PROBLEMA 7.4

Consideremos la función:

$$f(\varepsilon) = \operatorname{sen}(x+\varepsilon) = \operatorname{sen}(x)\cos(\varepsilon) + \cos(x)\operatorname{sen}(\varepsilon)$$

$$= \operatorname{sen}(x)\left\{1 - \frac{\varepsilon^2}{2} + o(\varepsilon^2)\right\} + \cos(x)\left\{\varepsilon + o(\varepsilon^2)\right\}$$

$$= \operatorname{sen}(x) + \varepsilon\cos(x) - \frac{\varepsilon^2}{2}\operatorname{sen}x + o(\varepsilon^2)$$

Por tanto:

$$\left|\operatorname{sen}(x+\varepsilon) - (\operatorname{sen}x + \varepsilon\cos x)\right| = \left|\frac{\varepsilon^2}{2}\operatorname{sen}x + o(\varepsilon^2)\right| \leq$$

$$\leq \frac{\varepsilon^2}{2}|\operatorname{sen}x| + o(\varepsilon^2) \leq$$

$$\leq \frac{\varepsilon^2}{2} + o(\varepsilon^2)$$

$$\Rightarrow \boxed{\left|\operatorname{sen}(x+\varepsilon) - (\operatorname{sen}x + \varepsilon\cos x)\right| \leq \frac{\varepsilon^2}{2} + o(\varepsilon^2)}$$