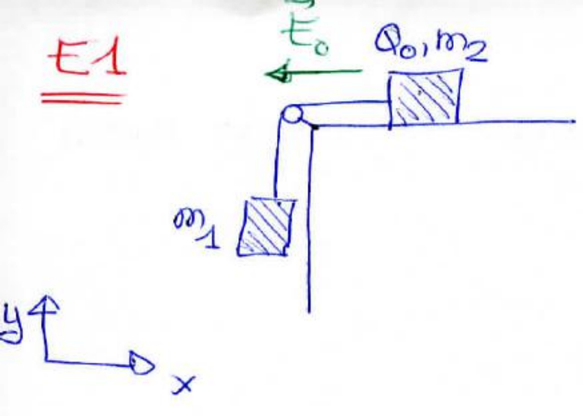
E1



$Q_0 = -2\cdot10^{-3}$ C

$m_1 = 8$ kg

$m_2 = 2$ kg

$\mu = 0.2$ (rough surface, there is friction acting on m_2)

$E_0 = 9\cdot10^{3}$ N/C

(a) The masses are attached so they move together, having the same acceleration.

The forces acting on the system are:

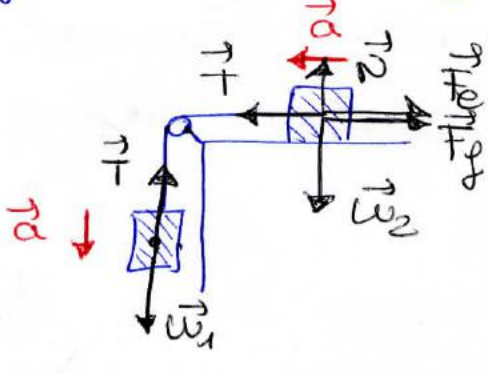
On m_1: the weight $\vec{w}_1 = m_1\vec{g} = 78.48\,\mathrm{N}\,(-\vec{j})$
the tension $\vec{T} = T\vec{j}$

On m_2: the weight $\vec{w}_2 = m_2\vec{g}$
the normal force $\vec{N} = N\vec{j}$
the tension $\vec{T} = -T\vec{i}$ (same tension on both masses)

the Coulomb force acting on the mass:
$\vec{F}_e = Q_0\vec{E}_0 = 18\,\mathrm{N}\,(\vec{i})$, as $Q_0 < 0$.

the friction force $|\vec{F}_f| = \mu\cdot N$

The system falls with $\vec{a} < \vec{g}$, moving as shown:



$|\vec{w}_1|$ is larger than $|\vec{F}_e|$ so m_1 will fall, dragging m_2 towards the left.

Applying the 2nd Newton's Law to each mass ($\sum_i \vec{F}_i = m\vec{a}$)

$\boxed{m_1}$ $m_1 g - T = m_1 a$ (Y-axis) ⊛

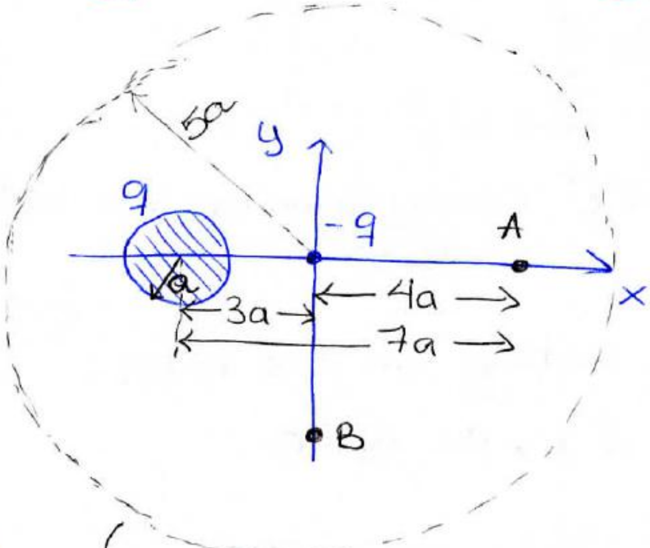
$\boxed{m_2}$ $\begin{cases} N = m_2 g \text{ (Y-axis)} \rightarrow N = 19.62\,N \rightarrow F_f = \mu N = \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad = 3.924\,N \\ T - F_{el} - F_f = m_2 a \text{ (X-axis)} \end{cases}$

$\hookrightarrow T - (18N) - (3.924N) = m_2 a \rightarrow T = m_2 a + 21.92\,N$

⊛ $m_1 g - (m_2 a + 21.92N) = m_1 a \rightarrow \boxed{a = 5.65\ m/s^2}$

(b) $\boxed{T = m_2 a + 21.92N = 33.23\,N}$

E2

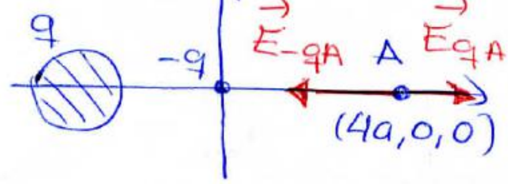


(a) Gaussian sphere of radius 5a centred on the point charge.

$\Phi = \oint \vec{E}\cdot d\vec{S} \rightarrow$ However, $\vec{E}$ is <u>not</u> constant along the chosen gaussian surface so this integral is not easy.

However, we also now that $\Phi = \frac{Q_{ins}}{\varepsilon_0}$

$Q_{ins} = q - q = 0 \rightarrow \boxed{\Phi = 0}$



(b) $\underline{\underline{\vec{E}_A}}$: $\vec{E}_A = \underbrace{\vec{E}_{qA}}_{\text{sphere}} + \underbrace{\vec{E}_{-qA}}_{\text{point charge}}$

$\vec{E}_{qA}$ can be found using Gauss' Law:

$\Phi = \oint \vec{E} \cdot d\vec{S} = E \cdot S = E \cdot 4\pi r^2$

we choose a surface (spherical) concentric with the sphere and passing through A. $\vec{E} \parallel d\vec{S}$ and constant along S.

"r" is the distance from the centre of the Gaussian surface to A $\rightarrow r = 7a$

$\Phi = E \cdot 4\pi (7a)^2 = \frac{Q_{ins}}{\varepsilon_0} = \frac{q}{\varepsilon_0} \rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{q}{(7a)^2}$

(magnitude)

$\vec{E}_{qA} = \frac{1}{4\pi\varepsilon_0} \frac{q}{(7a)^2} \vec{\imath}$ → "q" is outside the sphere, so $\vec{E}$ coincides with the one given by Coulomb's Law.

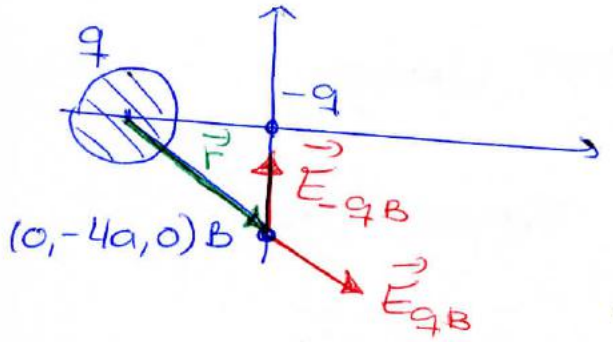
Moreover, $\vec{E}_{-qA}$ is given by Coulomb's Law:

$\vec{E}_{-qA} = \frac{1}{4\pi\varepsilon_0} \frac{q}{(4a)^2} (-\vec{\imath})$

So then, applying the superposition principle:

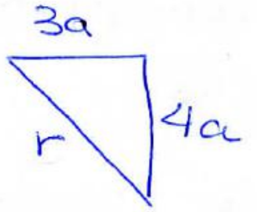
$$\vec{E}_A = \frac{q}{4\pi\varepsilon_0 a^2}\left(\frac{1}{49} - \frac{1}{16}\right)\vec{\imath} = -\frac{33q}{3136\pi\varepsilon_0 a^2}\,\vec{\imath}$$

$\underline{\underline{\vec{E}_B}}$: $\quad \vec{E}_B = \vec{E}_{qB} + \vec{E}_{-qB}$



$$\vec{E}_{-qB} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(4a)^2}\,\vec{\jmath}$$

$\vec{E}_{qB}$ is given by a similar expression (as B is also outside the charged sphere), being the distance "r" between the centre of the sphere and B : $r = 5a$, as:



$$\rightarrow r^2 = (3a)^2 + (4a)^2 \rightarrow r = 5a$$

So $\vec{E}_{qB} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(5a)^2}\,\vec{u}_r$, where $\vec{u}_r = \frac{\vec{r}}{r}$

$$\vec{r} = (0,-4a,0) - (-3a,0,0) = (3a,-4a,0)$$

$$\vec{E}_{qB} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(5a)^2}\left(3/5, -4/5, 0\right)$$

Finally, applying the superposition principle:

$$\vec{E}_B = \frac{q}{4\pi\varepsilon_0 a^2}(0.024, 0.031, 0)$$

(c)

$$\underset{A\to B}{W_{ext}} = \Delta \underset{\substack{\uparrow \\ \text{potential } \underline{\text{energy}}}}{U_{AB}} = Q\cdot(\Delta V_{AB}) = Q\,(V_B - V_A)$$

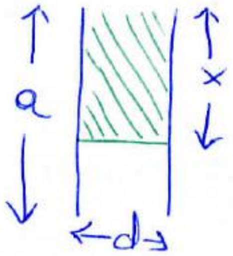
$$V_A = V_{qA} + V_{-qA} = k\,\frac{q}{7a} - k\,\frac{q}{4a} = -\frac{3q}{112\pi\varepsilon_0 a}\,(V)$$

$$V_B = V_{qB} + V_{-qB} = k\,\frac{q}{5a} - k\,\frac{q}{4a} = -\frac{q}{80\pi\varepsilon_0 a}\,(V)$$

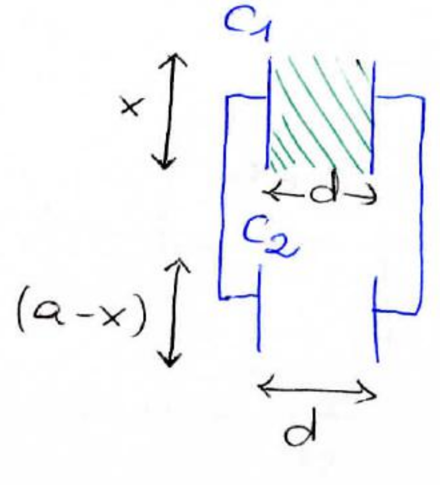
$$V_B - V_A = \frac{q}{70\pi\varepsilon_0 a} \longrightarrow \boxed{\underset{A\to B}{W_{ext}} = \frac{Qq}{70\pi\varepsilon_0 a}}$$

E3

(a) when the mass M is placed on the scales, the dielectric dscends a distance "x" into the capacitor, so that it fills part of the capacitor:



→ This system is equivalent to two capacitors connected in parallel:



The equivalent capacitance of this system is the sum of the two:

$$C_{eq} = C_1 + C_2 = \frac{\overset{2}{\varepsilon_r}\varepsilon_0 a x}{d} + \frac{\varepsilon_0 a (a-x)}{d} \rightarrow$$

$$\rightarrow C_{eq} = \frac{2\varepsilon_0 a x}{d} + \frac{\varepsilon_0 a^2}{d} - \frac{\varepsilon_0 a x}{d} = \frac{\varepsilon_0 a (a+x)}{d} \rightarrow$$

$$\rightarrow x = \frac{C_{eq} \cdot d}{\varepsilon_0 a} - a = 4.46 \text{ mm}$$

$$C_{eq} = 6.4 \cdot 10^{-13} \text{ F}$$
$$d = 0.2 \cdot 10^{-2} \text{ m}$$
$$a = 10^{-2} \text{ m}$$

Then, applying the equilibrium of forces:

$$F_{el} = W \rightarrow kx = Mg \rightarrow \boxed{M = \frac{kx}{g} = 36.4 \text{ g}}$$

(b) The capacitor is disconnected after charging, so the charge ($q_0 = 2 \cdot 10^{-10}$ C) remains constant. Then:

$$\boxed{\frac{V}{V_0} = \frac{q_0/C_{eq}}{q_0/C_{in}} = \frac{C_{in}}{C_{eq}} = 0.69}$$

$$C_{in} = \frac{\varepsilon_0 a^2}{d} = 4.425 \cdot 10^{-13} \text{ F}$$

E4

$\swarrow$ mass of a proton

$q_N = +2e$; $m_N = 2m_p$

$q_H = +2e$; $m_H = 2m_p + 2m_N \simeq 4m_p$

What is the trajectory described by the particls ?

→ Meanwhile they are inside the $\vec{B}$ region, they describe $\frac{1}{2}$ a circumference (as $\vec{v} \perp \vec{B}$).
The radii of the circls will depend on the particle as they masses are different, and:

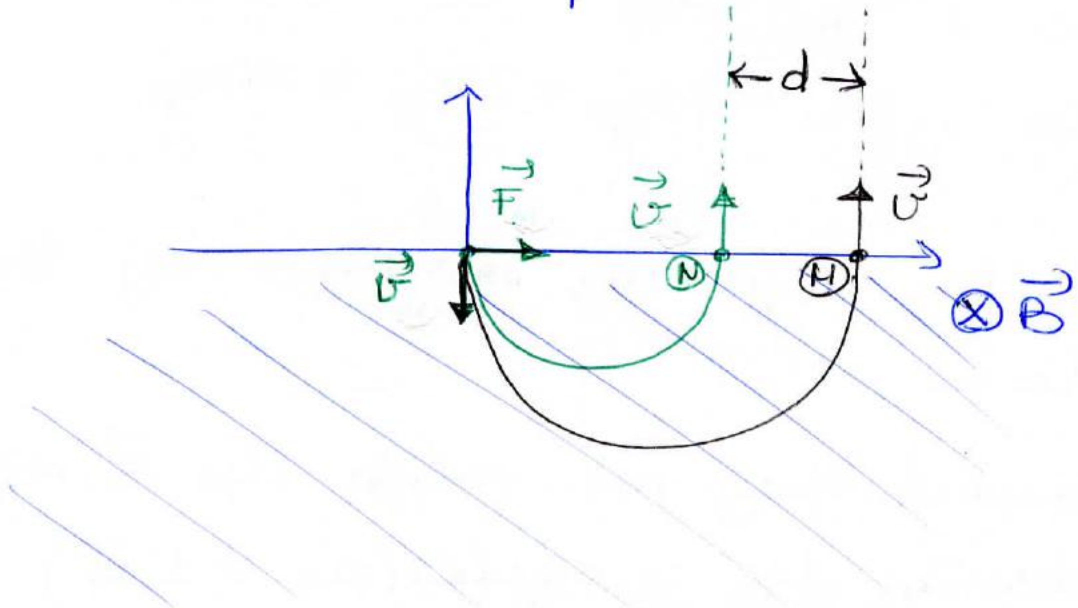
$$|\vec{F}_m| = qvB \underset{\substack{\uparrow \\ \text{uniform circular motion}}}{=} m a_c = m\frac{v^2}{R} \rightarrow R = \frac{mv}{qB}$$

There is no linear acceleration (only centripetal acceleration) so $|\vec{v}|$ is constant during the time that the particls remain in the region. This time is half of the period of the circumference:

$$t = \frac{T}{2} \underset{\substack{\uparrow \\ 2\pi R = v \cdot T}}{=} \frac{\pi R}{v}$$

→ When the particls go out of the $\vec{B}$ region, they will follow a linear trajectory with constant $\vec{v}$ (as there is no force acting on them).

so then, the trajectories would be:



(a) The distance between trajectories "d" is:

$$d = 2R_M - 2R_N = 2(R_M - R_N)$$

$$\left.\begin{aligned} R_M &= \frac{m_M v}{q_M B} = \frac{2 m_p v}{eB} \\ R_N &= \frac{m_N v}{q_N B} = \frac{m_p v}{eB} \end{aligned}\right\} \rightarrow \boxed{d = 33.4\,\text{mm}}$$

(b) $\boxed{t_M = \frac{T_M}{2} = \frac{\pi R_M}{v} = 2.62 \cdot 10^{-8}\,\text{s}}$

$\boxed{t_N = \frac{\pi R_N}{v} = 1.31 \cdot 10^{-8}\,\text{s}}$

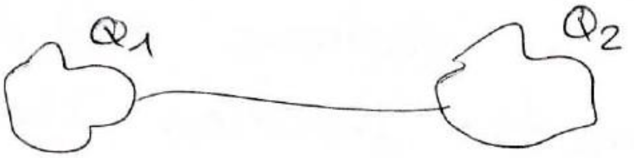
(c) The kinetic energy is constant through all the motion, and remains constant when the particls go out of the $\vec{B}$ region, as "v" does not change. So:

$$\boxed{E_c^N = \frac{1}{2} m_N \cdot v^2 = 2.67 \cdot 10^{-14}\,J}$$

$$\boxed{E_c^H = 2E_c^N = 5.34 \cdot 10^{-14}\,J}$$

<u>Q-1</u>

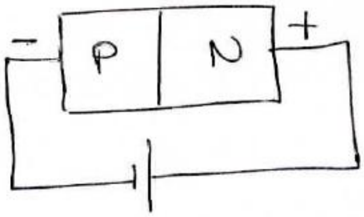
(a)



When they are put in contact, there will be a transference of charge until their potentials <u>equal</u>. The charge will be redistributed, being the total charge of the system: $Q_T = Q_1 + Q_2$.

(b) When an $\vec{E}$ goes through a dielectric, the ions feel it and when $\vec{E}$ is large enough, it rips the ions in the material, which becomes a conductor. When this happens, it is said that the dielectric suffers a breakdown. The dielectric strength is the maximum $\vec{E}$ that the material can withstand without breaking down.

Q2



When the p-type semiconductor is connected to the negative terminal and the N-type to the positive one, the electrons and hols are pushed away from the junction. This increases the width of the depletion region, and nearly no current flows through the junction.