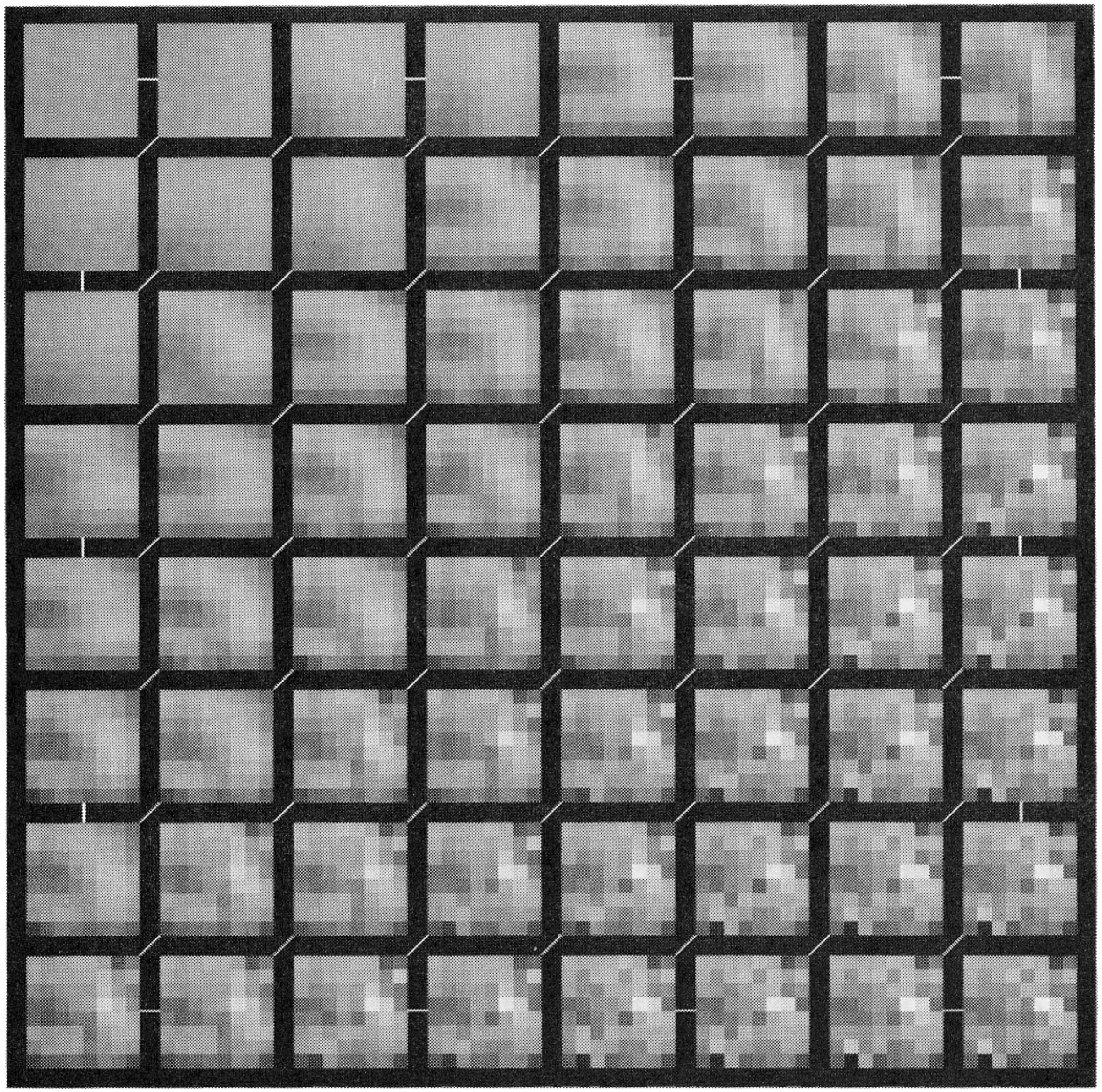
$$\alpha(u)=\begin{cases}\sqrt{1/N} & \text{for} \quad u=0\\ \sqrt{2/N} & \text{for} \quad u=1,2,\ldots,N-1\end{cases}$$

2D DCT



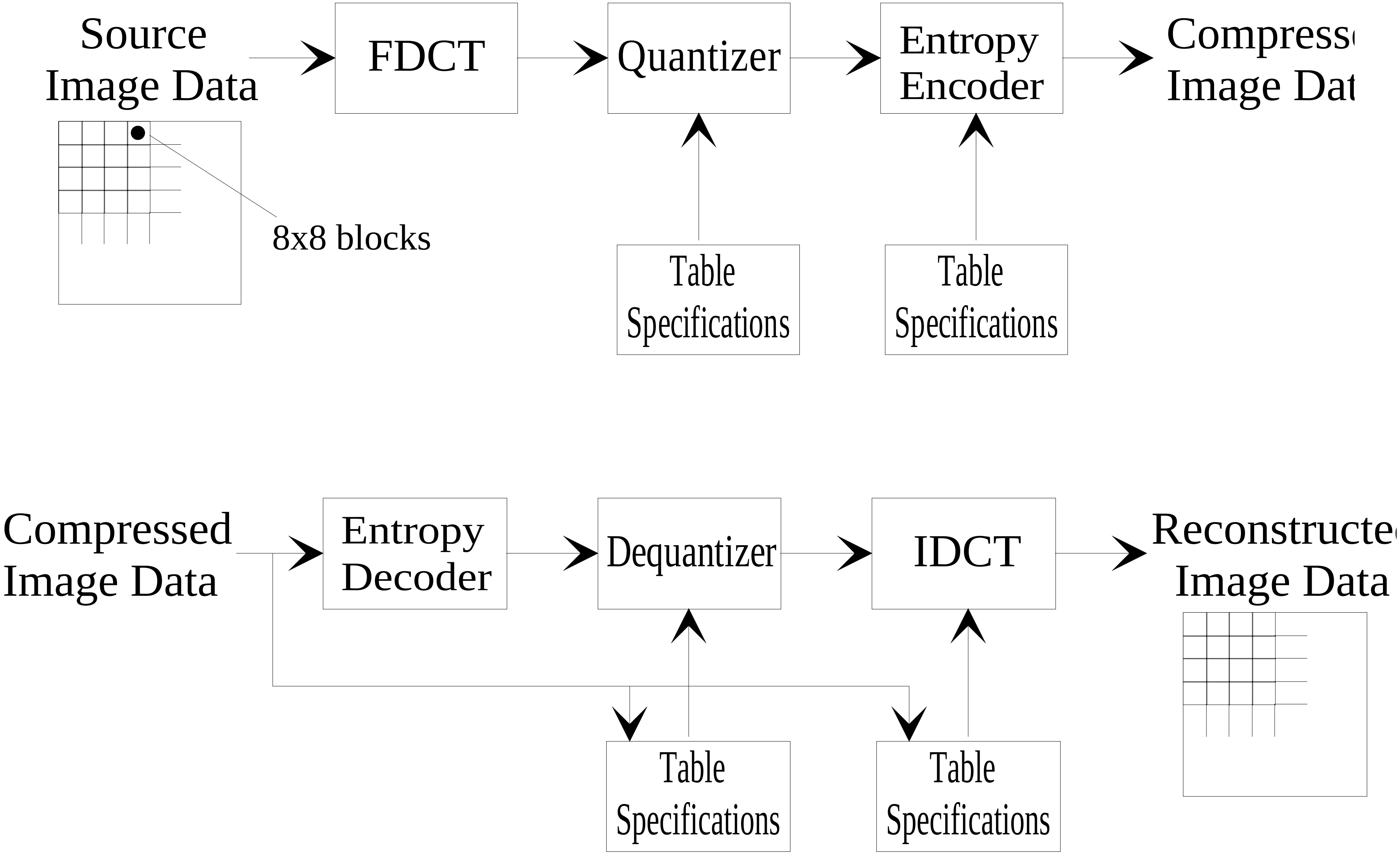
DCT basis

2D DCT



DCT reconstruction

JPG (Encoding & Decoding)



JPG

luminance

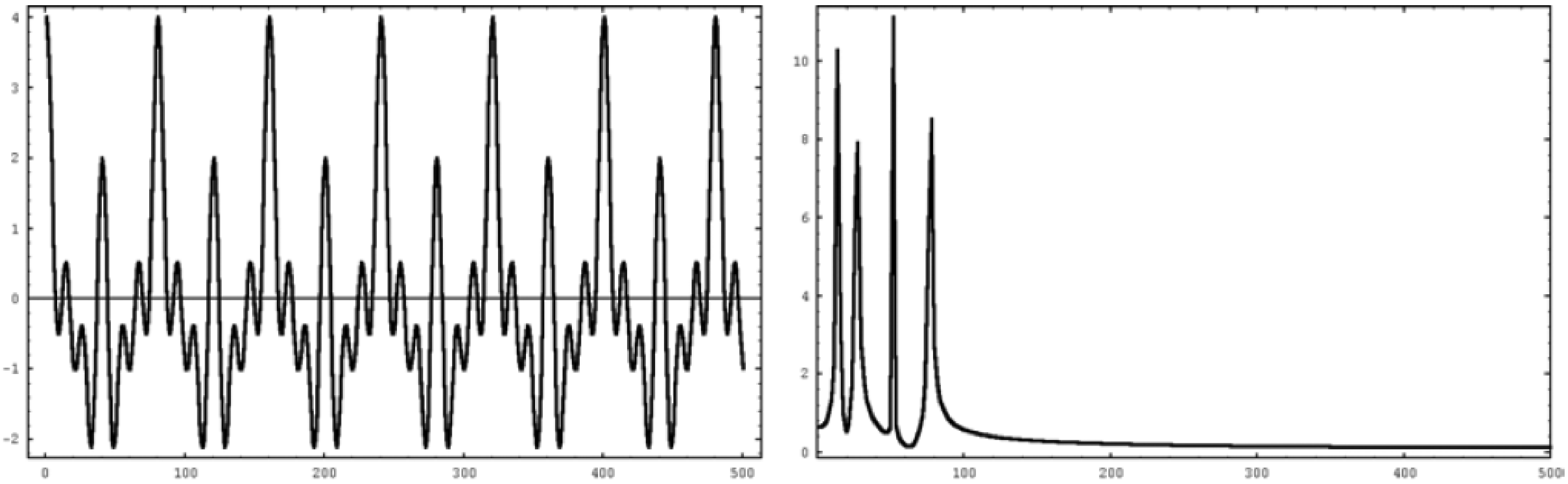
16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

chrominance

17	18	24	47	99	99	99	99
18	21	26	66	99	99	99	99
24	26	56	99	99	99	99	99
47	66	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99

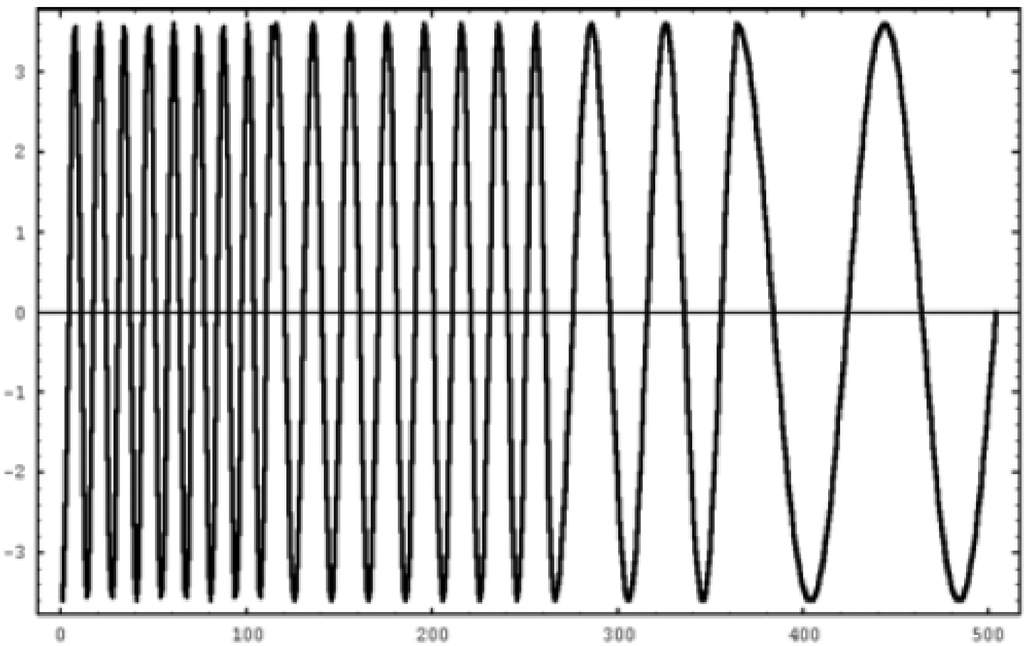
Standard quantization tables

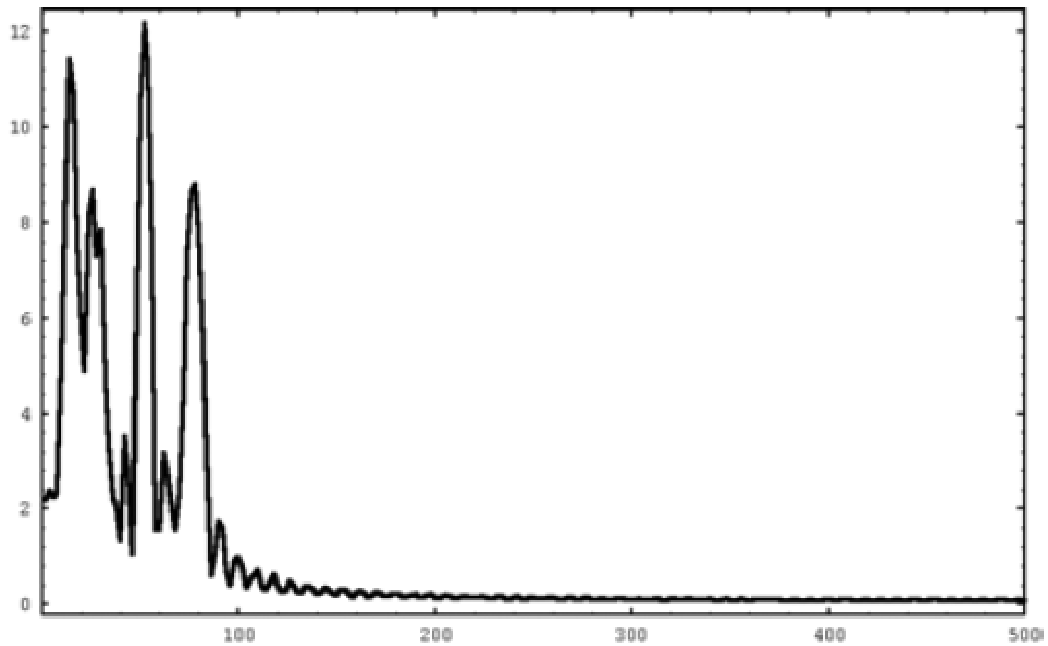
DCT



Un segnale stazionario e la sua trasformata di Fourier

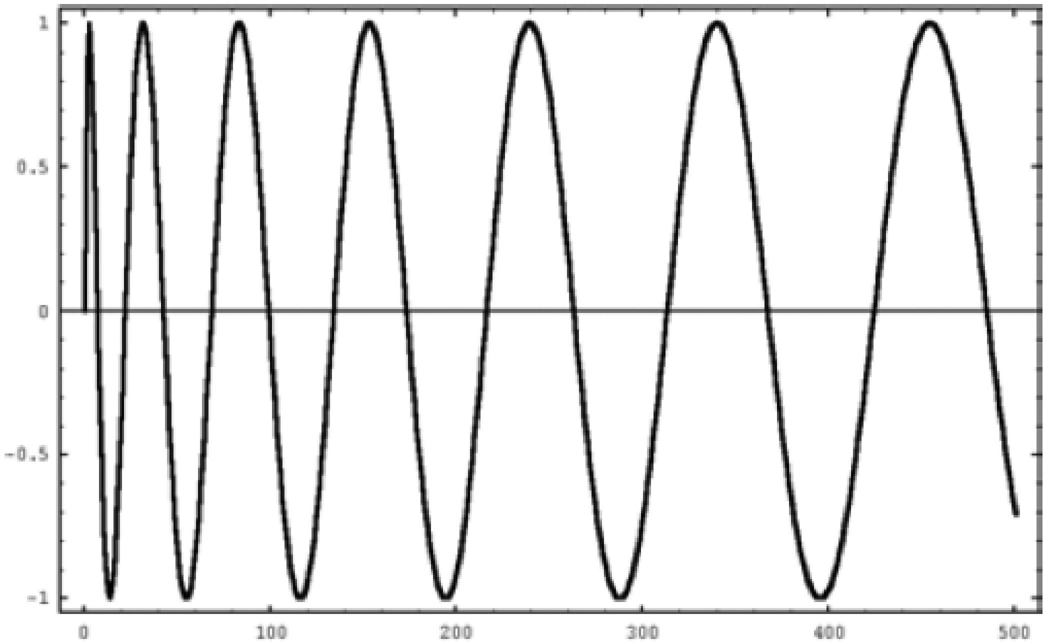
DCT

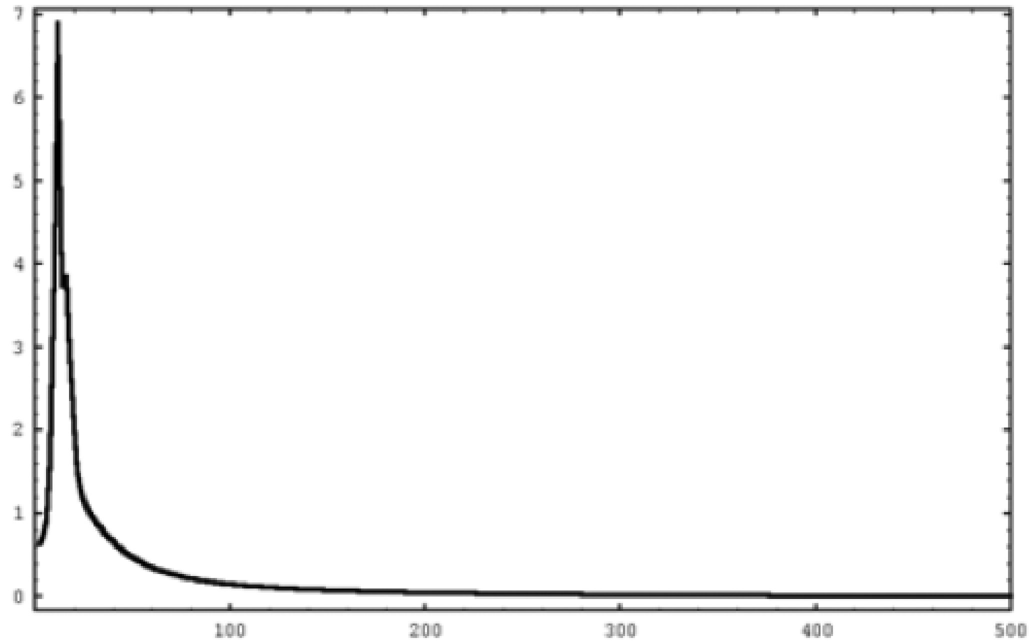




Un segnale composto e la sua trasformata di Fourier

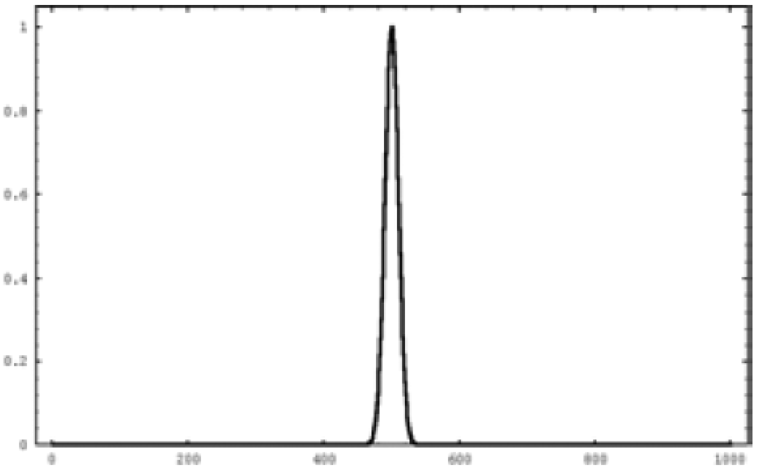
DCT



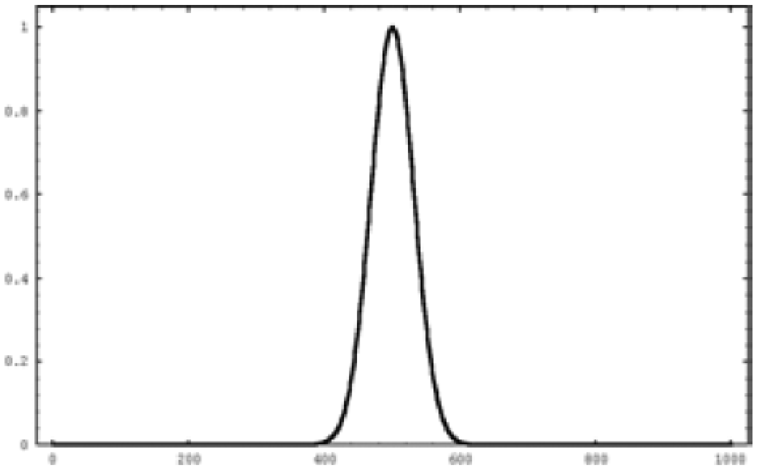


Un segnale non stazionario e la sua trasformata di Fourier

STFT



a=0.01



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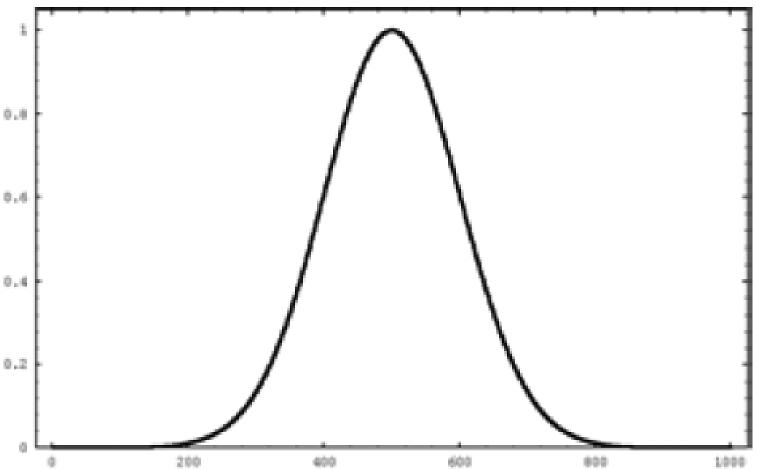

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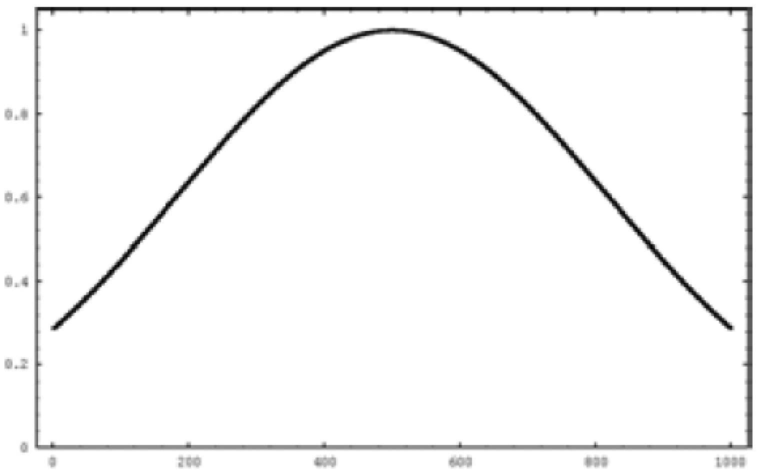

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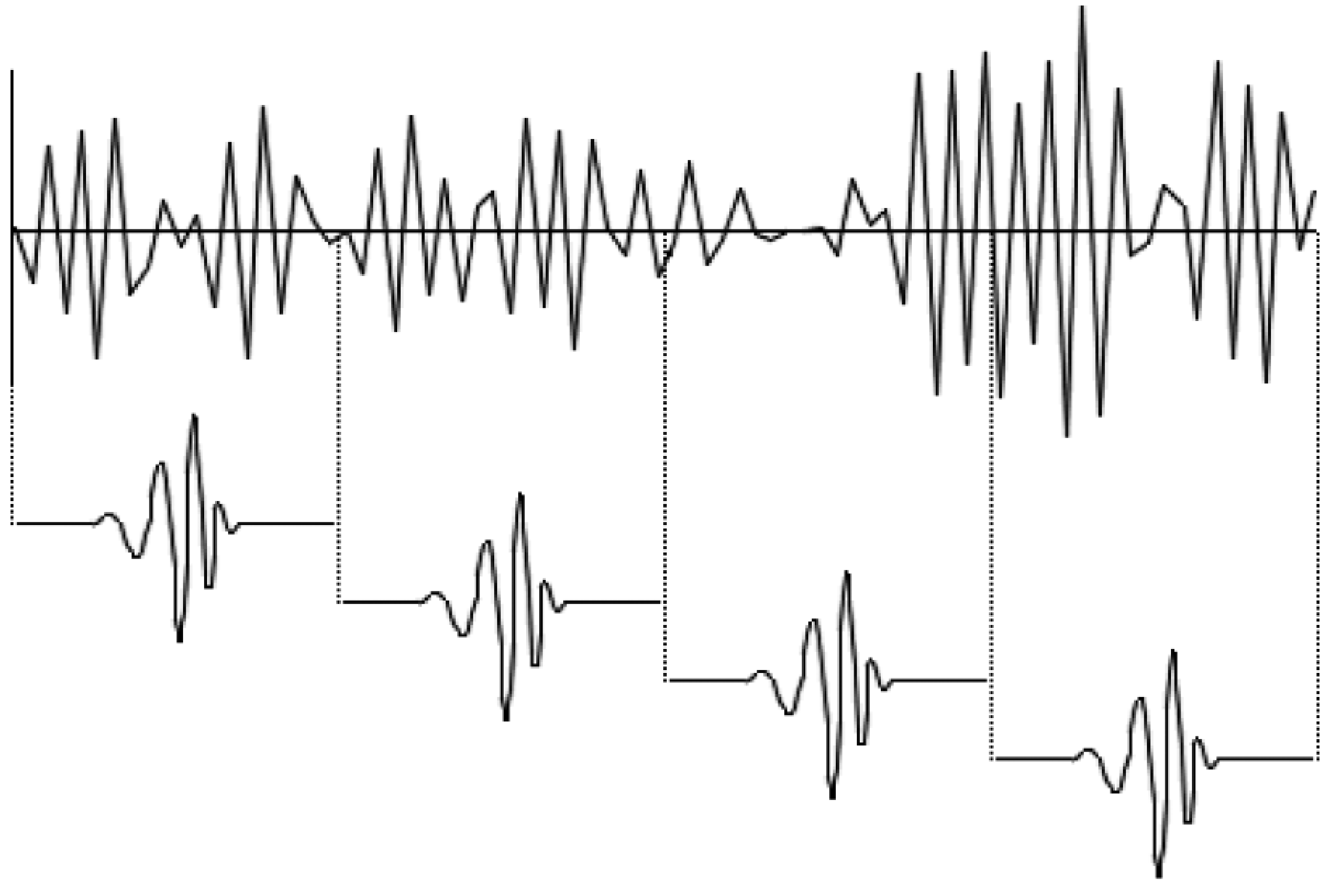
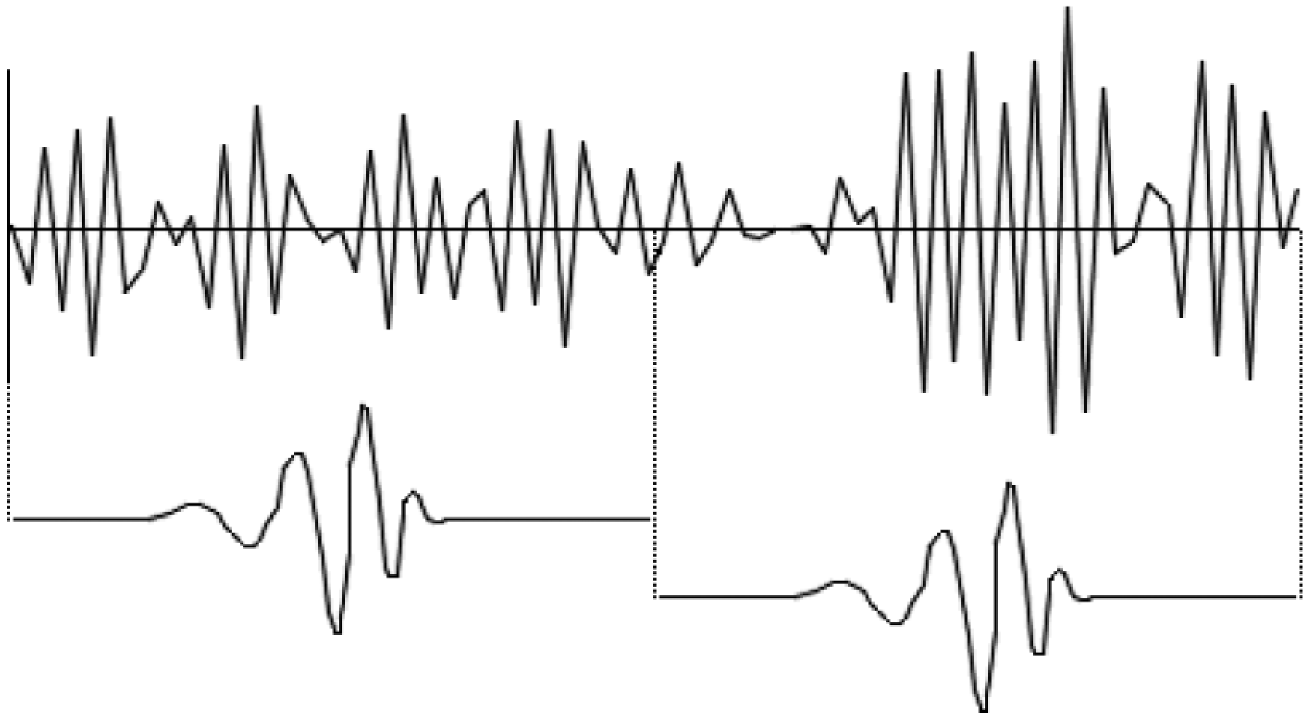
DWT



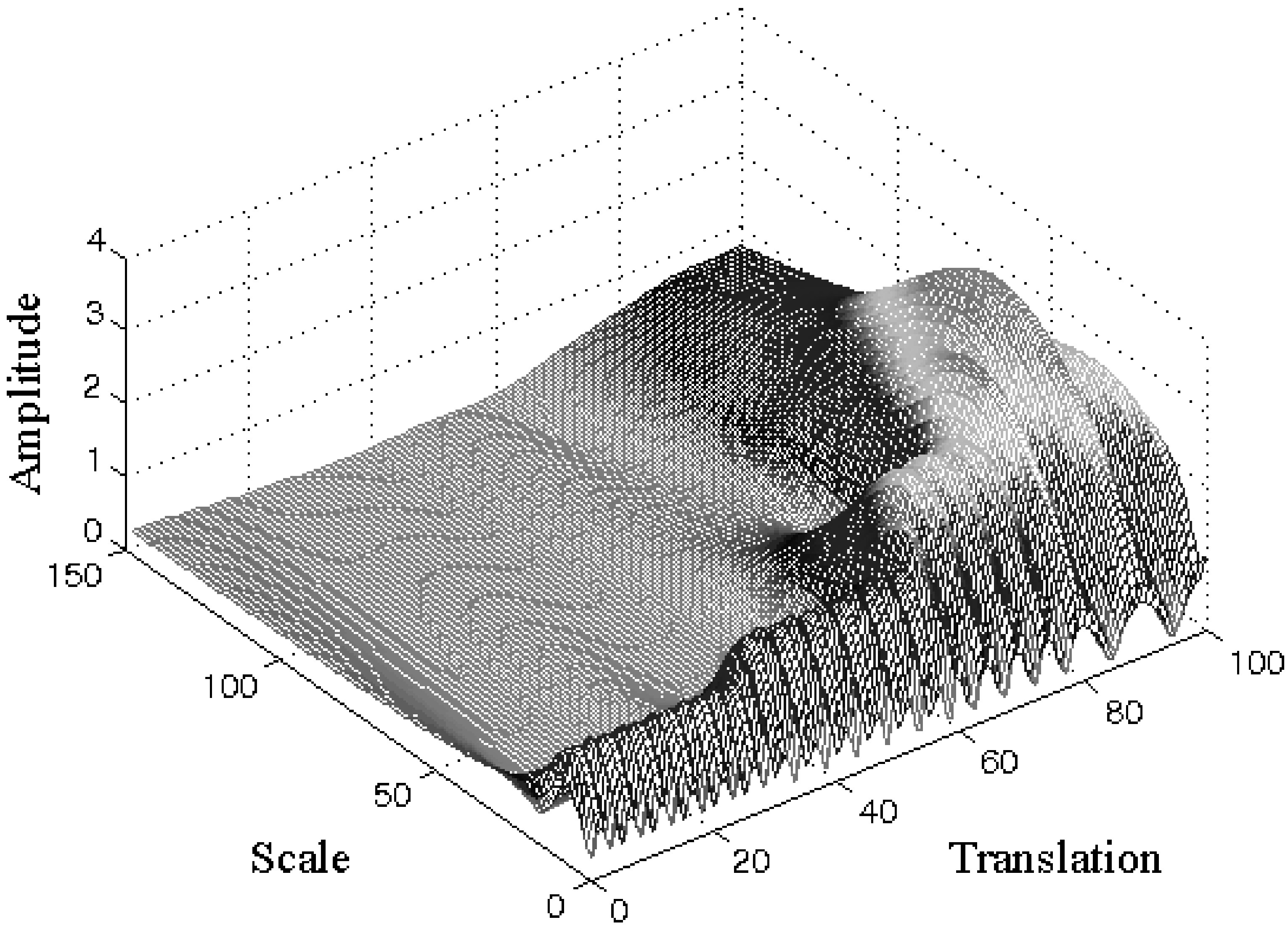
Scale-shifting process

DWT



Scale-shifting process

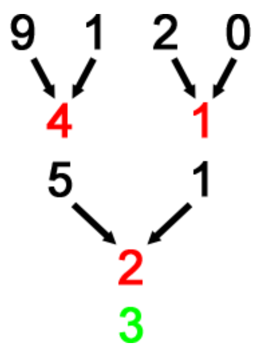
DWT



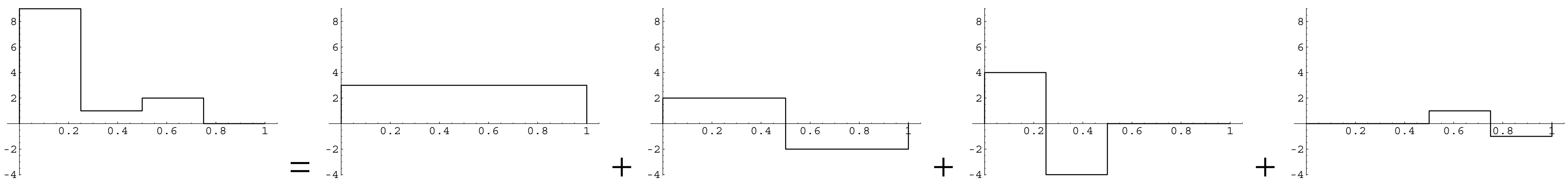
Trasformata wavelet del precedente segnale composto

Haar

La prima trasformata wavelet è stata introdotta da Haar (1909) e si presta bene all’analisi di segnali ad onda quadra. Inoltre, è veloce da realizzare poiché si basa solo su semisomme e semidifferenze.



$$\varphi(t) = \begin{cases} 1 & 0 \le t < 1, \\ 0 & \text{otherwise.} \end{cases} \qquad \psi(t) = \begin{cases} 1 & 0 \le t < 1/2, \\ -1 & 1/2 \le t < 1, \\ 0 & \text{otherwise.} \end{cases}$$



$x(t)$ = $3\,\varphi(t)$ + $2\,\psi(t)$ + $4\,\psi(2t)$ + $1\,\psi(2t\text{-}1)$

Esempio di trasformata di Haar

Haar

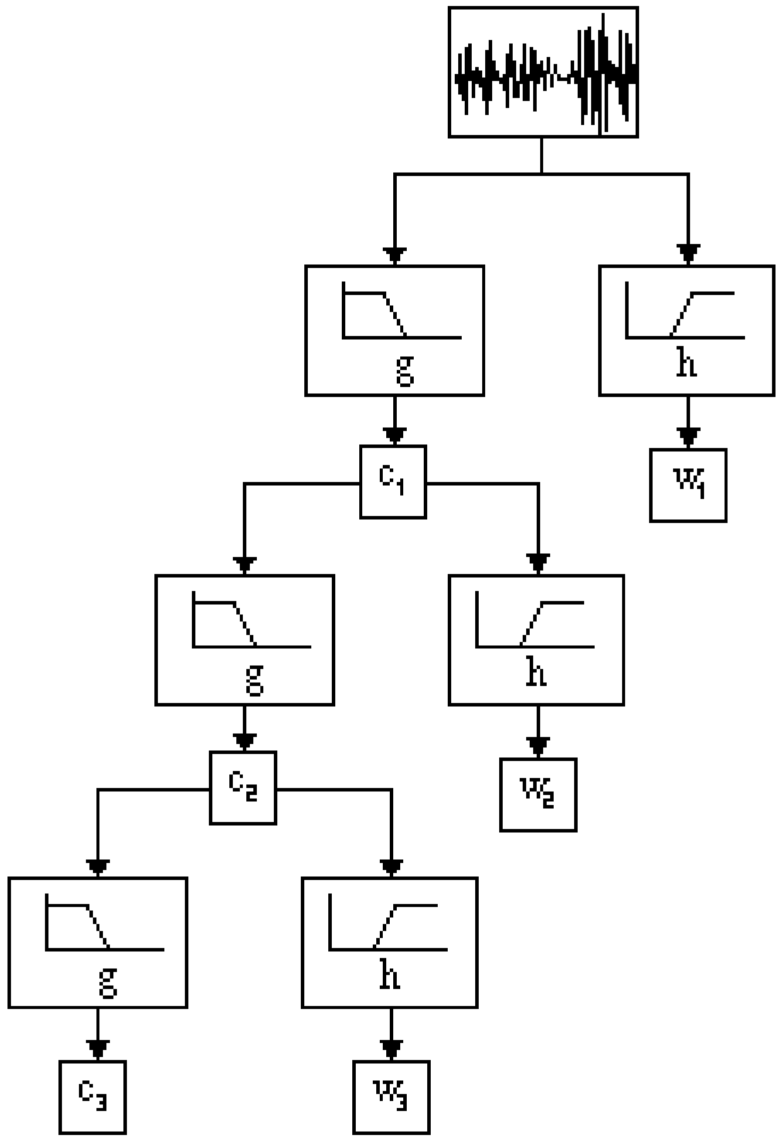
$$\frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 2 & -2 & 0 & 0 \\ 0 & 0 & 2 & -2 \end{pmatrix}\begin{pmatrix} 9 \\ 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 4 \\ 1 \end{pmatrix}$$

The Haar matrix multiplication

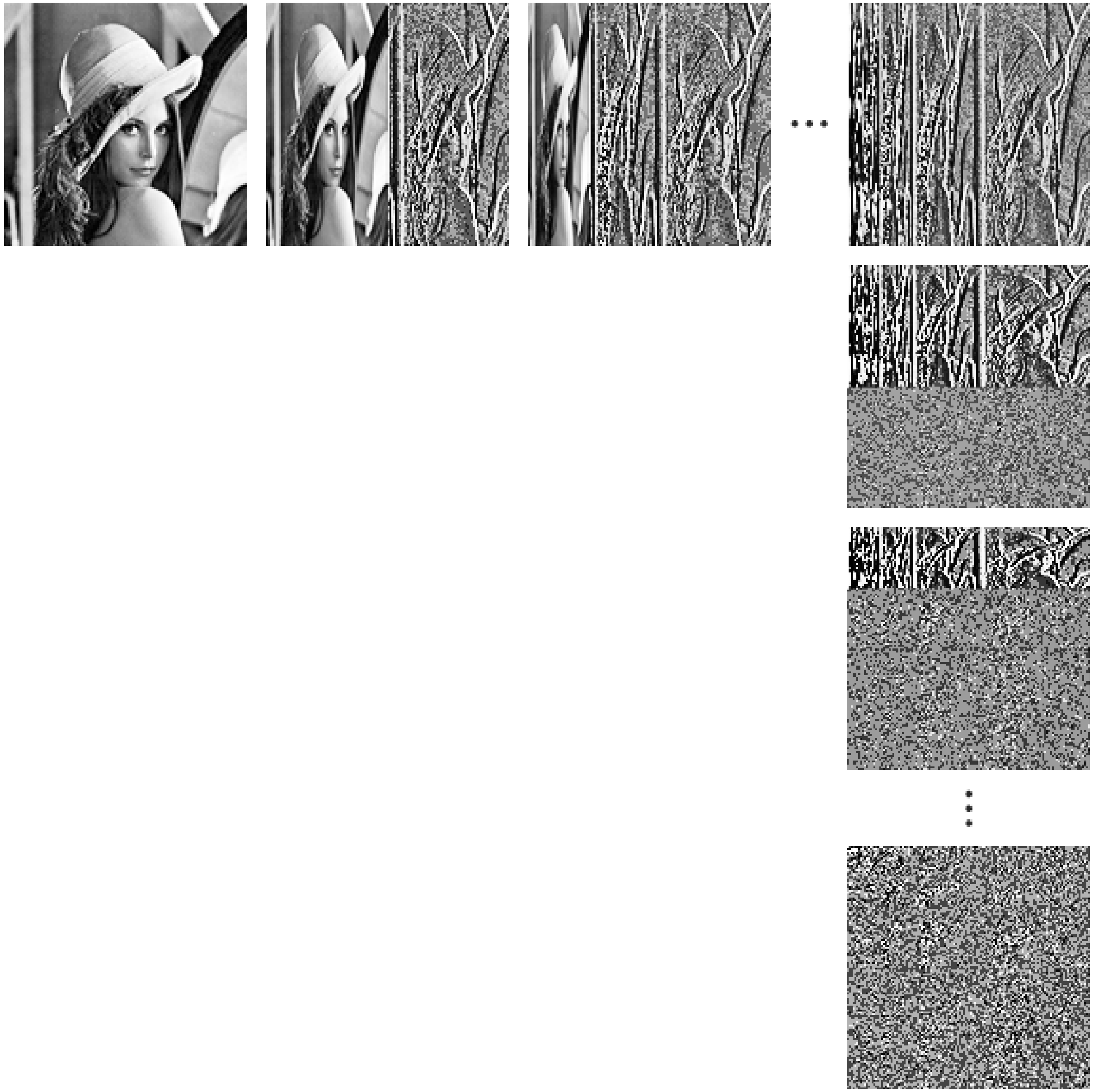
$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 2 \\ 0 \end{pmatrix}$$

Esempio di scomposizione matriciale della trasformata di Haar

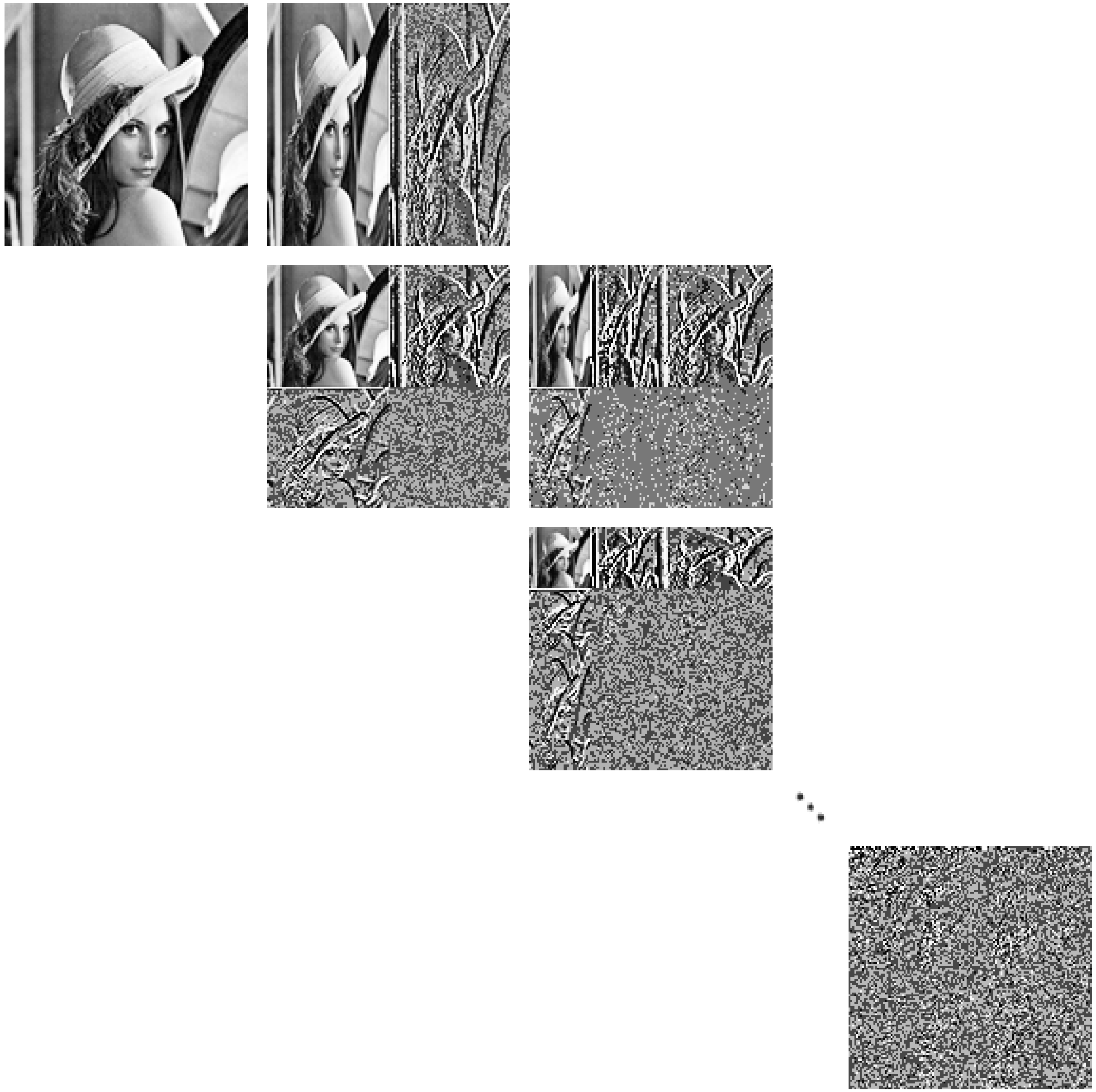
Filter Bank



Decomposizione standard



Decomposizione non standard

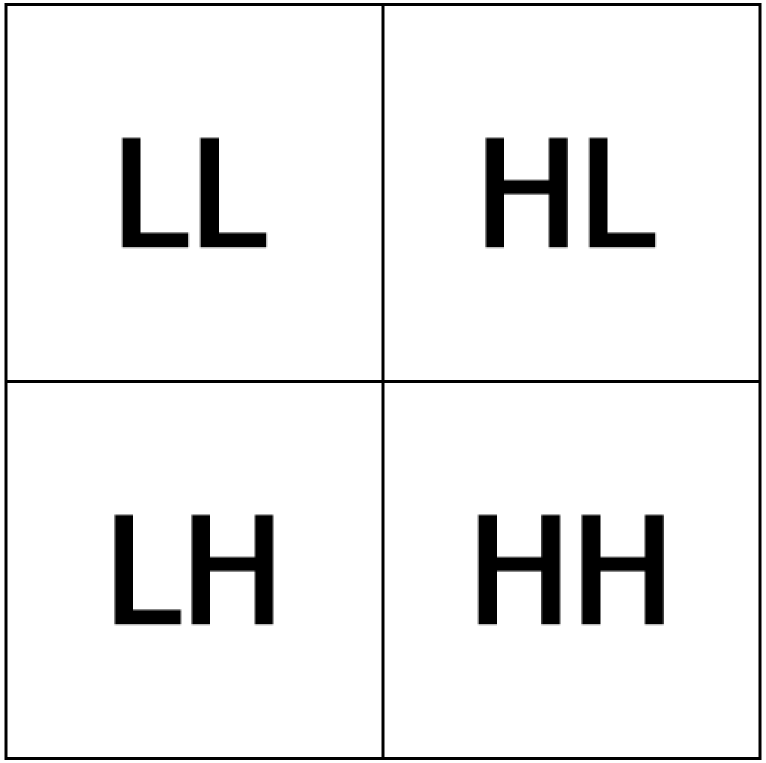


Analisi multirisoluzione (MRA)

Level 0

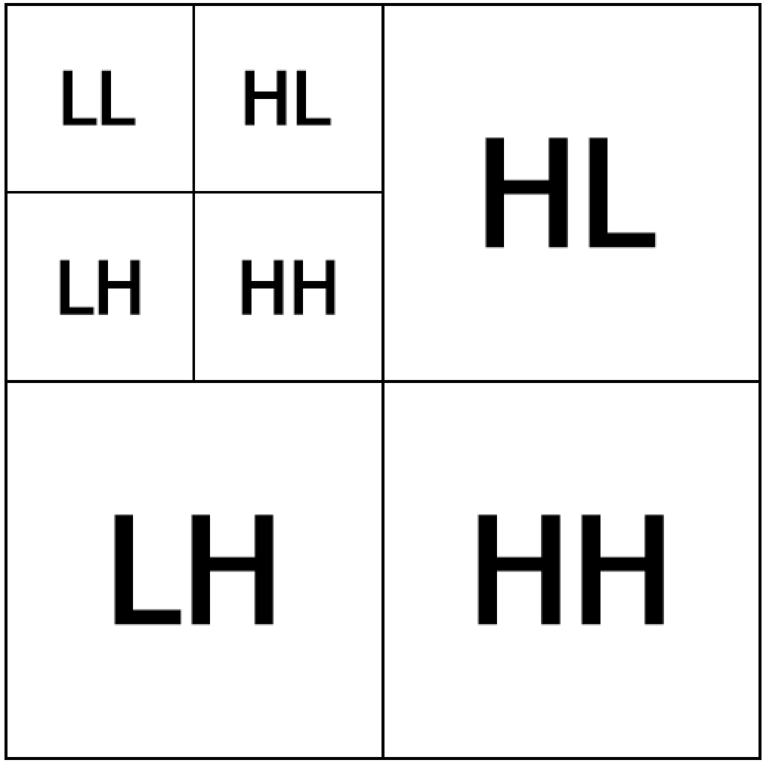
Analisi multirisoluzione (MRA)

Level 1

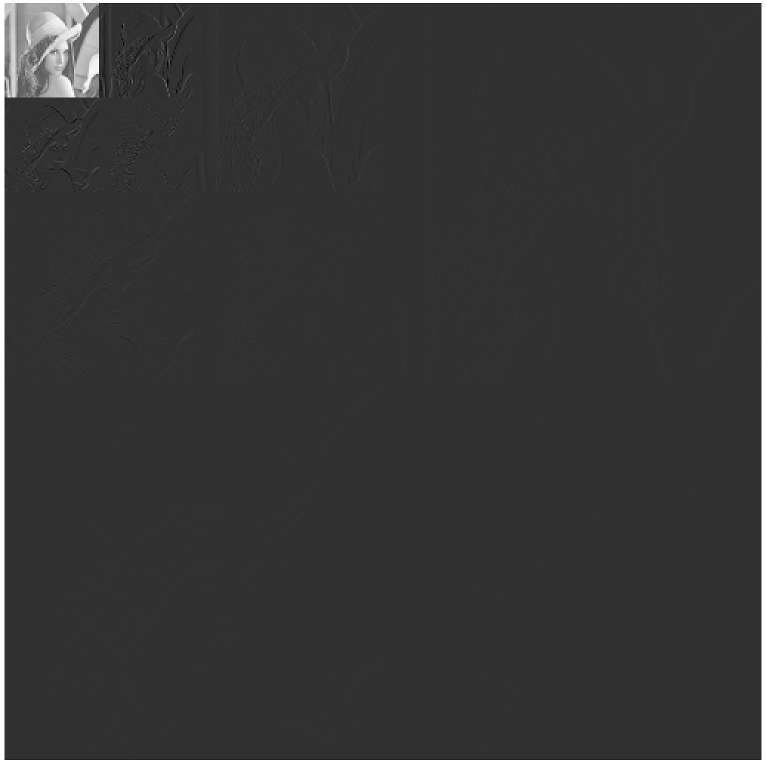


Analisi multirisoluzione (MRA)

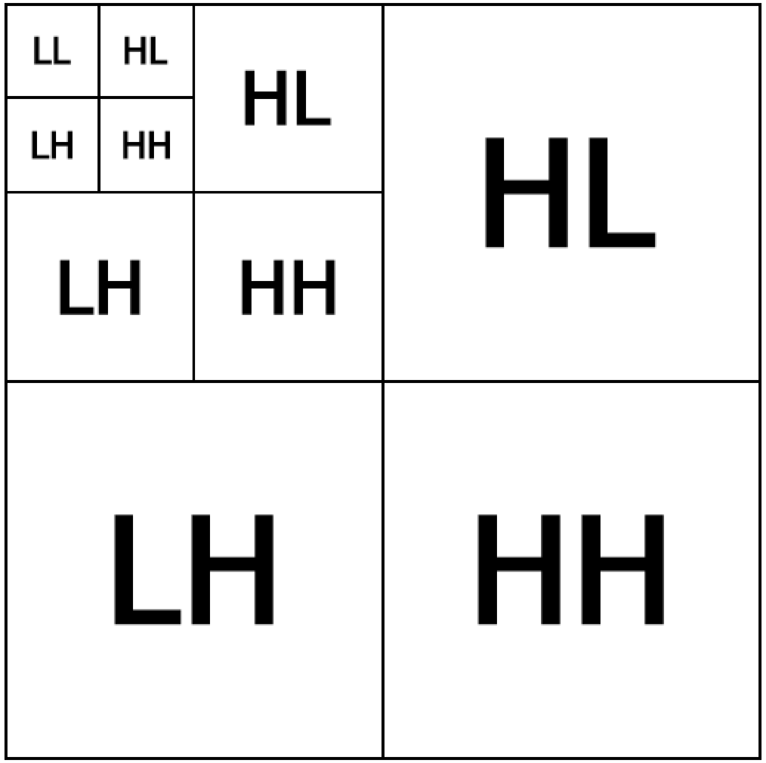
Level 2



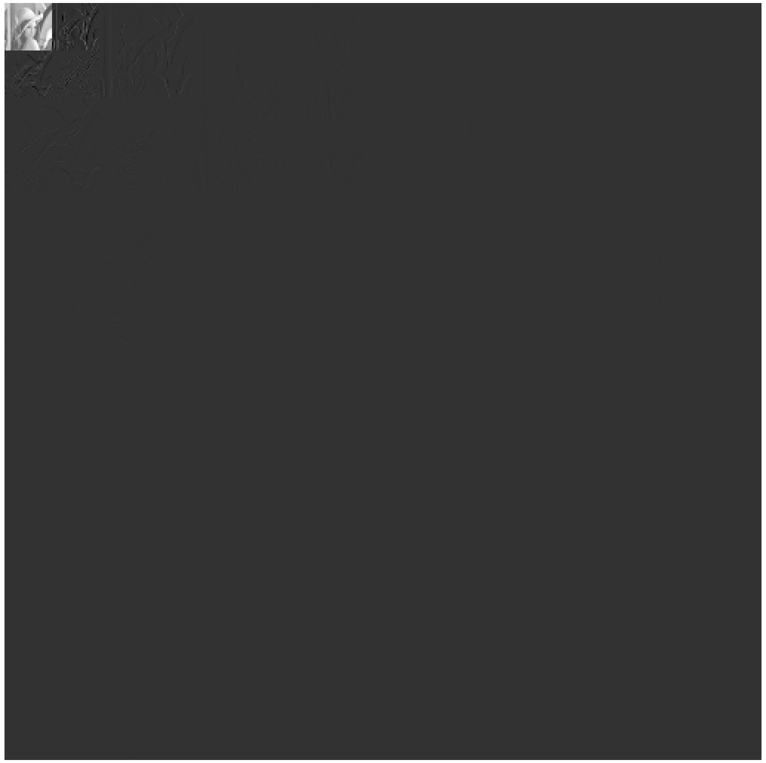
Analisi multirisoluzione (MRA)



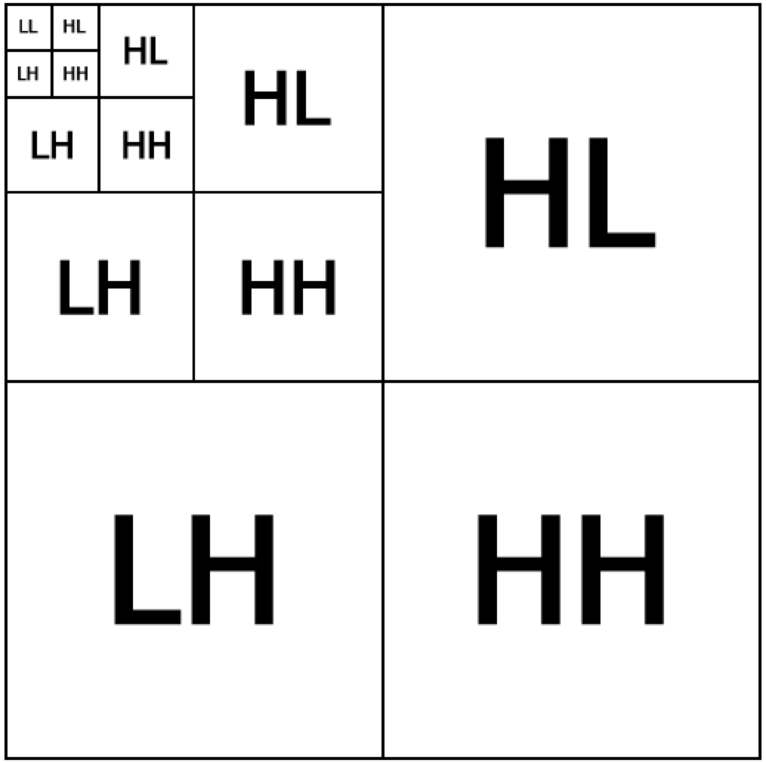
Level 3



Analisi multirisoluzione (MRA)

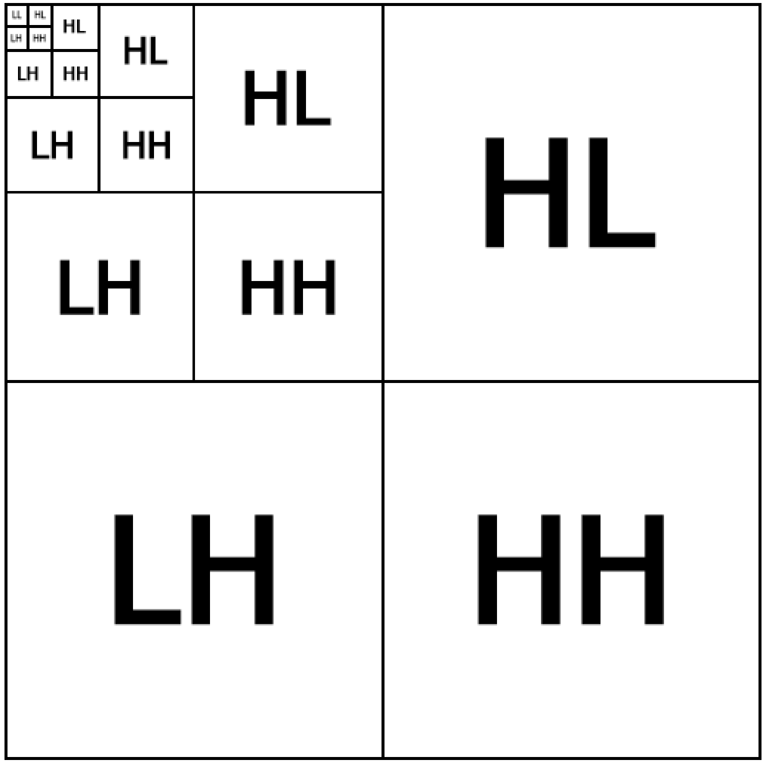


Level 4

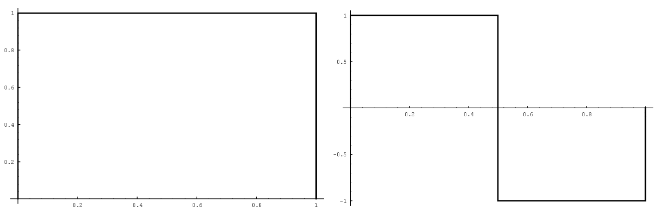


Analisi multirisoluzione (MRA)

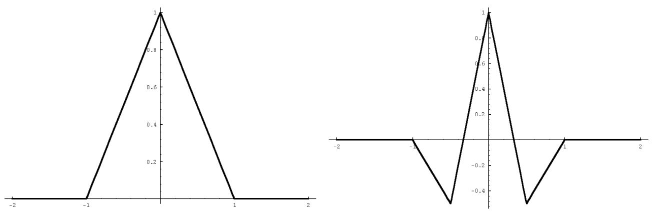
Level 5



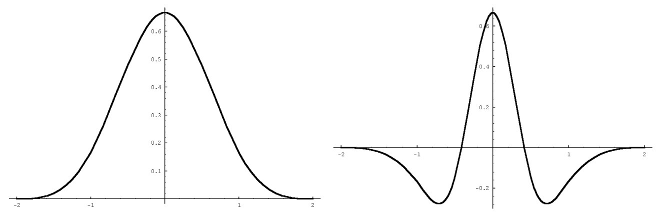
Funzioni di scalatura e wavelet madri



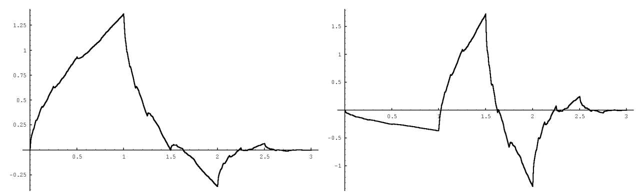
Haar scaling function φ and wavelet ψ.



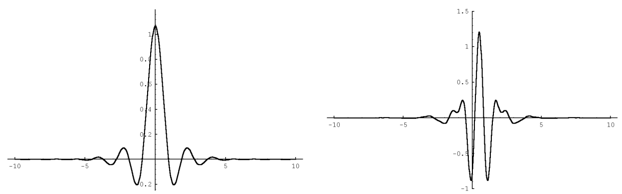
B_1-spline scaling function φ and wavelet ψ.



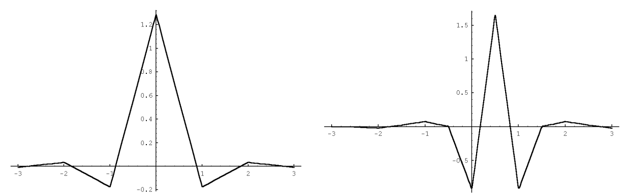
B_3-spline scaling function φ and wavelet ψ.



Daubechies scaling function φ and wavelet ψ.



Meyer scaling function φ and wavelet ψ.



Battle-Lemarié scaling function φ and wavelet ψ.

À trous

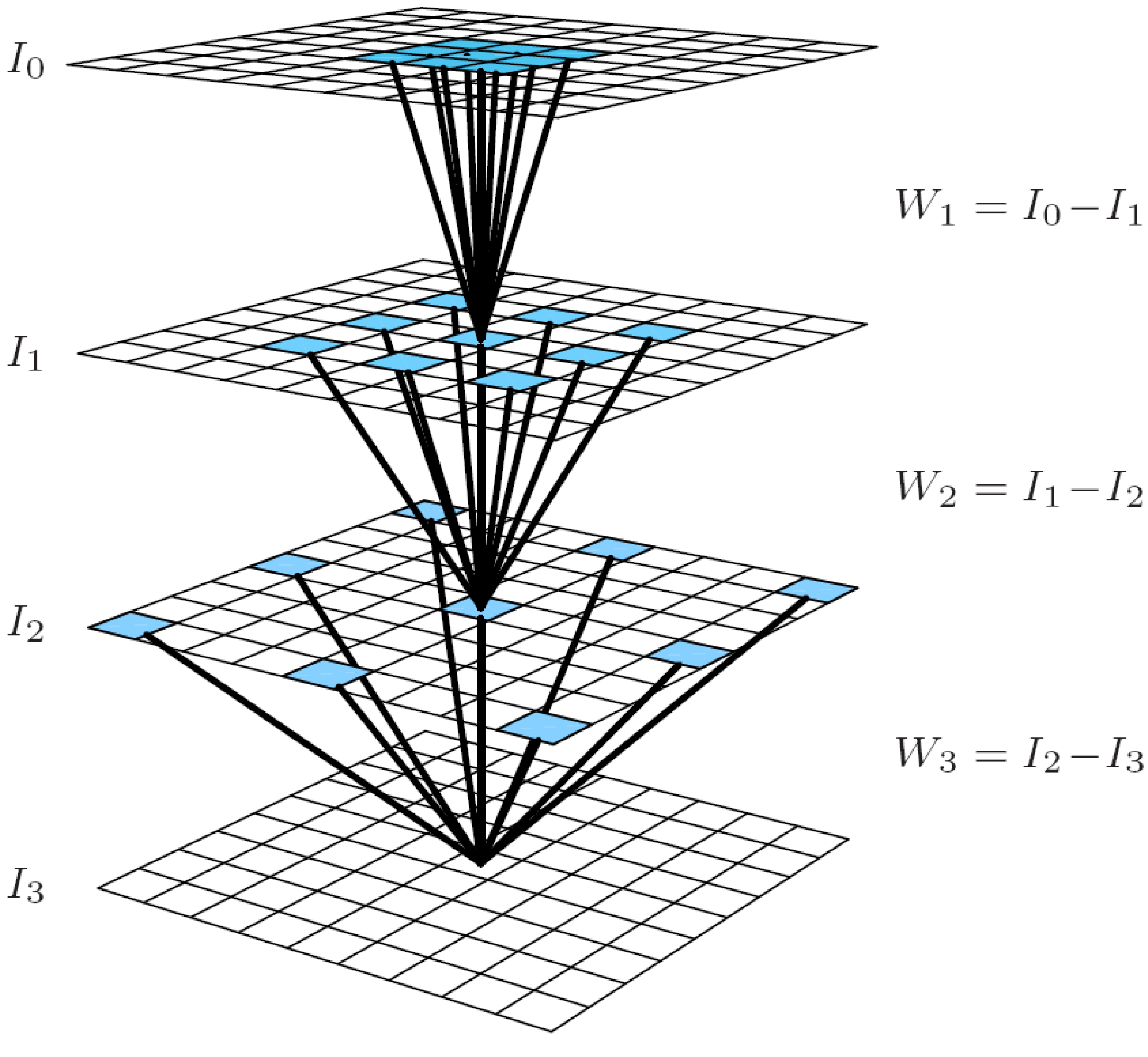
$$I_0(\mathbf{p}) = I(\mathbf{p})$$

$$I_i(\mathbf{p}) = I_{i-1}(\mathbf{p}) \otimes \ell_i$$

$$W_i(\mathbf{p}) = I_{i-1}(\mathbf{p}) - I_i(\mathbf{p})$$

$$\ell_i(2^{i-1}\mathbf{q}) = \ell(\mathbf{q})$$

À trous

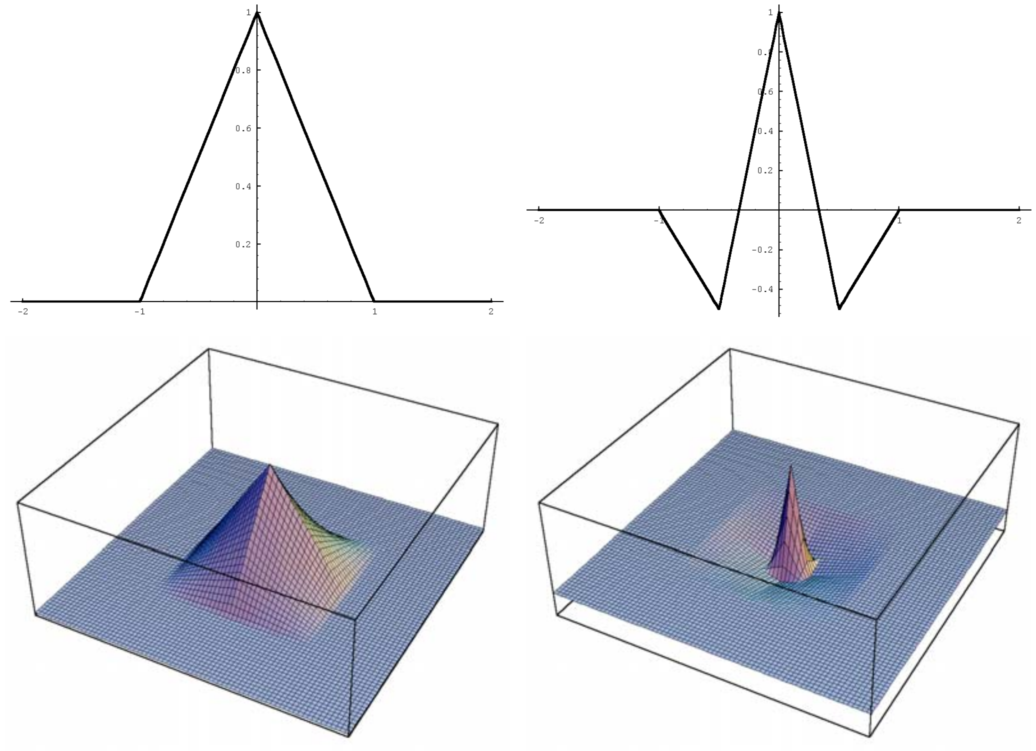


Funzioni di scalatura e wavelet madri

$$B_s(z) = \frac{1}{2s!} \sum_{t=0}^{s+1} (-1)^t \begin{pmatrix} t \\ s+1 \end{pmatrix} \left| z + t - \frac{s+1}{2} \right|^s$$

Funzioni di scalatura e wavelet madri

$$B_1(x) = \frac{1}{2}\left(\left|x-1\right| - 2\left|x\right| + \left|x+1\right|\right)$$



B_1-*spline* scaling function ϕ and wavelet ψ

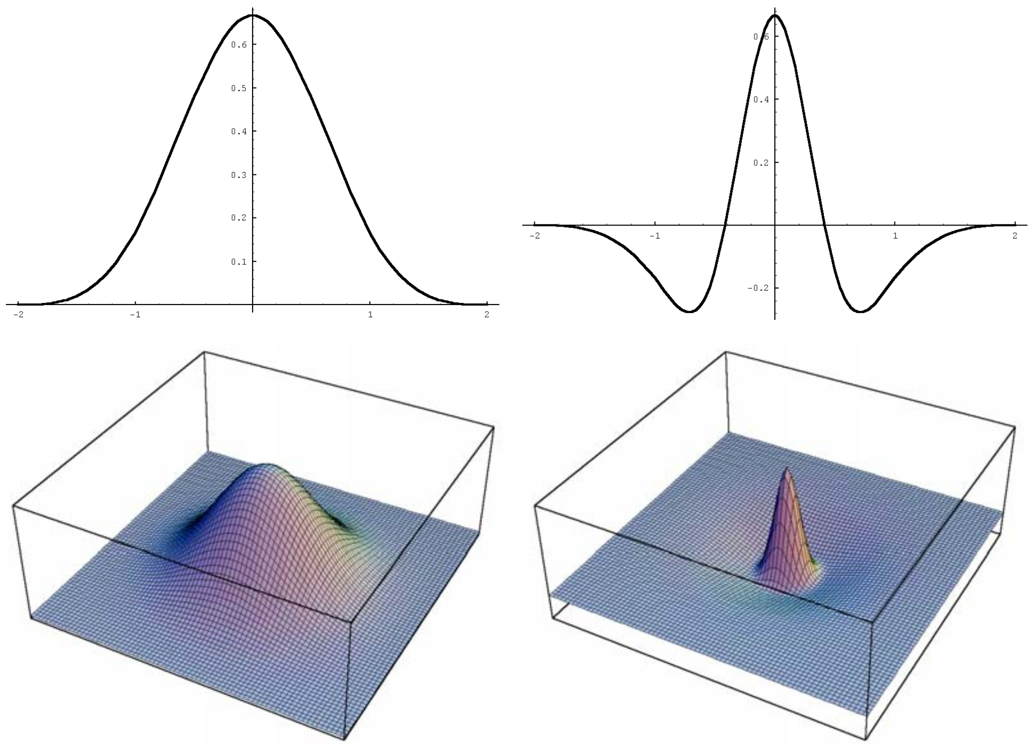
Funzioni di scalatura e wavelet madri

$$\ell = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\ell = \frac{1}{16}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Funzioni di scalatura e wavelet madri

$$B_3(x)=\frac{1}{12}\left(\left|x-2\right|^3-4\left|x-1\right|^3+6\left|x\right|^3-4\left|x+1\right|^3+\left|x+2\right|^3\right)$$



B_3-*spline* scaling function φ and wavelet ψ

JPG vs JPG2000

JPG (bpp = 0.3; *MSE* = 150; *PSNR* = 26.2)

JPG2K (bpp = 0.3; *MSE* = 73; *PSNR* = 29.5dB)

JPG vs JPG2000

JPG (bpp = 0.2; *MSE* = 320; *PSNR* = 23.1dB)

JPG2K (bpp = 0.2; *MSE* = 113; *PSNR* = 27.6dB)