Problema 5.1 Supondremos f y g derivables en todo $\mathbb{R}$.

1) $F_1(x) = \sqrt{f^2(x) + g^2(x)} = \left(f^2(x) + g^2(x)\right)^{1/2}$

$$F_1'(x) = \frac{1}{2}\left(f^2(x) + g^2(x)\right)^{-1/2} \cdot \left(2f'(x)f(x) + 2g'(x)g(x)\right)$$

$$F_1'(x) = \frac{f'(x)f(x) + g'(x)g(x)}{\sqrt{f^2(x) + g^2(x)}}$$

$x: \ f^2(x) + g^2(x) \neq 0$

Obs: En los puntos $x_0 : f^2(x_0) + g^2(x_0) = 0$ la derivabilidad de $F_1(x)$ depende de la forma que tengan $f(x)$ y $g(x)$:

- $f(x) = x \ ; \ g(x) = 0$

  $\Rightarrow F_1(x) = \sqrt{x^2} = |x|$ no es derivable en $x_0 = 0$

  $(x_0 = 0 : \ f^2(x_0) + g^2(x_0) = 0)$

- $f(x) = x^2 \ ; \ g(x) = 0$

  $\Rightarrow F_1(x) = \sqrt{x^4} = x^2$ derivable en todo $\mathbb{R}$

  (incluido $x_0 = 0$)

2) $F_2(x) \underset{\uparrow}{=} \arctan\left(\frac{f(x)}{g(x)}\right)$

$(x : g(x) \neq 0)$

$F_2'(x) \underset{\uparrow}{=} \arctan'\left(\frac{f(x)}{g(x)}\right) \cdot \frac{f'(x)g(x) - g'(x)f(x)}{g^2(x)}$

$(x : g(x) \neq 0)$

$$= \frac{1}{1 + f^2(x)/g^2(x)} \cdot \frac{f'(x)g(x) - g'(x)f(x)}{g^2(x)}$$

$$= \frac{f'(x)g(x) - g'(x)f(x)}{f^2(x) + g^2(x)}$$

$$F_2'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{f^2(x) + g^2(x)} \ ; \left(x : g(x) \neq 0\right)$$

3) $F_3(x) = e^{f(x)} g(f(x)) \ , \ \forall x \in \mathbb{R}.$

$$F_3'(x) = f'(x)e^{f(x)} g(f(x)) + $$
$$+ e^{f(x)} g'(f(x)) f'(x)$$
$$= f'(x)e^{f(x)} \left(g(f(x)) + g'(f(x)) \right)$$

$$\Rightarrow F_3'(x) = f'(x)e^{f(x)} \left(g(f(x)) + g'(f(x)) \right)$$
$\forall x \in \mathbb{R}$

4) $F_4(x) = \log(g(x)\cos(f(x)))$

Necesitamos que x : $g(x)\cos(f(x)) > 0$

En ese caso:

$$F_4'(x) = \frac{g'(x)\cos(f(x)) - f'(x)g(x)\operatorname{sen}(f(x))}{g(x)\cos(f(x))}$$

5) $$F_5(x) = (g(x))^{f(x)} \underset{g(x)>0}{=} e^{f(x)\log(g(x))}$$

$$F_5'(x) = e^{f(x)\log(g(x))} \cdot \left(f'(x)\log(g(x)) + \frac{f(x)g'(x)}{g(x)} \right)$$

$$= (g(x))^{f(x)} \cdot \left(f'(x)\log(g(x)) + \frac{f(x)g'(x)}{g(x)} \right)$$

$$= \log(g(x))\,(g(x))^{f(x)} \cdot f'(x) +$$

$$+ f(x)\cdot (g(x))^{f(x)-1} \cdot g'(x)$$

Obs: En particular:

- Si $f(x) = x$; $g(x) = b > 0$ constante:
  $F_5(x) = b^x \Longrightarrow F_5'(x) = \log(b)\, b^x \quad \forall x \in \mathbb{R}.$

- Si $f(x) = a$; $g(x) = x$
  $F_5(x) = x^a \Longrightarrow F_5'(x) = a x^{a-1} \quad (x > 0)$

6) $$F_6(x) = \frac{1}{\log\left(f^2(x)+g^2(x)\right)}$$

$x:\ f^2(x)+g^2(x) \neq 0 \ \& \ f^2(x)+g^2(x) \neq 1$

$$F_6'(x) = -\frac{1}{\log^2\left(f^2(x)+g^2(x)\right)} \cdot \frac{2f'(x)f(x)+2g'(x)g(x)}{f^2(x)+g^2(x)}$$

$$F_6'(x) = -\frac{2f'(x)f(x)+2g'(x)g(x)}{\left(f^2(x)+g^2(x)\right)\log^2\left(f^2(x)+g^2(x)\right)}$$

$x:\ f^2(x)+g^2(x) \neq 0 \ \& \ f^2(x)+g^2(x) \neq 1$