Soluciones # 7

Forma normal de una transformación lineal

Problema 7.1

1. Las matrices de cambio de base son

$$T_{B_0B} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad T_{BB_0} = \begin{pmatrix} -1 & 2 & -5 \\ 0 & 0 & 1 \\ 1 & -1 & 2 \end{pmatrix}.$$

2. $[v]_B = (-12, 1, 7)^t$.

3. $w = (3, -2, 1)^t$.

Problema 7.2

1. La matriz $T_{B_1B_0}$ es

$$T_{B_1B_0} = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 \\ 2 & 2 & 2 & 1 \\ -1 & -1 & -1 & -1 \end{pmatrix}.$$

2. La matriz $T_{B_2B_0}$ es

$$T_{B_2B_0} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

3. La matriz $T_{B_2B_1}$ es

$$T_{B_2B_1} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 2 \end{pmatrix}.$$

4. $[p]_{B_0} = (0,-3,0,1)^t$, $\quad [p]_{B_1} = (-2,-5,-5,2)^t$, $\quad [p]_{B_2} = (-3,3,1,-1)^t$.

Problema 7.3

1. La matriz $A_{T,B}$ es

$$A_{T,B} = \frac{1}{7}\begin{pmatrix} -9 & 13 \\ -31 & 23 \end{pmatrix}.$$

2. $A_{T,B_2B_1} = I_2$, $\quad B_1 = B_0$, $\quad B_2 = ((1,-1)^t,(3,1)^t)$.

Problema 7.4

1. La matriz A_{T,B_0B} es

$$A_{T,B_0B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}.$$

2. $B_1 = B, \quad B_2 = ((1,0,0,1)^t, (0,1,1,0)^t, (0,1,-1,0)^t, (0,0,0,1)^t) \quad$ y

$$A_{T,B_2B_1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

Problema 7.5

1. La matriz A_{T,B_0B} es

$$A_{T,B_0B} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}.$$

2. Las bases que dan lugar a la forma canónica de T son

$$\begin{aligned} B_1 &= \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right), \\ B_2 &= \left((1,0,1)^t, (0,1,0)^t, (1,0,0)^t\right), \\ A_{T,B_1B_2} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \end{aligned}$$