**Problema 10.4** Calcula los siguientes límites:

a) $\lim\limits_{x\to 0} \frac{1}{x}\int_0^x \frac{|\cos(t^3)|}{t^2+1}\,dt$

Indeterminación $\frac{0}{0}$. Usando LH:

$$\lim_{x\to 0} \frac{\int_0^x \frac{|\cos(t^3)|}{t^2+1}\,dt}{x} \underset{\frac{0}{0}}{\overset{L'H}{=}} \lim_{x\to 0} \frac{|\cos(x^3)|}{x^2+1} = 1$$

b) $\lim\limits_{x\to \infty} \frac{1}{x}\int_0^x \frac{|\cos(t^3)|}{t^2+1}\,dt$

Analicemos $\lim\limits_{x\to\infty} \int_0^x \frac{|\cos(t^3)|}{t^2+1}\,dt$:

$$0 \le \frac{|\cos(t^3)|}{t^2+1} \le \frac{1}{t^2+1}$$

$$\Rightarrow 0 \le \int_0^x \frac{|\cos(t^3)|}{t^2+1}\,dt \le \int_0^x \frac{dt}{t^2+1} = \arctan(x)$$

$$\Rightarrow 0 \le \lim_{x\to\infty} \int_0^\infty \frac{|\cos(t^3)|}{t^2+1}\,dt \le \lim_{x\to\infty} \arctan(x) = \frac{\pi}{2}$$

Por tanto:

$$0 \le \lim_{x\to\infty} \frac{1}{x}\int_0^x \frac{|\cos(t^3)|}{t^2+1}\,dt \le \lim_{x\to\infty} \frac{\arctan(x)}{x} = 0$$

$$\Longrightarrow \lim_{x\to\infty} \frac{1}{x}\int_0^x \frac{|\cos(t^3)|}{t^2+1}\,dt = 0$$

No hay indeterminación