Problema 3.2

① $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2+k}$

$$\left.\begin{array}{l} \left|\frac{(-1)^{k+1}}{k^2+k}\right| = \frac{1}{k^2+k} \\ \& \\ \sum_{k=1}^{\infty} \frac{1}{k^2+k} \text{ converge} \end{array}\right\} \Rightarrow \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2+k} \text{ converge absolutamente}$$

② $\sum_{k=1}^{\infty} \frac{\cos k}{5^k}$

$$\left.\begin{array}{l} 0 \leq \left|\frac{\cos k}{5^k}\right| \leq \frac{1}{5^k} \\ \& \\ \sum_{k} \frac{1}{5^k} \text{ converge} \end{array}\right\} \Rightarrow \sum_{k=1}^{\infty} \frac{\cos k}{5^k} \text{ converge absolutamente}$$

③ $\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ no converge absolutamente (ya que $\sum_k \frac{1}{k}$ diverge)

pero sí converge condicionalmente (criterio de LEIBNIZ)

- CRITERIO DE LEIBNIZ:

$a_k \geq 0$ & DECRECIENTE & $a_k \xrightarrow[k \to \infty]{} 0$

$\Rightarrow \sum (-1)^k a_k$ CONVERGE

(4) $\sum_{k=1}^{\infty} \frac{(-4)^k}{4+k!}$

$a_k = \frac{(-4)^k}{4+k!}$

$\lim_{k\to\infty} \frac{|a_{k+1}|}{|a_k|} = \lim_{k\to\infty} \frac{4^{k+1}}{4+(k+1)!} \cdot \frac{4+k!}{4^k} =$

$= 4 \lim_{k\to\infty} \frac{4+k!}{4+(k+1)!} = 0 < 1$

$\sum_{k=1}^{\infty} \frac{(-4)^k}{1+k!}$ converge por el CRITERIO DEL COCIENTE (absolutamente)

CRITERIO DEL COCIENTE:

$r = \lim_{k\to\infty} \frac{|a_{k+1}|}{|a_k|}$: Si $r<1$ $\sum_k a_k$ converge absolut.

Si $r>1$ $\sum_k a_k$ diverge

(ó $r=\infty$)

(5) $\sum_{k=1}^{\infty} (-1)^k 3^k 5^{-\sqrt{k}}$

$a_k = (-1)^k 3^k 5^{-\sqrt{k}}$

$|a_k| = 3^k 5^{-\sqrt{k}}$; $\sqrt[k]{|a_k|} = 3 \cdot 5^{-\frac{1}{\sqrt{k}}} \xrightarrow[k\to\infty]{} 3 > 1$

$\Rightarrow \sum_{k=1}^{\infty} (-1)^k 3^k 5^{-\sqrt{k}}$ diverge por el CRITERIO DE LA RAÍZ

CRITERIO DE LA RAÍZ

$r = \lim_{k\to\infty} \sqrt[k]{|a_k|}$: Si $r<1$: $\sum_k a_k$ converge absolut.

Si $r>1$: $\sum_k a_k$ diverge

(ó $r=\infty$)

⑥ $\sum_{k=2}^{\infty} \frac{1}{(\log k)^k}$

$$a_k = \frac{1}{(\log k)^k} \Rightarrow \sqrt[k]{|a_k|} = \frac{1}{|\log k|} \xrightarrow[k\to\infty]{} 0 < 1$$

Por el CRITERIO DE LA RAÍZ:

$\sum_{k=2}^{\infty} \frac{1}{(\log k)^k}$ converge

⑦ $\sum_{k=1}^{\infty} \frac{k^a}{b^k}$; $a_k = \frac{k^a}{b^k}$ $(a>0 ; b \neq 0)$

$$\frac{|a_{k+1}|}{|a_k|} = \frac{(k+1)^a}{|b|^{k+1}} \cdot \frac{|b|^k}{k^a} = \left(1+\frac{1}{k}\right)^a \cdot \frac{1}{|b|}$$

$$\Rightarrow \lim_{k\to\infty} \frac{|a_{k+1}|}{|a_k|} = \frac{1}{|b|}$$

- Si $\frac{1}{|b|} < 1 \Rightarrow |b| > 1$ converge
- Si $\frac{1}{|b|} > 1 \Rightarrow |b| < 1$ diverge
- Si $b = \pm 1 \Rightarrow \sum_{k=1}^{\infty} (\pm 1)^k k^a$ diverge $\forall a > 0$

  ya que $(\pm 1)^k k^a \not\to 0$

⑧ $\sum_{k=1}^{\infty} \frac{b^k}{k!}$

$$a_k = \frac{b^k}{k!} \quad ; \quad \frac{|a_{k+1}|}{|a_k|} = \frac{|b|^{k+1}}{(k+1)!} \cdot \frac{k!}{|b|^k} = \frac{|b|}{k+1}$$

$$\Rightarrow \lim_{k\to\infty} \frac{|a_{k+1}|}{|a_k|} = \lim_{k\to\infty} \frac{|b|}{k+1} = 0 < 1$$

$\sum_{k=1}^{\infty} \frac{b^k}{k!}$ converge absolutamente $\forall b$

(y, por tanto, $\frac{b^k}{k!} \xrightarrow[k\to\infty]{} 0$)

⑨ $\sum_{k=1}^{\infty} \frac{k!}{k^k} \quad ; \quad |a_k| = \frac{k!}{k^k}$

$$\frac{|a_{k+1}|}{|a_k|} = \frac{(k+1)!}{(k+1)^{k+1}} \cdot \frac{k^k}{k!} =$$

$$= \frac{(k+1)!}{k!} \, \frac{k^k}{(k+1)^{k+1}} = \frac{(k+1)\, k^k}{(k+1)^{k+1}}$$

$$= \frac{k^k}{(k+1)^k} = \frac{1}{(1+1/k)^k} \xrightarrow[k\to\infty]{} \frac{1}{e} < 1$$

$\Rightarrow \sum_{k=1}^{\infty} \frac{k!}{k^k}$ converge

⑩ $$\sum_{k=1}^{\infty} \log\left(\frac{k}{k+1}\right) = \sum_{k=1}^{\infty} \left(\log k - \log(k+1)\right) =$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\log k - \log(k+1)\right) =$$

$$\underset{\text{TELESCÓPICA}}{=} \log 1 - \lim_{n\to\infty} \log(n+1) = -\infty$$

DIVERGE.