PROBLEMA 7.17 $\quad \text{sen}(1/2)$, error $< 10^{-12}$

$$\text{sen}(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

Además

$$f(x) = \text{sen}(x) \Rightarrow |f^{(k)}(x)| = \begin{cases} |\text{sen}\, x| & \text{si } k \text{ PAR} \\ |\cos x| & \text{si } k \text{ IMPAR} \end{cases}$$

por lo que:

$$|R_{2n+2}(x|\text{sen},0)| = \frac{|\text{sen}\, c|}{(2n+3)!} x^{2n+3} \leq \frac{x^{2n+3}}{(2n+3)!}$$

En particular:

$$|R_{2n+2}(1/2|\text{sen},0)| \leq \frac{1}{2^{2n+3}(2n+3)!}$$

Imponiendo:

$$\frac{1}{2^{2n+3}(2n+3)!} < 10^{-12} \Rightarrow n \geq 5$$

[y, por tanto, $2n+1 \geq \underline{\underline{11}}$]

se tiene que:

$$\boxed{\text{sen}(1/2) \simeq 1 - 0.5 + \frac{(0.5)^3}{3!} - \frac{(0.5)^5}{5!} + \frac{(0.5)^7}{7!} - \frac{(0.5)^9}{9!} + \frac{(0.5)^{11}}{11!}}$$

es una aproximación de sen(0.5) cuyo error es $< 10^{-12}$