PROBLEMA 7.16 $\quad f(x) = e^x$

$$f(x) = e^x \Rightarrow f^{(k)}(x) = e^x \quad \forall k = 0,1,2,3,\ldots$$

$$R_n(x|\exp,0) = \frac{e^c}{(n+1)!} x^{n+1} \quad \text{con } c \in (0,x)$$

$$\Rightarrow \text{Error}(x) = \left| \frac{e^c}{(n+1)!} x^{n+1} \right| = \frac{e^c}{(n+1)!} |x|^{n+1}$$

- Si $x \in [-1,1] \Rightarrow |x| \le 1$ y podemos escribir:

$$\text{Error}(x) = \frac{e^c}{(n+1)!} |x|^{n+1} \le \frac{e^c}{(n+1)!} \quad \text{con } c \in (0,x)$$

$$\underset{\substack{c \in (0,x) \\ \& \\ x \in [-1,1]}}{<} \frac{e}{(n+1)!} < \frac{3}{(n+1)!}$$

$$\Rightarrow \text{Error}(x) < \frac{3}{(n+1)!}$$

- Si queremos que la aproximación proporcione 3 cifras decimales exactas basta imponer:

$$\frac{3}{(n+1)!} < \frac{1}{2} \cdot 10^{-3} \iff (n+1)! > 6000$$

Por tanto basta tomar $n = 7$:

$$\boxed{e^x \simeq 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!}}$$

para $x \in [-1,1]$ proporciona 3 cifras decimales exactas.