

PROBLEMA 7.5

$$f(x) = \log(\cos x)$$

$$\cos z = 1 - \frac{z^2}{2} + \frac{z^4}{4!} + o(z^4)$$

$$\log(1-z) = -z - \frac{z^2}{2} + o(z^2)$$

$$\begin{aligned}\Rightarrow f(x) &= \log(\cos x) = \log\left(1 - \left(\frac{x^2}{2} - \frac{x^4}{4!} + o(x^4)\right)\right) \\&= -\frac{x^2}{2} + \frac{x^4}{4!} + o(x^4) - \frac{1}{2}\left(\frac{x^2}{2} - \frac{x^4}{4!} + o(x^4)\right)^2 + o(x^4) \\&= -\frac{x^2}{2} + \left(\frac{1}{4!} - \frac{1}{8}\right)x^4 + o(x^4) \\&= -\frac{x^2}{2} - \frac{x^4}{12} + o(x^4)\end{aligned}$$