PROBLEMA 10.10

a) $F(x) = \int_0^x (1 + \text{sen}(\text{sen}\, t))\, dt$

- $F'(x) = 1 + \text{sen}(\text{sen}(x)) \geq 0$
- $F'(x) = 0 \Leftrightarrow \text{sen}(\text{sen}(x)) = -1$

$\Leftrightarrow \text{sen}(x) = -\frac{\pi}{2} + 2\pi k$

Como $\left| -\frac{\pi}{2} + 2\pi k \right| > 1 \quad \forall k \in \mathbb{Z}$

$\nexists x \in \mathbb{R} : \text{sen}(x) = -\frac{\pi}{2} + 2\pi k$

$\nexists x \in \mathbb{R} : F'(x) = 0$

- Por tanto: $\underline{\underline{F'(x) > 0 \quad \forall x \in \mathbb{R}}} \Rightarrow F$ es monótona creciente.

$\Rightarrow F$ es inyectiva.

- Puesto que $F(0) = 0 \Rightarrow F^{-1}(0) = 0$. Por tanto:

$$(F^{-1})'(0) = \frac{1}{F'(0)} = \frac{1}{1 + \text{sen}(\text{sen}(0))} = 1$$

b) $G(x) = \int_1^x \operatorname{sen}(\operatorname{sen} t)\, dt \;;\quad x \in \mathbb{R}.$

$$G(x) \doteq \int_1^x \operatorname{sen}(\operatorname{sen} t)\, dt =$$

$$= \underbrace{\int_1^0 \operatorname{sen}(\operatorname{sen} t)\, dt}_{\text{constante } G_0} + \underbrace{\int_0^x \operatorname{sen}(\operatorname{sen} t)\, dt}_{H(x)}$$

Analicemos $H(x) := \int_0^x \operatorname{sen}(\operatorname{sen} t)\, dt$:

$$H(-x) = \int_0^{-x} \operatorname{sen}(\operatorname{sen} t)\, dt$$

c.v. $\boxed{\begin{array}{l} u = -t \\ du = -dt \end{array}}$

$$= \int_0^x \operatorname{sen}(\operatorname{sen}(-u)) \cdot (-du)$$

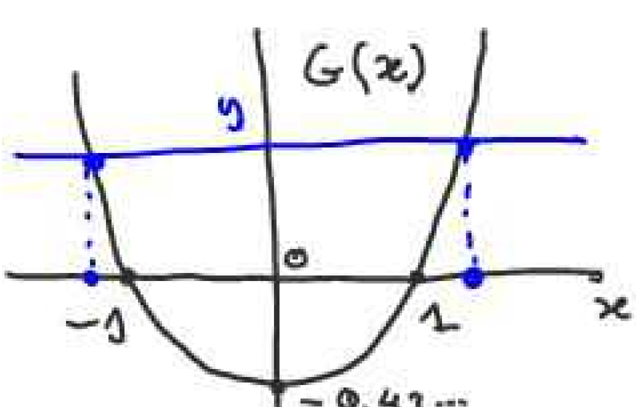
$$= \int_0^x \operatorname{sen}(\operatorname{sen} u)\, du = H(x)$$

Función PAR.

$$\Rightarrow G(x) = G_0 + H(x) \Rightarrow G(-x) = G_0 + H(-x) =$$

$$= G_0 + H(x) = G(x)$$

Función PAR



$\nexists\, G^{-1}$