

PROBLEMA 7.8

$$f(x) = \log(1 + \cos x)$$

$$f(x) = \log(1 + \cos x) \quad ; \quad x \in (-\pi, \pi)$$

$$f'(x) = \frac{-\operatorname{sen} x}{1 + \cos x}$$

$$f''(x) = \frac{-\cos x (1 + \cos x) - \operatorname{sen}^2 x}{(1 + \cos x)^2} = -\frac{1}{1 + \cos x}$$

$$f'''(x) = -\frac{\operatorname{sen} x}{(1 + \cos x)^2}$$

$$\text{Por tanto: } \left. \begin{array}{l} f(0) = \log 2 \\ f'(0) = 0 \\ f''(0) = -\frac{1}{2} \end{array} \right\} P_2(x|f, 0) = \log 2 - \frac{x^2}{4}$$

$$\log(1 + \cos x) = \log 2 - \frac{x^2}{4} + R_2(x|f, 0)$$

$$\text{donde } R_2(x|f, 0) = -\frac{\operatorname{sen} c}{(1 + \cos c)^2 3!} x^3$$

$$\text{con } c \in (0, x) \text{ ó } c \in (x, 0)$$

$$\bullet \text{ Error}(x) := \left| \log(1 + \cos x) - \log 2 + \frac{x^2}{4} \right| =$$

$$= |R_2(x|f, 0)| =$$

$$= \frac{|\operatorname{sen} c|}{6 (1 + \cos c)^2} |x|^3 \quad \text{donde } x \in (-\pi, \pi) \setminus \{0\}$$

$$c \in (0, x)$$

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$$c \in (x, 0)$$

$$\bullet \text{ Error}(0) = 0$$