Problema 3.1

(1.) $$\sum_{k=1}^{\infty} \frac{1}{k^2+k} = 1$$

$$\sum_{k=1}^{\infty} \frac{1}{k^2+k} = \sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right) =$$

$$= \lim_{n\to\infty} \underbrace{\sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1} \right)}_{\left(1-\cancel{\frac{1}{2}}\right)+\left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)+\cdots+\left(\cancel{\frac{1}{n}}-\frac{1}{n+1}\right)} = \lim_{n\to\infty} \left(1 - \frac{1}{n+1} \right) = 1$$

SERIE TELESCÓPICA:

$$\sum_{k=1}^{\infty} (b_k - b_{k+1}) = \lim_{n\to\infty} (b_1 - b_{n+1}) =$$

$$= b_1 - \lim_{n\to\infty} b_{n+1}$$

(2) $$\sum_{k=1}^{\infty} \frac{1+\operatorname{sen} k}{k^2+k}$$

Puesto que $$0 \le \frac{1+\operatorname{sen} k}{k^2+k} \le \frac{2}{k^2+k}$$

&

$$\sum_{k=1}^{\infty} \frac{2}{k^2+k} \text{ converge}$$

$$\Rightarrow \sum_{k=1}^{\infty} \frac{1+\operatorname{sen} k}{k^2+k} \text{ converge}$$

CRITERIO DE COMPARACIÓN (SERIES TERMINOS NO NEG.)

$$\left.\begin{array}{c} 0 \le a_k \le b_k \\ \& \\ \sum_k b_k \text{ converge} \end{array}\right\} \Rightarrow \sum_k a_k \text{ converge}$$

③ $\displaystyle\sum_{k=1}^{\infty} \frac{k+1}{k^2}$

Puesto que $\quad 0 \leq \dfrac{1}{k} \leq \dfrac{k+1}{k^2}$

&

$\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$ diverge

$\Rightarrow \displaystyle\sum_{k=1}^{\infty} \frac{k+1}{k^2}$ diverge

COMPARACIÓN: $\left.\begin{array}{c} 0 \leq a_k \leq b_k \\ \& \\ \sum_k a_k \text{ diverge} \end{array}\right\} \Rightarrow \sum_k b_k$ diverge

④ $\displaystyle\sum_{k=1}^{\infty} \frac{7\sqrt{k}+323}{k^2+\cos k}$

$$\frac{7\sqrt{k}+323}{k^2+\cos k} \underset{k\uparrow\infty}{\sim} \frac{7\sqrt{k}}{k^2} = 7k^{-3/2}$$

$$\left(\lim_{k\to\infty} \frac{7\sqrt{k}+323}{k^2+\cos k} \cdot \frac{k^2}{7\sqrt{k}} = 1\right)$$

Puesto que $\displaystyle\sum_{k=1}^{\infty} k^{-3/2}$ converge $\Rightarrow \displaystyle\sum_{k=1}^{\infty} \frac{7\sqrt{k}+323}{k^2+\cos k}$ converge

CRITERIO DE COMPARACIÓN (LÍMITE)

Si $a_k, b_k \geq 0$ & $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ se tiene que

$$\sum_k a_k \text{ converge} \Leftrightarrow \sum_k b_k \text{ converge}$$

Si $a_k, b_k \geq 0$ & $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ se tiene que

$$\sum_k b_k \text{ converge} \Rightarrow \sum_k a_k \text{ converge}$$

⑤ $\displaystyle\sum_{k=1}^{\infty} \frac{\arctan k}{k^2+7}$

$$\frac{\arctan k}{k^2+7} \sim \frac{1}{k^2} \qquad \left(\lim_{k\to\infty} \frac{\arctan k}{k^2+7}\cdot k^2 = \pi/2 \right)$$

Puesto que $\displaystyle\sum_{k=1}^{\infty} k^{-2}$ converge $\Rightarrow \displaystyle\sum_{k=1}^{\infty} \frac{\arctan k}{k^2+7}$ converge

⑥ $\displaystyle\sum_{k=1}^{\infty} \frac{1}{3^k+(-1)^k}$

$$\frac{1}{3^k+(-1)^k} \sim \frac{1}{3^k} \qquad \left(\lim_{k\to\infty} \frac{1}{3^k+(-1)^k}\cdot 3^k = 1 \right)$$

Puesto que $\displaystyle\sum_{k=1}^{\infty} \frac{1}{3^k}$ converge $\Rightarrow \displaystyle\sum_{k=1}^{\infty} \frac{1}{3^k+(-1)^k}$ converge

$$\sum_{k=1}^{\infty} \frac{1}{3^k} = \frac{1/3}{1-1/3} = 1/2 \quad \text{(geométrica)}$$

⑦ $\displaystyle\sum_{k=1}^{\infty} \frac{\log k}{k^4}$

$$\lim_{k\to\infty} \frac{\log k}{k^4}\cdot k^3 = \lim_{k\to\infty} \frac{\log k}{k} = 0$$

&

$\displaystyle\sum_k k^{-3}$ converge

$\Rightarrow \displaystyle\sum_{k=1}^{\infty} \frac{\log k}{k^4}$ converge

(8) $\displaystyle\sum_{k=1}^{\infty} \frac{\log k}{k}$

$$\left.\begin{array}{l} 0 \leq \dfrac{1}{k} \leq \dfrac{\log k}{k}, \ \forall k \geq 4 \\ \displaystyle\sum_k \frac{1}{k} \text{ diverge} \end{array}\right\} \Rightarrow \sum_k \frac{\log k}{k} \text{ diverge}$$

(9) $\displaystyle\sum_{k=1}^{\infty} \frac{\log k}{k^2}$

$$\lim_{k\to\infty} \frac{\log k}{k^2} \cdot k^{3/2} = \lim_{k\to\infty} \frac{\log k}{\sqrt{k}} = 0$$

&

$\sum k^{-3/2}$ converge

$$\Rightarrow \sum_{k=1}^{\infty} \frac{\log k}{k^2} \text{ converge}$$

(10) $\displaystyle\sum_{k=1}^{\infty} \frac{(k+1)^k}{k^{k+1}}$

$$\left.\begin{array}{c} \dfrac{(k+1)^k}{k^{k+1}} \geq \dfrac{k^k}{k^{k+1}} = \dfrac{1}{k} \geq 0 \\ \& \\ \displaystyle\sum_k \frac{1}{k} \text{ diverge} \end{array}\right\} \Rightarrow \sum_k \frac{(k+1)^k}{k^{k+1}} \text{ diverge}$$