PROBLEMA 7.15 $f(x) = \cos x + e^x$

$f(x) = \cos x + e^x$ $\Rightarrow$ $f(0) = 2$

$f'(x) = -\operatorname{sen} x + e^x$ $\Rightarrow$ $f'(0) = 1$

$f''(x) = -\cos x + e^x$ $\Rightarrow$ $f''(0) = 0$

$f'''(x) = \operatorname{sen} x + e^x$ $\Rightarrow$ $f'''(0) = 1$

$f^{(4)}(x) = \cos x + e^x$

$$\Rightarrow \begin{cases} P_3(x\mid f, 0) = 2 + x + \dfrac{x^3}{6} \\[3mm] R_3(x\mid f, 0) = \dfrac{\cos(c) + e^c}{4!} x^4 \quad \text{con } c \in (0, x) \end{cases}$$

$$\boxed{\cos x + e^x = 2 + x + \frac{x^3}{6} + \frac{\cos(c) + e^c}{4!} x^4 \\ \text{con } c \in (0, x)}$$

Aproximación: $\cos x + e^x \approx 2 + x + \dfrac{x^3}{6}$

Error: $E(x) := \left| \dfrac{\cos c + e^c}{4!} x^4 \right| \leq \dfrac{1 + e^c}{4!} x^4$

$$\text{con } c \in (0, x)$$

- Si $x \in [-1/4, 1/4]$:

$$E(x) \leq \underbrace{\frac{1 + e^c}{4!} x^4}_{\substack{\uparrow \\ x \in [-1/4, 1/4] \\ c \in (0, x)}} \leq \frac{1 + 2}{4!} \left(\frac{1}{4}\right)^4 = \frac{1}{2^{11}}$$

$\Rightarrow$ Si $x \in [-1/4, 1/4]$ el error es menor que $\dfrac{1}{2^{11}}$