PROBLEMA 7.10

Obs: $\left.\begin{array}{l} f(x) = o(x^p) \\ \text{cuando } x \to 0 \end{array}\right| \Leftrightarrow \lim_{x\to 0} \frac{f(x)}{x^p} = 0$

$\left.\begin{array}{l} f(x) = o(x^p) \\ \text{cuando } x \to \infty \end{array}\right| \Leftrightarrow \lim_{x\to \infty} \frac{f(x)}{x^p} = 0$

- $\forall a<1$: $\operatorname{sen} x = o(x^a)$ cuando $x \to 0$:

$$\lim_{x\to 0} \frac{\operatorname{sen} x}{x^a} = \lim_{x\to 0} \frac{x + o(x)}{x^a} = \underbrace{\lim_{x\to 0} x^{1-a}}_{0} + \underbrace{\lim_{x\to 0} \frac{o(x)}{x^a}}_{0}$$

ya que $1-a>0$

$$= 0$$

- $\log(1+x^2) = o(x)$ cuando $x \to 0$:

$$\lim_{x\to 0} \frac{\log(1+x^2)}{x} = \lim_{x\to 0} \frac{x^2 + o(x^2)}{x} = 0$$

$\log(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \cdots$

- $\log x = o(x)$ cuando $x \to \infty$

$$\lim_{x\to \infty} \frac{\log x}{x} \underset{\frac{\infty}{\infty}}{\overset{\text{L'H}}{=}} \lim_{x\to\infty} \frac{1/x}{1} = 0$$

• $\tan(x) - \operatorname{sen}(x) = o(x^2)$ cuando $x \longrightarrow 0$

$$\lim_{x\to 0} \frac{\tan(x) - \operatorname{sen}(x)}{x^2} = \lim_{x\to 0} \frac{x + o(x^2) - x + o(x^2)}{x^2} = 0$$

$$\tan x = x + \frac{x^3}{3} + o(x^3)$$

$$\operatorname{sen} x = x - \frac{x^3}{3!} + o(x^3)$$