$\boxed{1.1}$

(1) $x^2 + \dfrac{2}{x} > 3 \qquad (x \neq 0)$

$$\Updownarrow$$

$$\frac{x^2 - 3x + 2}{x} > 0$$

$$\Updownarrow$$

$$\frac{(x+2)(x-1)^2}{x} > 0$$

$\underline{\text{obs:}}$

$$\begin{array}{r|rrrr} & 1 & 0 & -3 & 2 \\ -2 & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & \boxed{0} \end{array}$$

$$x^2 - 3x + 2 = (x+2)(x-1)^2$$

$\overset{x>0}{\Updownarrow}$

$$(x+2)(x-1)^2 > 0$$

$$\Updownarrow$$

$$x+2 > 0 \ \& \ x \neq 1 \ \& \ x > 0$$

$$\Updownarrow$$

$$x \in (0,1) \cup (1,\infty)$$

$\overset{x<0}{\Updownarrow}$

$$(x+2)(x-1)^2 < 0$$

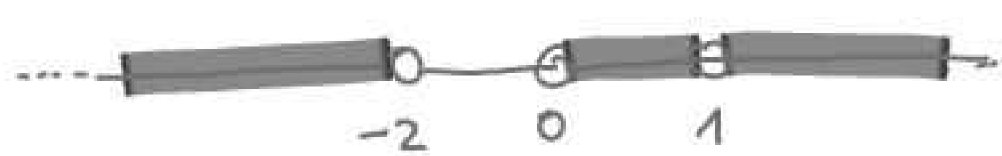
$$\Updownarrow$$

$$x+2 < 0 \ \& \ x \neq 1 \ \& \ x < 0$$

$$\Updownarrow$$

$$x \in (-\infty, -2)$$

Por tanto: $\quad x^2 + \dfrac{2}{x} > 3 \iff \boxed{x \in (-\infty,-2) \cup (0,1) \cup (1,\infty)}$



② $|\sqrt{x} - 2| \leq 3 \qquad (x \geq 0)$

$$\Updownarrow$$

$$-3 \leq \sqrt{x} - 2 \leq 3$$

$$\begin{cases} -3 \leq \sqrt{x} - 2 \Leftrightarrow -1 \leq \sqrt{x} \ : \text{cierto } \forall x \geq 0 \\ \sqrt{x} - 2 \leq 3 \Leftrightarrow \sqrt{x} \leq 5 \Leftrightarrow 0 \leq x \leq 25 \end{cases}$$

Por tanto $|\sqrt{x} - 2| \leq 3 \Leftrightarrow \boxed{x \in [0, 25]}$

③ $-8 \leq |x-5| - |x+3| \leq 8$

<u>Obs:</u>

$$|x| - 5 \leq |x-5| \leq |x| + 5 \quad \forall x$$

$$|x| - 3 \leq |x-3| \leq |x| + 3 \quad \forall x$$

$$-|x| - 3 \leq -|x-3| \leq 3 - |x| \quad \forall x$$

$$\Rightarrow |x| - 5 + (-|x| - 3) \leq |x-5| - |x-3| \leq$$

$$\leq |x| + 5 + 3 - |x| \quad \forall x$$

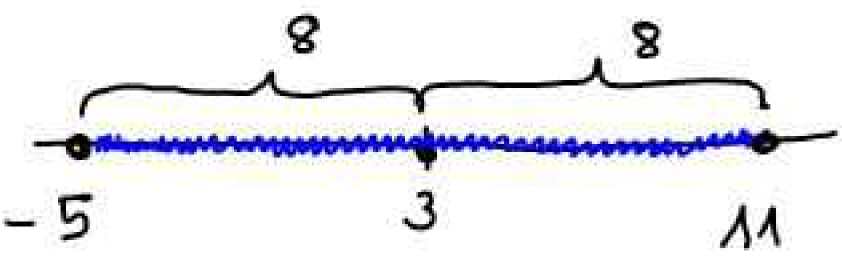
$$\Rightarrow \boxed{-8 \leq |x-5| - |x-3| \leq 8 \quad \forall x \in \mathbb{R}}$$

④ $|x-3| \leq 8 \iff -8 \leq x-3 \leq 8$

$-8 \leq x-3 \iff -5 \leq x$

$x-3 \leq 8 \iff x \leq 11$

Por tanto: $|x-3| \leq 8 \iff x \in [-5, 11]$



⑤ $0 < |x-2| < \frac{1}{2}$

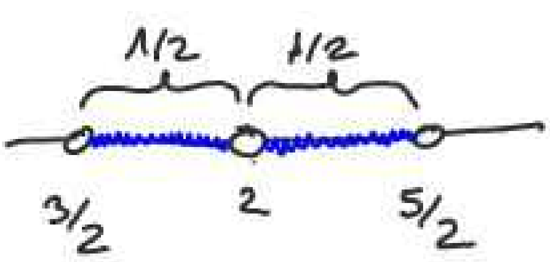
$0 < |x-2| \iff x \neq 2$

$|x-2| < \frac{1}{2} \iff -\frac{1}{2} < x-2 < \frac{1}{2}$

$-\frac{1}{2} < x-2 \iff \frac{3}{2} < x$

$x-2 < \frac{1}{2} \iff x < \frac{5}{2}$

$0 < |x-2| < \frac{1}{2} \iff \boxed{x \in \left(\frac{3}{2}, 2\right) \cup \left(2, \frac{5}{2}\right)}$



⑥ $x^2 - 5x + 6 \geq 0$

Obs: $x^2 - 5x + 6 = 0 \Leftrightarrow x = \dfrac{5 \pm \sqrt{25-24}}{2} \begin{array}{l} \nearrow 3 \\ \searrow 2 \end{array}$

$\Rightarrow x^2 - 5x + 6 = (x-2)(x-3)$

$x^2 - 5x + 6 \geq 0 \Leftrightarrow (x-2)(x-3) \geq 0$

$\Leftrightarrow \quad x \leq 2 \ \& \ x \leq 3$

ó

$x \geq 2 \ \& \ x \geq 3$

Por tanto $x^2 - 5x + 6 \geq 0 \Leftrightarrow \boxed{x \in (-\infty, 2] \cup [3, \infty)}$

⑦ $x^3 (x+3)(x-5) > 0$

$+++$
$--+$
$-+-$
$+--$

- $+++$: $x > 0$; $x > -3$; $x > 5 \Leftrightarrow \underline{x > 5}$
- $--+$: $x < 0$; $x < -3$; $x > 5$ $\nexists$
- $-+-$: $x < 0$; $x > -3$; $x < 5 \Leftrightarrow -3 < x < 0$
- $+--$: $x > 0$; $x < -3$; $x < 5$ $\nexists$

$\boxed{x \in (-3, 0) \cup (5, \infty)}$

(8) $$\frac{2x+8}{x^2+8x+7} > 0$$

Obs: $x^2+8x+7=0 \Leftrightarrow x=-1$ ó -7

$$x^2+8x+7=(x+1)(x+7)$$

$$\frac{2x+8}{x^2+8x+7} > 0 \qquad (x \neq -1, -7)$$

$$\frac{2(x+4)}{(x+1)(x+7)} > 0 \qquad \frac{\oplus}{\oplus\,\oplus} ; \frac{\oplus}{\ominus\,\ominus} ; \frac{\ominus}{\oplus\,\ominus} ; \frac{\ominus}{\ominus\,\oplus}$$

$\frac{\oplus}{\oplus\,\oplus}$: $x>-4$; $x>-1$; $x>-7 \Leftrightarrow \underline{x>-1}$

$\frac{\oplus}{\ominus\,\ominus}$: $x>-4$; $x<-1$; $x<-7$ $\nexists$

$\frac{\ominus}{\oplus\,\ominus}$: $x<-4$; $x>-1$; $x<-7$ $\nexists$

$\frac{\ominus}{\ominus\,\oplus}$: $x<-4$; $x<-1$; $x>-7 \Leftrightarrow \underline{-7<x<-4}$

$$\boxed{x \in (-7,-4) \cup (-1,\infty)}$$

9 $|x-1| + |x-2| > 1$

- Si $x-1>0$ & $x-2>0$ (es decir, si $x>2$)

$x-1+x-2>1 \iff 2x>4 \iff x>2$

se cumple $\forall x>2$

- Si $x-1>0$ & $x-2<0$ (es decir, si $1<x<2$)

$\cancel{x}-1+2-\cancel{x}>1 \iff 1>1$ ¡¡

- Si $x-1<0$ & $x-2>0$ ¡¡

- Si $x-1<0$ & $x-2<0$ (es decir, si $x<1$)

$1-x+2-x>1 \iff 2>2x \iff$

$\iff 1>x$

se cumple $\forall x<1$.

$$\boxed{x \in (-\infty, 1) \cup (2, \infty)}$$

(10) $|x-1||x+2| = 3$

- Si $x \geq 1$ & $x \geq -2$ (es decir, si $x \geq 1$)

$(x-1)(x+2) = 3$

$x^2 + x - 2 = 3 \iff x^2 + x - 5 = 0$

$\iff x = \dfrac{\sqrt{21} - 1}{2}$; $\cancel{x = -\dfrac{1+\sqrt{21}}{2}}$

No cumple $x \geq 1$

- Si $x \geq 1$ & $x \leq -2$!!
- Si $x \leq 1$ & $x \geq -2$ (es decir, si $x \in [-2, 1]$):

$(1-x)(x+2) = 3 \iff -x^2 - x + 2 = 3$

$\iff x^2 + x + 1 = 0 \quad \nexists x \in \mathbb{R}.$

- Si $x \leq 1$ & $x \leq -2$ (es decir, si $x \leq -2$)

$(1-x)(-x-2) = 3 \iff x^2 + x - 2 = 3$

$\iff x^2 + x - 5 = 0$

$\iff x = -\dfrac{1+\sqrt{21}}{2}$; $\cancel{x = \dfrac{\sqrt{21} - 1}{2}}$

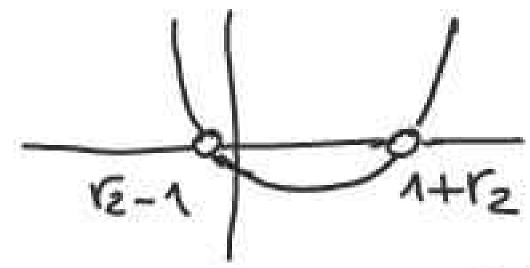
No cumple $x \leq -2$

$$\boxed{x = -\frac{1+\sqrt{21}}{2} \ \& \ x = \frac{\sqrt{21} - 1}{2}}$$

(11) $|x^2-2x|<1 \Leftrightarrow |x(x-2)|<1$

• Si $x\geq 0$ & $x\geq 2$ (es decir, si $x\geq 2$)

$x^2-2x<1 \Rightarrow x^2-2x-1<0$



$x\in(\sqrt{2}-1,1+\sqrt{2})$ & $x\geq 2$

$x\in[2,1+\sqrt{2})$

• Si $x\leq 0$ & $x\leq 2$ (es decir, si $x\leq 0$)

el mimo razonamiento nos dice que $x\in(\sqrt{2}-1,0]$

• Si $x\leq 0$ & $x\geq 2$ ¡¡

• Si $x\geq 0$ & $x\leq 2$

$2x-x^2<1 \Leftrightarrow x^2-2x+1>0$

$(x-1)^2>0 \Leftrightarrow x\neq 1$

con $x\geq 0$ & $x\leq 2$

$x\in[0,1)\cup(1,2]$

Por tanto:

$$x\in(\sqrt{2}-1,1)\cup(1,1+\sqrt{2})$$