PROBLEMA 11.1

- $\int_0^{\log 2} \sqrt{e^t - 1}\, dt$

$$\int_0^{\log 2} \sqrt{e^t - 1}\, dt \underset{\substack{\uparrow \\ \text{c.v.}}}{=} \int_0^1 u \frac{2u}{1+u^2}\, du$$

$$\begin{cases} u = \sqrt{e^t - 1} \Rightarrow e^t = 1 + u^2 \\ du = \dfrac{e^t}{2\sqrt{e^t - 1}}\, dt \Rightarrow dt = \dfrac{2u}{1+u^2}\, du \end{cases}$$

Ahora bien:

$$\int_0^1 \frac{2u^2}{u^2+1}\, du = \int_0^1 \frac{2u^2+2-2}{u^2+1}\, du = \int_0^1 \left(2 - \frac{2}{u^2+1}\right) du$$

$$= \left(2u - 2\arctan(u)\right)\Big|_{u=0}^{u=1} = 2 - \frac{\pi}{2}$$

$$\Rightarrow \boxed{\int_0^{\log(2)} \sqrt{e^t - 1}\, dt = 2 - \frac{\pi}{2}}$$

$$\int_1^2 \frac{\sqrt{t^2-1}}{t}\,dt$$

$$\int_1^2 \frac{\sqrt{t^2-1}}{t}\,dt = \int_0^{\sqrt{3}} \frac{u}{\sqrt{u^2+1}}\cdot\frac{u}{\sqrt{u^2+1}}\,du = \int_0^{\sqrt{3}} \frac{u^2}{u^2+1}\,du$$

C.V. $u=\sqrt{t^2-1} \Rightarrow t=\sqrt{u^2+1}$

$$du = \frac{t}{\sqrt{t^2-1}}\,dt \Rightarrow dt = \frac{u}{\sqrt{u^2+1}}\,du$$

Ahora bien:

$$\int_1^{\sqrt{3}} \frac{u^2}{u^2+1}\,du = \int_1^{\sqrt{3}} \left(1-\frac{1}{u^2+1}\right)du$$

$$= u-\arctan(u)\Big|_{u=0}^{u=\sqrt{3}} = \sqrt{3}-\frac{\pi}{3}$$

$$\Rightarrow \boxed{\int_1^2 \frac{\sqrt{t^2-1}}{t}\,dt = \sqrt{3}-\frac{\pi}{3}}$$

$$\int \cos(\log x)\, dx$$

$$\int \cos(\log x)\, dx \underset{\uparrow}{=} \int \cos(u) e^u\, du$$

$$\boxed{\begin{array}{l} u = \log x \Rightarrow x = e^u \\ du = \dfrac{dx}{x} \Rightarrow dx = e^u\, du \end{array}}$$

Ahora bien:

$$\int e^u \cos(u)\, du \underset{\uparrow}{=} e^u \operatorname{sen}(u) - \int e^u \operatorname{sen}(u)\, du$$

PARTES:

$$\boxed{\begin{array}{l|l} U = e^u & dU = e^u\, du \\ dV = \cos(u)\, du & V = \operatorname{sen} u \end{array}}$$

$$\underset{\uparrow}{=} e^u \operatorname{sen}(u) + e^u \cos(u) - \int e^u \cos(u)\, du$$

PARTES:

$$\boxed{\begin{array}{l|l} U = e^u & dU = e^u\, du \\ dV = \operatorname{sen}(u)\, du & V = -\cos(u) \end{array}}$$

$$\Rightarrow \int e^u \cos(u)\, du = \frac{e^u}{2}\Big(\cos(u) + \operatorname{sen}(u) \Big) + c$$

$$\Rightarrow \boxed{\int \cos(\log x)\, dx \underset{\substack{\uparrow \\ \boxed{u = \log x}}}{=} \frac{x}{2}\Big(\cos(\log x) + \operatorname{sen}(\log x) \Big) + c}$$

$$\int \cos^2(\log x)\,dx$$

$$\int \cos^2(\log x)\,dx \underset{\uparrow}{=} \int \cos^2(u)\,e^u\,du$$

$$\boxed{\begin{array}{l} u = \log x \Rightarrow x = e^u \\ du = \frac{dx}{x} \Rightarrow dx = e^u du \end{array}}$$

Usando: $\cos(2u) = \cos^2(u) - \text{sen}^2(u) = 2\cos^2(u) - 1$

$$\Rightarrow \cos^2(u) = \frac{1}{2}\cos(2u) + 1/2$$

se tiene qe:

$$\int \cos(\log x)\,dx = \int \cos^2(u)\,e^u\,du =$$

$$= \frac{1}{2}\int \cos(2u)\,e^u\,du + \frac{1}{2}\int e^u\,du$$

$$= \frac{1}{2}\int \cos(2u)\,e^u\,du + \frac{1}{2}e^u$$

$$\underset{\substack{\uparrow \\ \text{PARTES}}}{=} \frac{1}{10}e^u\left(\cos(2u) + 2\,\text{sen}(2u)\right) + \frac{1}{2}e^u + C$$

$$\underset{\substack{\uparrow \\ u = \log x}}{=} \frac{x}{10}\left(\cos(2\log x) + 2\,\text{sen}(2\log x)\right) + \frac{x}{2} + C$$

$$\Rightarrow \boxed{\int \cos^2(\log x)\,dx = \frac{x}{10}\cos(2\log x) + \frac{x}{5}\text{sen}(2\log x) + \frac{x}{2} + C}$$

$$\int \frac{dx}{(x+2)\sqrt{1+x}}$$

$$\int \frac{dx}{(x+2)\sqrt{1+x}} = \int \frac{2u\,du}{(u^2+1)\,u} = \int \frac{2\,du}{u^2+1}$$

$$u = \sqrt{1+x} \Rightarrow x = u^2 - 1$$

$$du = \frac{dx}{2\sqrt{1+x}} \Rightarrow dx = 2u\,du$$

$$= 2\int \frac{du}{u^2+1} = 2\arctan(u) + c$$

$$= 2\arctan\left(\sqrt{1+x}\right) + c$$

$$\Rightarrow \boxed{\int \frac{dx}{(x+2)\sqrt{1+x}} = 2\arctan\left(\sqrt{1+x}\right) + c}$$

$$\int \frac{dx}{1+\sqrt[3]{1+x}}$$

$$\int \frac{dx}{1+(1+x)^{1/3}} \underset{\uparrow}{=} \int \frac{3u^2}{1+u}\,du$$

$$\boxed{\begin{array}{l} u=(1+x)^{1/3} \\ du = \frac{1}{3}(1+x)^{-2/3}dx \Rightarrow dx = 3u^2du \end{array}}$$

Usando:

$$\begin{array}{l|l} \cancel{u^2}+0\cdot u+0 & u+1 \\ \hline \cancel{u^2}+u & u-1 \\ \hline \quad -\cancel{u}+0 & \\ \quad -\cancel{u}-1 & \\ \hline \qquad 1 & \end{array} \Rightarrow \frac{u^2}{u+1} = u-1+\frac{1}{u+1}$$

$$\Rightarrow \int \frac{dx}{1+(1+x)^{1/3}} = 3\int \frac{u^2du}{u+1} =$$

$$= 3\int \left(u-1+\frac{1}{u+1}\right)du$$

$$= \frac{3}{2}u^2 - 3u + 3\log|u+1| + C$$

$$\underset{\substack{\uparrow \\ u=(1+x)^{1/3}}}{=} \frac{3}{2}(1+x)^{2/3} - 3(1+x)^{1/3} + 3\log\left|1+(1+x)^{1/3}\right| + C$$

$$\Rightarrow \boxed{\int \frac{dx}{1+(1+x)^{1/3}} = \frac{3}{2}(1+x)^{2/3} - 3(1+x)^{1/3} + 3\log\left|1+(1+x)^{1/3}\right| + C}$$

$$\int \frac{dx}{\sqrt{e^{2x}-1}}$$

$$\int \frac{dx}{\sqrt{e^{2x}-1}} = \int \frac{1}{u} \cdot \frac{u}{u^2+1}\, du = \int \frac{du}{u^2+1}$$

$$\boxed{\begin{array}{l} u = \sqrt{e^{2x}-1} \Rightarrow e^{2x} = u^2+1 \\ du = \dfrac{e^{2x}}{\sqrt{e^{2x}-1}}\, dx \Rightarrow dx = \dfrac{u}{u^2+1}\, du \end{array}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{e^{2x}-1}} = \arctan(u) + c = \arctan\left(\sqrt{e^{2x}-1}\right) + c$$

$$\boxed{\int \frac{dx}{\sqrt{e^{2x}-1}} = \arctan\left(\sqrt{e^{2x}-1}\right) + c}$$

$$\int \frac{dx}{3x^2+4x+2}$$

El polinomio $3x^2+4x+2$ no tiene raíces reales:

$$3x^2+4x+2=0 \Leftrightarrow x=\frac{-4\pm\sqrt{16-24}}{6}\notin \mathbb{R}.$$

$$\Rightarrow 3x^2+4x+2 = 3\left\{x^2+\frac{4}{3}x+\frac{2}{3}\right\} =$$

completar cuadrados

$$= 3\left\{\left(x+\frac{2}{3}\right)^2-\frac{2^2}{3^2}+\frac{2}{3}\right\}$$

$$= 3\left\{\left(x+\frac{2}{3}\right)^2+\frac{2}{3^2}\right\}$$

$$\Rightarrow \int\frac{dx}{3x^2+4x+2} = \frac{1}{3}\int\frac{dx}{(x+2/3)^2+2/3^2} =$$

$$= \frac{3}{2}\int\frac{dx}{\frac{3^2}{2}(x+2/3)^2+1}$$

$$= \frac{3}{2}\int\frac{dx}{\left(\frac{3}{\sqrt{2}}(x+2/3)\right)^2+1}$$

$$= \frac{1}{\sqrt{2}}\int\frac{du}{u^2+1} = \frac{1}{\sqrt{2}}\arctan(u)+c$$

c.v.

$$u=\frac{3}{\sqrt{2}}\left(x+\frac{2}{3}\right)$$

$$dx=\frac{\sqrt{2}}{3}\,du$$

$$\boxed{\int\frac{dx}{3x^2+4x+2} = \frac{1}{\sqrt{2}}\arctan\left(\frac{3}{\sqrt{2}}x+\sqrt{2}\right)+c}$$

$$\int \frac{e^{4x}}{e^{2x}+2e^{x}+2}\,dx$$

$$\int \frac{e^{4x}}{e^{2x}+2e^{x}+2}\,dx = \int \frac{u^3}{u^2+2u+2}\,du$$

$$\boxed{\begin{array}{l} u = e^{x} \\ du = e^{x}dx \Rightarrow dx = \dfrac{du}{u} \end{array}}$$

Dividiendo: $\dfrac{u^3}{u^2+2u+2} = u-2+\dfrac{2u+4}{u^2+2u+2}$

$$\Rightarrow \int \frac{u^3}{u^2+2u+2}\,du = \frac{u^2}{2}-2u+\int \frac{2u+4}{u^2+2u+2}\,du$$

$$= \frac{u^2}{2}-2u+\int \frac{2u+2}{u^2+2u+2}\,du+\int \frac{2\,du}{u^2+2u+2}$$

$$\boxed{d(u^2+2u+2)=(2u+2)du}$$

$$= \frac{u^2}{2}-2u+\log\left(u^2+2u+2\right)$$

$$+\int \frac{2\,du}{u^2+2u+2}$$

Usando: $u^2+2u+2 \underset{\text{completar cuadrados}}{=} (u+1)^2+1$

$$\Rightarrow \int \frac{du}{u^2+2u+2} = \int \frac{du}{(u+1)^2+1} = \arctan(u+1)+C$$

$$\boxed{\int \frac{e^{4x}\,dx}{e^{2x}+2e^{x}+2} = \frac{e^{2x}}{2}-2e^{x}+\log\left(e^{2x}+e^{x}+2\right)+ \\ +2\arctan\left(e^{x}+1\right)+C}$$