Physics Exam 10^{th} Jan 2011

E1

$\vec{v}_0 = 0$

$|\vec{E}| = 640\ N/C$

$|\vec{v}(t)| = 1.2 \cdot 10^6\ m/s$

(a) t ?

The proton is accelerated due to $\vec{F}_e = e\vec{E} \Rightarrow$

$\Rightarrow$ Using the 2nd Newton's law:

$$eE = ma \rightarrow a = \frac{eE}{m} = \frac{1.6 \cdot 10^{-19} \cdot 640}{1.67 \cdot 10^{-27}}\ (m/s^2) =$$

$$= 6.13 \cdot 10^{10}\ m/s^2$$

As the force is uniform, the acceleration is uniform too $\rightarrow$ motion with uniform acc:

$$\vec{r} = \cancel{\vec{r}_0} + \cancel{\vec{v}_0} t + \frac{1}{2}\vec{a}t^2 \quad ; \quad \vec{v}(t) = \cancel{\vec{v}_0} + \vec{a}t$$

Supposing for example that the direction of $\vec{E}$ is along the x-axis, then:

$$x = \frac{1}{2}at^2 \quad ; \quad v(t) = at$$

From the 2nd equation we can derive "t": $v(t) = 1.2 \cdot 10^{6}\ m/s = 6.13 \cdot 10^{10}\ \frac{m}{s^2} \cdot t$

$$\rightarrow \boxed{t = 1.96 \cdot 10^{-5}\ s}$$

(b) x ? Using the 1st equation:

$$\boxed{x = \frac{1}{2} a t^2 = 11.77\ m}$$

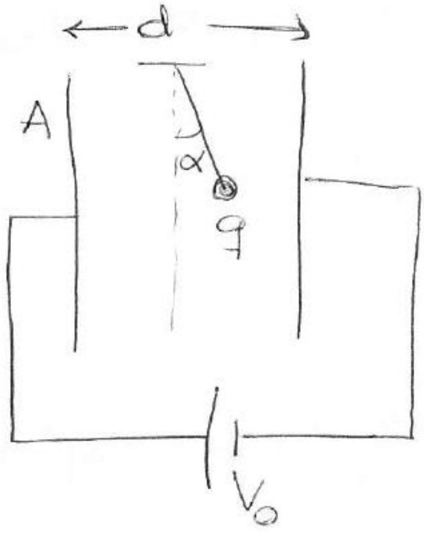
$1eV = 1.6 \cdot 10^{-19} J$

(c) $K = \frac{1}{2} m v^2 \underset{\uparrow}{=} 1.2 \cdot 10^{-15} J \overset{\downarrow}{=} 7515 eV$

$v(t) = 1.2 \cdot 10^{6}\ m/s$

$m_p = 1.67 \cdot 10^{-27}\ kg$

E2



(a) C, E, Q

$$\boxed{C = \varepsilon_0 \frac{A}{d} = \varepsilon_0 \cdot \frac{1.13\,m^2}{0.04\,m} = 2.5 \cdot 10^{-10}\,F = 250\,pF}$$

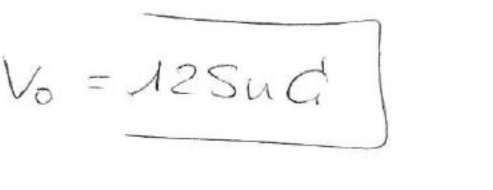
$$V_0 = E \cdot d \rightarrow \boxed{E = \frac{V_0}{d} = \frac{500\,V}{0.04\,m} = 12.5 \cdot 10^3\,V/m}$$

$$\boxed{Q = C \cdot V_0 = 125\,nC}$$

(b) The pendulum is in equilibrium; let us draw the forces acting on the system:

$|\vec{T}_x| = T\sin\alpha$

$|\vec{T}_y| = T\cos\alpha$



$\Rightarrow$ In principle, we do NOT know if $\vec{F}_e$ is directed along this or the opposite direction, as we do not know the sign of q. However, in order to have an equilibrium of forces, $\vec{F}_e$ must be directed along the direction indicated, so that:

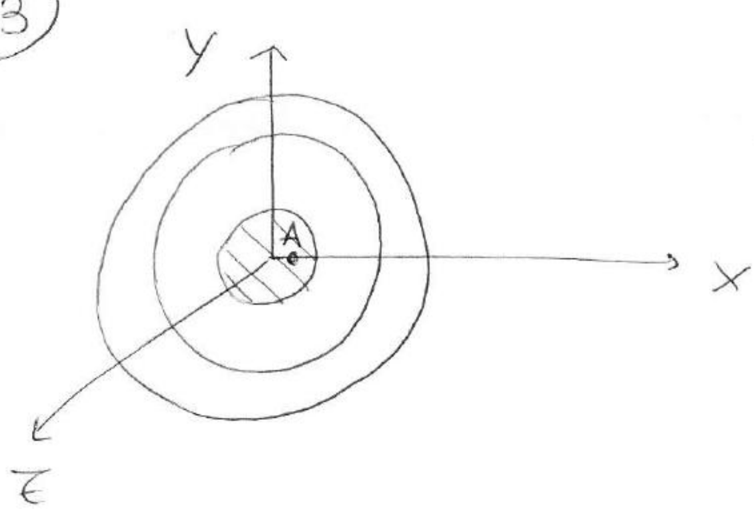
$$\vec{T}_x = \vec{F}_e \quad \text{and} \quad \vec{T}_y = m\vec{g}$$

$$T\cos\alpha = mg \rightarrow T = \frac{mg}{\cos\alpha}$$

$$T\sin\alpha = qE \rightarrow qE = \frac{mg}{\cos\alpha}\sin\alpha = mg\,tg\,\alpha \ ;$$

$$q = \frac{mg}{E}\,tg\,\alpha = \frac{2\cdot 10^{-3}\,\text{kg}\cdot 9{,}81\,\text{m/s}^2}{12.5\cdot 10^{3}\,\text{N/C}}\,tg\,30^\circ = 0.9\,\mu C$$

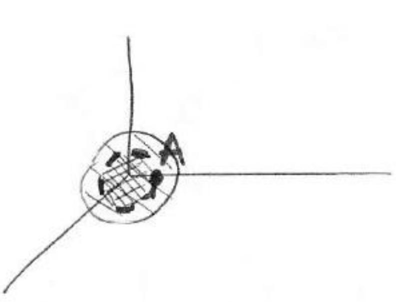
E3



solid sphere, R_1, ρ_0
metallic shell, R_2, R_3, Q

$$\left\{ \begin{array}{l} R_1 = 0.1\,\text{m} \\ R_2 = 0.3\,\text{m} \\ R_3 = 0.4\,\text{m} \end{array} \right.$$

A (5,0,0) cm → Inside the solid sphere.



We draw a gaussian surface passing through A. Only the charge <u>inside</u> that surface will contribute to $\vec{E}$.

$$Q_{ins} = \rho_0 \cdot V_{ins} = \rho_0 \cdot \frac{4}{3}\pi r^3 \quad ; \quad r = 0.05\,\text{m}$$

$$\oint \vec{E}\cdot d\vec{S} = \frac{Q_{ins}}{\varepsilon_0} \longrightarrow \oint \vec{E}\cdot d\vec{S} = E \cdot \oint dS = E \cdot 4\pi r^2$$

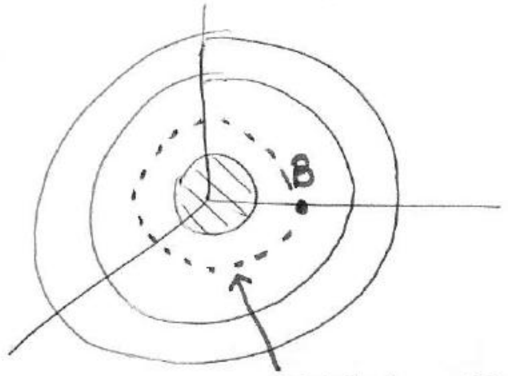
$$E \cdot \cancel{4\pi} r^{\cancel{2}} = \frac{1}{\varepsilon_0} \cdot \rho_0 \frac{\cancel{4}}{3} \cancel{\pi} r^{\cancel{3}} \longrightarrow \vec{E} = \frac{\rho_0 r}{3\varepsilon_0}\vec{e_x}$$

$\rho_0 < 0$ so finally, $\vec{E} = |\vec{E}|(-\vec{e}_x)$

$$\vec{E} = \frac{-4.8\cdot 10^{-3} \cdot 0{,}05}{3\varepsilon_0}\vec{e}_x = -90.4\cdot 10^{5}\,\vec{e}_x \;(N/C)$$

$\underline{\underline{B}}$ (20,0,0) cm

$r = 0.2\,m \rightarrow$ This point is located between the two spheres.



gaussian surface

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{ins}}{\varepsilon_0} \; ; \quad \oint \vec{E} \cdot d\vec{S} = E \cdot 4\pi r^2 \; ; \quad Q_{ins} = \rho_0 \cdot \frac{4}{3}\pi R_1^3$$

$$E \cdot 4\pi r^2 = \frac{\rho_0 \, 4/3 \, \pi R_1^3}{\varepsilon_0} \rightarrow \vec{E} = \frac{\rho_0 R_1^3}{3\varepsilon_0 r^2} \vec{e}_x$$

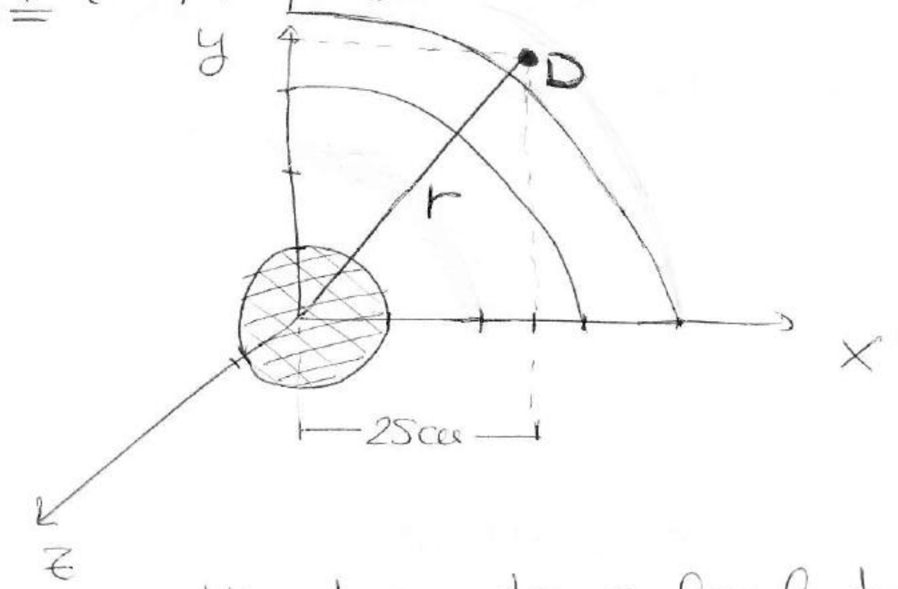
$$\vec{E} = -\frac{4.8 \cdot 10^{-3} \cdot (0{,}1)^3}{3\,\varepsilon_0\,(0{,}2)^2} \vec{e}_x = -45 \cdot 10^5 \vec{e}_x \; (N/C)$$

$\underline{\underline{C}}$ (38,0,0) cm

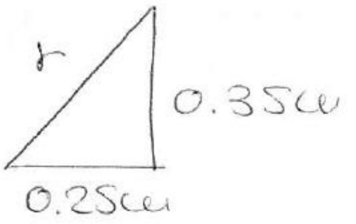
$r = 0.38\,m \rightarrow$ This point is located inside the metallic sphere.

$\vec{E} = 0$ (conductor)

$\underline{D}$ (25, 35, 0) cm



We have to calculate the distance to D:



$$r = \sqrt{0.35^2 + 0.25^2} =$$

$$= 0.43 \text{cm} \Rightarrow$$

$\Rightarrow$ Point D is external to all spheres.

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{ins}}{\varepsilon_0}$$

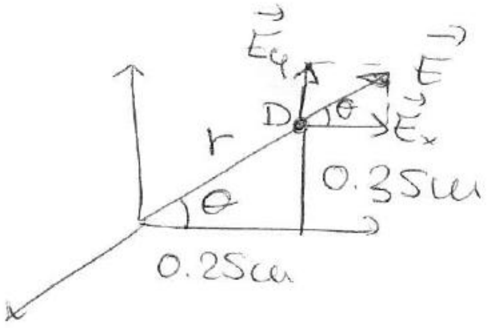
$$Q_{ins} = \left[\rho_0 \cdot \frac{4}{3} \pi R_1^3 + Q\right] = \left[-4.8 \cdot 10^{-3} \frac{4}{3} \pi (0.1)^3 + \right.$$

all the charge

$$\left. + 30 \cdot 10^{-6}\right] = -2.01 \cdot 10^{-5} + 30 \cdot 10^{-6} = 9.89 \cdot 10^{-6} \text{ C}$$

$$E \cdot 4\pi (0.43)^2 = \frac{9.89 \cdot 10^{-6} \text{ C}}{\varepsilon_0} \rightarrow \vec{E} = 4.8 \cdot 10^5 \vec{e}_r \left(\frac{N}{C}\right)$$

However, we have to express this in rectangular coordinats, so:



$$tg\theta = \frac{0.35}{0.25} = 1.4 \rightarrow \theta = 54.46^{o}$$

$$\vec{E} = \vec{E_x} + \vec{E_y} = |\vec{E}| \cdot \overbrace{\cos\theta}^{0.58}\, \vec{e_x} + |\vec{E}| \cdot \overbrace{\sin\theta}^{0.81}\, \vec{e_y}$$

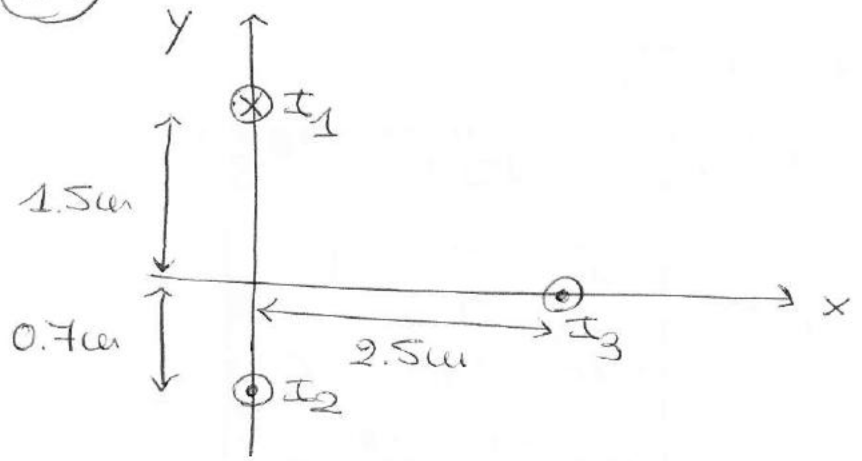
$$\vec{E} = 4.8 \cdot 10^{5} \left(0.58\, \vec{e_x} + 0.81\, \vec{e_y} \right) =$$

$$= \left(2.79\, \vec{e_x} + 3.9\, \vec{e_y} \right) \cdot 10^{5} \left(N/C \right)$$

Or, alternatively: $\vec{E} = |\vec{E}| \cdot \vec{u}$, where:

$\vec{u} = \frac{\vec{r}}{|\vec{r}|} = \frac{(0.25, 0.35, 0)}{0.43}$ → unit vector in the direction of the field.

(E4)



$I_1 = 2A$
$I_2 = 5A$
$I_3 = 10A$

(a) $\vec{B}(0,0,0) = \vec{B}_1(0,0,0) + \vec{B}_2(0,0,0) + \vec{B}_3(0,0,0)$

Infinite wires → We can apply Ampère's Law:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_c \rightarrow B \cdot 2\pi r = \mu_0 I_c$$

$\boxed{\vec{B}_1}$ $B_1 \cdot 2\pi(1.5) = \mu_0 \cdot I_1 \rightarrow \vec{B}_1 = \frac{4\pi \cdot 10^{-7} \cdot \cancel{2}}{\cancel{2}\pi \cdot (1.5)}(-\vec{e}_x)$

$\vec{B}_1 = 2.67 \cdot 10^{-7}(-\vec{e}_x)\ T$

$\boxed{\vec{B}_2}$ $B_2 \cdot 2\pi(0.7) = \mu_0 I_2 \rightarrow \vec{B}_2 = \frac{\overset{2}{\cancel{4}}\cancel{\pi} \cdot 10^{-7} \cdot 5}{\cancel{2}\cancel{\pi} \cdot (0.7)}(-\vec{e}_x)$

$\vec{B}_2 = 14.29 \cdot 10^{-7}(-\vec{e}_x)\ T$

$\boxed{\vec{B}_3}$ $B_3 \cdot 2\pi \cdot (2.5) = \mu_0 I_3 \rightarrow \vec{B}_3 = \frac{2\cancel{4}\cancel{\pi} \cdot 10^{-7} \cdot 10}{\cancel{2}\cancel{\pi} \cdot (2.5)}(-\vec{e}_y)$

$\vec{B}_3 = 8 \cdot 10^{-7}(-\vec{e}_y)\ T$

So: $\vec{B}(0,0,0) = (-16.96\,\vec{e}_x - 8\,\vec{e}_y) \cdot 10^{-7}\ T$

(b)

electron $\rightarrow$ $q_e = -1.6 \cdot 10^{-19}$ C

$\vec{v}_e = 3 \cdot 10^4 \vec{e}_x + 5 \cdot 10^4 \vec{e}_y$ (m/s)

$$\vec{F} = q \vec{v} \wedge \vec{B} = -1.6 \cdot 10^{-19} \cdot \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 \cdot 10^4 & 5 \cdot 10^4 & 0 \\ -16.96 \cdot 10^{-7} & -8 \cdot 10^{-7} & 0 \end{vmatrix} =$$

$$= 1.6 \cdot 10^{-19} \cdot 10^4 \cdot 10^{-7} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 5 & 0 \\ 16.96 & 8 & 0 \end{vmatrix} =$$

$$= -97.3 \cdot 10^{-22} \vec{k} \ (N)$$

Q1

In a semiconductor, the valence band (last band filled with e^-) and the conduction band (next band, where e^- can move freely, thus contributing to conductivity) are separated by a small ($\sim 1eV$) forbidden energy gap:

conduction energy band | CB

$\updownarrow E_g \rightarrow$ forbidden gap.

valence energy band | VB

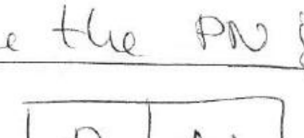
At T=0K, the solid is frozen and all valence e^- are participating in covalent bonds (no free e^-). Thinking in terms of band theory, this means that all e^- are on the VB, being the CB empty.

At T>0K, the lattice is vibrating some bonds break, so those e^- are now free to move. In terms of band theory, this means that some e^- in the VB have reveiced enough thermal energy to be able to jump through E_g onto the CB, being now free to contribute to conduction.

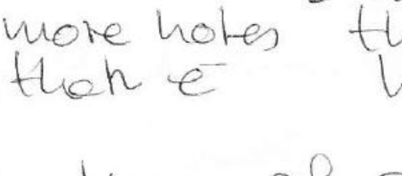
As T $\uparrow$ more and more, more e^- will jump through E_g and the conductivity $\uparrow$

Q2

A semic. diode is based on the PN junction.

When we have a PN junction: 

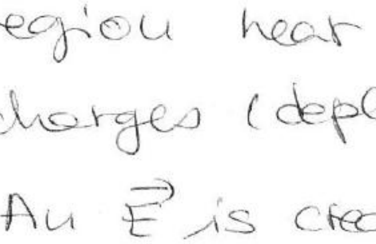
There is 1st diffusion of holes towards the N-side and e^- " " P-side. This induces the creation of a region near the junction depleted in free charges (depletion region):



An $\vec{E}$ is created on this region. It balances diffusion, and equilibrium is reached.

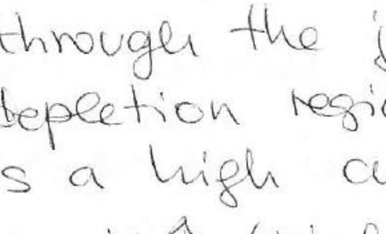
If we apply an external voltage, two situations can arise:

① Forward bias: { P-side connected to ⊕ terminal
N- " " " ⊖ "



As a consequence, the external $\vec{E}$ makes charge carriers to move through the junction, and the width of the depletion region decreases. In this situation, there is a high current passing through. The I ↑ as V ↑ (right side of the curve).

② Reverse bias: { P-side connected to ⊖ terminal
N-side " " ⊕ terminal



In this case, $\vec{E}_{ext}$ pushes e^- and holes away from the junction, and the width of the depletion region increases too. In this case, there is nearly no current passing through (left side of the curve).