PROBLEMA 7.14 $\sqrt[3]{28}$

Consideremos
$$f(x) = (27+x)^{1/3} \Rightarrow f(0) = 3$$
$$f'(x) = \frac{1}{3}(27+x)^{-2/3} \Rightarrow f'(0) = 1/3^3$$
$$f''(x) = -\frac{2}{3^2}(27+x)^{-5/3} \Rightarrow f''(0) = -2/3^7$$
$$f'''(x) = \frac{2.5}{3^3}(27+x)^{-8/3}$$

$$\Rightarrow \begin{cases} P_2(x|f,0) = 3 + x/3^3 - x^2/3^7 \\ R_2(x|f,0) = \dfrac{5x^3}{3^4(27+c)^{8/3}} \quad \text{con } c \in (0,x) \end{cases}$$

$$\boxed{(27+x)^{1/3} = 3 + \frac{x}{3^3} - \frac{x^2}{3^7} + \frac{5x^3}{3^4(27+c)^{8/3}} \quad \text{con } c \in (0,x)}$$

En particular, tomando $x = 1$:
$$28^{1/3} = 3 + \frac{1}{3^3} - \frac{1}{3^7} + \frac{5}{3^4(27+c)^{8/3}} \quad \text{con } c \in (0,1)$$

$$\Rightarrow \begin{cases} 28^{1/3} \simeq 3 + \dfrac{1}{3^3} - \dfrac{1}{3^7} = 3.0365797\cdots \\ \text{Error} = \left|\dfrac{5}{3^4(27+c)^{8/3}}\right| < \dfrac{5}{3^{12}} < \dfrac{1}{2}\cdot 10^{-4} \end{cases}$$

$c \in (0,1)$
$\Rightarrow$ tomar $c = 0$

Obs: $28^{1/3} = \underbrace{3.03658}_{\text{cifras exactas de la aproximación}}89718\cdots$