Universidad Carlos III de Madrid

Digital Electronics. 1st Partial Exam. October 2015

Group 29

REMARKS:

- **Answers to problems must be returned separately. If you do not answer a problem, you must return at least a blank sheet for the problem.**
- **Please make sure your NAME and GROUP appears in every sheet you return.**
- **Calculators are not permitted.**
- **Time: 1h40’**

<u>Problem 1.1</u> (0.85 p.)

For the following logic function

$$f(a,b,c,d) = \sum_4 (1,5,6,7,9,11,13,15)$$

a) Find the most simplified logic expression as a **sum of products**
b) Find the most simplified logic expression as a **product of sums**
c) Implement the logic function with only 2-input NAND gates
d) Implement f with a MUX4 (multiplexer with 4 data inputs) and additional logic gates

<u>Problem 1.2</u> (0.3 p.)

Let A = 10001011_2 and B = 19_{10}.

a) If A is a number in two’s complement, which is the integer value of A?
b) Represent B in two’s complement. How many bits are needed?
c) Perform the operations A+B and A-B in two’s complement using 8 bits. Point out if there is overflow in any of these operations and why.

<u>Problem 1.3</u> (0.85 p.)

Design an up/down module-6 counter. The counter will go from 0 to 5 or from 5 to 0. The counter has an input A that is used to select the counting direction, up (A = 0) or down (A = 1). The counting direction can be changed at any time.

The counter shall have a Cout output that activates in state 5 when counting up and in state 0 when counting down.

a) Draw the State Transition Graph of the circuit.
b) The circuit shall be implemented using T flip-flops. Find simplified expressions for the implementation of the circuit.
c) Draw a circuit diagram.

$\boxed{1.1}$

a/

ab \ cd	00	01	11	10
00		1		
01		1	1	1
11		1	1	
10		1	1	

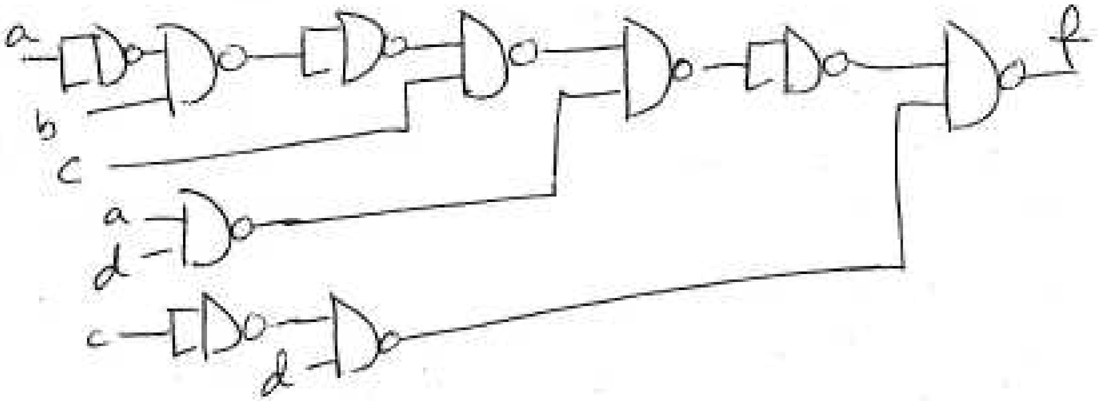
$f = \bar{a}bc + ad + \bar{c}d$

b/

ab \ cd	00	01	11	10
00	0		0	0
01	0			
11	0			0
10	0			0

$f = (a+b+\bar{c})(\bar{a}+d)(c+d)$

c/ $f = \overline{\overline{\bar{a}bc + ad + \bar{c}d}} = \overline{\overline{\bar{a}bc}\;\;\overline{ad}\;\;\overline{\bar{c}d}}$



d/

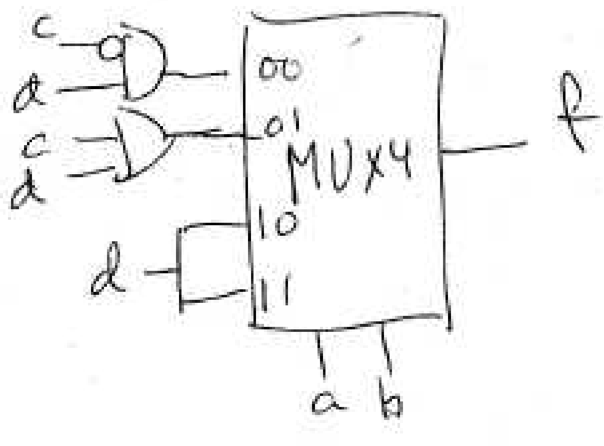
abcd	
0000	0
0001	1
0010	0
0011	0
0100	0
0101	1
0110	1
0111	1
1000	0
1001	1
1010	0
1011	1
1100	0
1101	1
1110	0
1111	1

$\bar{c}d$

$c+d$

d

d



$\boxed{1.2}$

a) $A = 10001011_2 = -117_{10}$

b) $B = 19_{10} = 0001\ 0011_2$ (needs at least 6 bits = 010011)

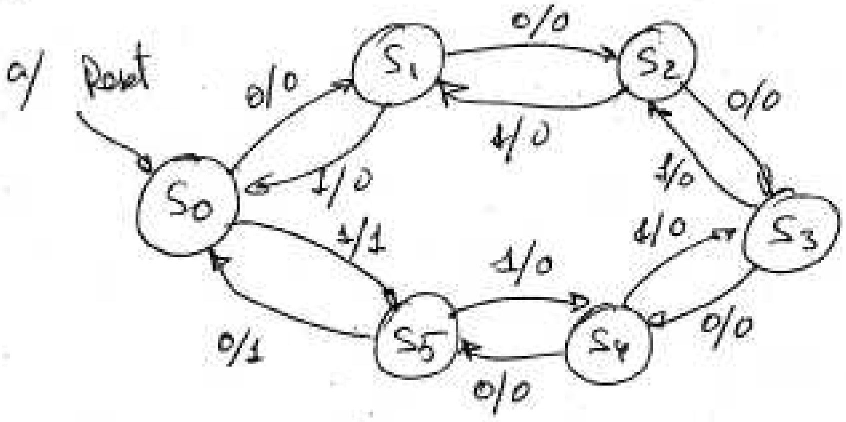
c)

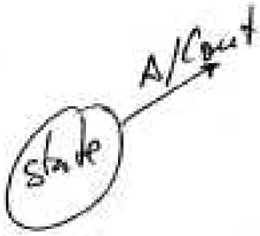
$$
\begin{array}{rl}
 & \scriptstyle 11 \\
A = & 1000\ 1011 \\
+ & \\
B = & 0001\ 0011 \\
\hline
 & 1001\ 1110 = -98
\end{array}
\qquad
\begin{array}{rl}
 & \scriptstyle 1\ \ 111 \\
A = & \boxed{1}000\ 1011 \\
+ & \\
-B = & \boxed{1}110\ 1101 \\
\hline
\cancel{1} & \boxed{0}111\ 1000 = 120 \\
 & \qquad\qquad\ \text{wrong}
\end{array}
$$

There is overflow because the signs of the operands do not coincide with the sign of the result

1.3

a/ Reset





b/

A	q_2	q_1	q_0	q_2^+	q_1^+	q_0^+	T_2	T_1	T_0	Cout
0	0	0	0	0	0	1	0	0	1	0
0	0	0	1	0	1	0	0	1	1	0
0	0	1	0	0	1	1	0	0	1	0
0	0	1	1	1	0	0	1	1	1	0
0	1	0	0	1	0	1	0	0	1	0
0	1	0	1	0	0	0	1	0	1	1
0	1	1	0	x	x	x	x	x	x	x
0	1	1	1	x	x	x	x	x	x	x
1	0	0	0	1	0	1	1	0	1	1
1	0	0	1	0	0	0	0	0	1	0
1	0	1	0	0	0	1	0	1	1	0
1	0	1	1	0	1	0	0	0	1	0
1	1	0	0	0	1	1	1	1	1	0
1	1	0	1	1	0	0	0	0	1	0
1	1	1	0	x	x	x	x	x	x	x
1	1	1	1	x	x	x	x	x	x	x

$Aq_2 \backslash q_1q_0$	00	01	11	10
00			1	
01		1	X	X
11	1		X	X
10	1			

$$T_2 = q_1 q_0 \bar{A} + \bar{A} q_2 q_0 + A \bar{q}_1 \bar{q}_0$$

$Aq_2 \backslash q_1q_0$	00	01	11	10
00		1	1	
01			X	X
11	1		X	X
10				1

$$T_1 = \bar{A} \bar{q}_2 q_0 + A q_2 \bar{q}_0 + A q_1 \bar{q}_0$$

$Aq_2 \backslash q_1q_0$	00	01	11	10
00	1	1	1	1
01	1	1	X	X
11	1	1	X	X
10	1	1	1	1

$$T_0 = 1$$

$Aq_2 \backslash q_1q_0$	00	01	11	10
00				
01		1	X	X
11			X	X
10	1			

$$C_{out} = A\bar{q}_2\bar{q}_1\bar{q}_0 + \bar{A}q_2q_0$$

c)