Universidad Carlos III de Madrid

Digital Electronics. 1st Partial Exam. October 2014

Group 29

REMARKS:

- **Answers to problems must be returned separately. If you do not answer a problem, you must return at least a blank sheet for the problem.**
- **Please make sure your NAME and GROUP appears in every sheet you return.**
- **Calculators are not permitted.**
- **Time: 1h40'**

Problem 1.1 (0.85 p.)

For the following logic function

$$f(a,b.c.d) = (a+b)c\overline{d} + \overline{b}(a\overline{d} + \overline{c})$$

a) Find the canonical forms of the function
b) Find the most simplified logic expression as a **product of sums**
c) Implement the logic function with only 2-input NOR gates
d) Implement f with a decoder and additional logic gates.
e) Implement f with a MUX4 (multiplexer with 4 data inputs) and additional logic gates

Problem 1.2 (0.3 p.)

Let $A = E3_{16}$ and $B = 96_{10}$.

a) If A represents a number in sign-magnitude, which is the integer value of A?
b) If A represents a number in two's complement, which is the integer value of A?
c) Represent B in two's complement. How many bits are needed?
d) Using two's complement representations for the numbers, perform the operations A+B and A-B. Point out if there is overflow in any of these operations and why.

Problem 1.3 (0.85 p.)

Design a sequential circuit that has a single input X and a single output Z. The circuit must comply with the following requirements:

- Initially, Z = 0.
- The output will be activated (Z = 1) when X = 1 for at least 3 consecutive clock cycles. Once Z = 1, the output will remain activated until X = 0 for at least 3 consecutive clock cycles.

Example:

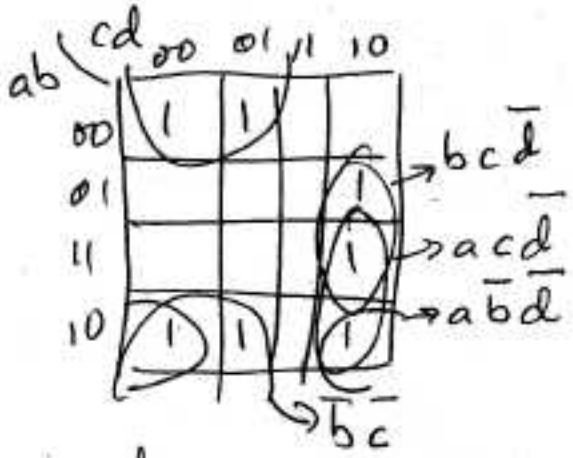
X: 0 0 0 1 1 0 1 1 1 0 1 0 0 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 0

Z: 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0

a) Draw the State Transition Graph of the circuit
b) Consider an implementation using D flip-flops and encoding the states in natural binary. How many flip-flops are necessary?
c) Write the transitions table
d) Find simplified expressions for the implementation of the circuit
e) Draw a circuit diagram

1.1

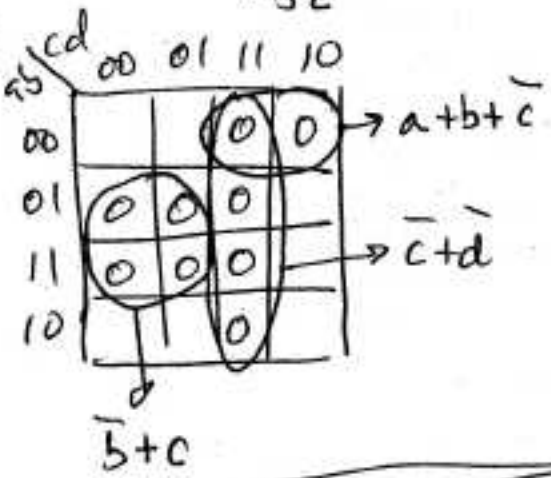
a) $f = (a+b)\,c\,\bar{d} + \bar{b}\,(a\bar{d}+\bar{c}) = ac\bar{d} + bc\bar{d} + a\bar{b}\bar{d} + \bar{b}\bar{c}$



$f = \Sigma(0,1,6,8,9,10,14)$

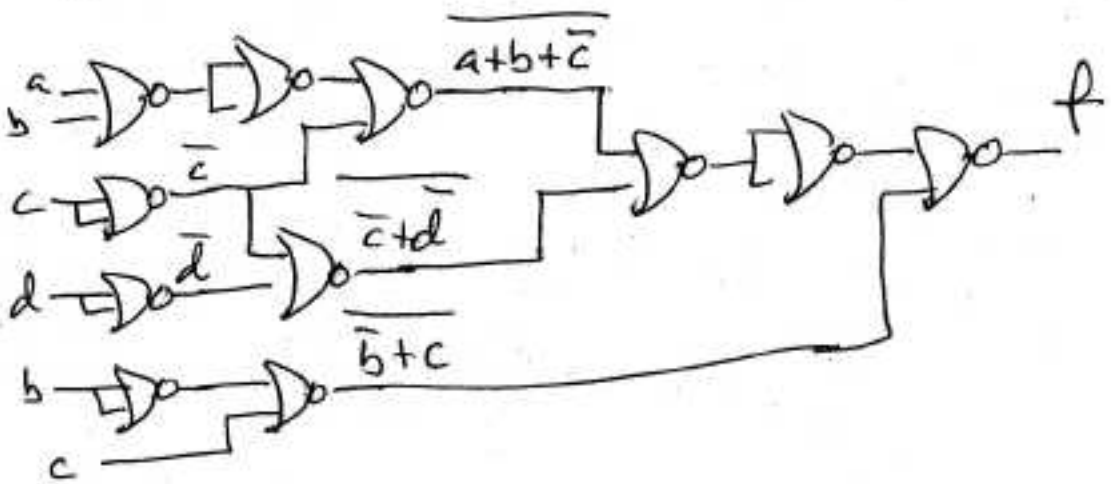
$= \Pi(2,3,4,5,7,11,12,13,15)$

b)

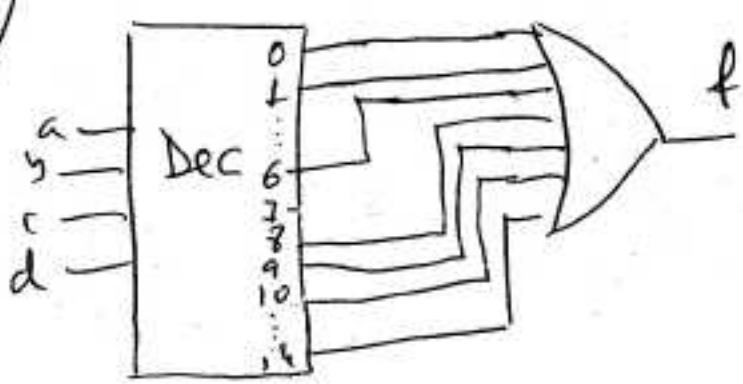


$f = (a+b+\bar{c})(\bar{c}+\bar{d})(\bar{b}+c)$

c) $f = \overline{(a+b+\bar{c})(\bar{c}+\bar{d})(\bar{b}+c)} = \overline{(a+b+\bar{c})} + \overline{(\bar{c}+\bar{d})} + \overline{\bar{b}+c}$

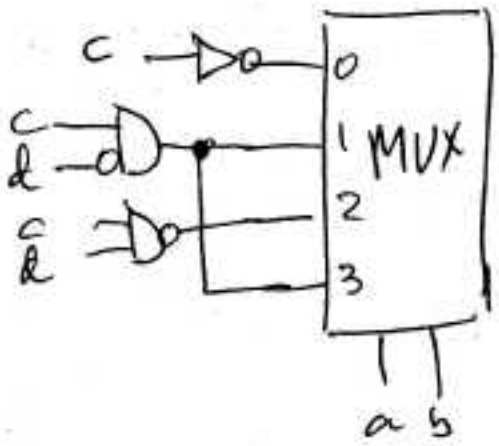


d)



e/

a b	c d	f	
0 0	0 0	1	$\bar{c}$
0 0	0 1	1	
0 0	1 0	0	
0 0	1 1	0	
0 1	0 0	0	$c\bar{d}$
0 1	0 1	0	
0 1	1 0	1	
0 1	1 1	0	
1 0	0 0	1	$\overline{cd} = \bar{c} + \bar{d}$
1 0	0 1	1	
1 0	1 0	1	
1 0	1 1	0	
1 1	0 0	0	$c\bar{d}$
1 1	0 1	0	
1 1	1 0	1	
1 1	1 1	0	



1.2

$A = E3_{16} = 1110\ 0011$

$B = 96_{10} = 0110\ 0000$

a) $A_{SH} = \begin{cases} 1 \text{ negative} \\ 110\ 0011 = 99_{10} \end{cases}$ $\quad A_{SH} = -99_{10}$

b) $A_{C2} = 1110\ 0011$

$-A = 0001\ 1101 = 29$

$A_{C2} = -29$

c) $B = 96 = 110\ 0000$

We need an additional bit for the sign: total = 8 bits

$B_{C2} = 0110\ 0000$

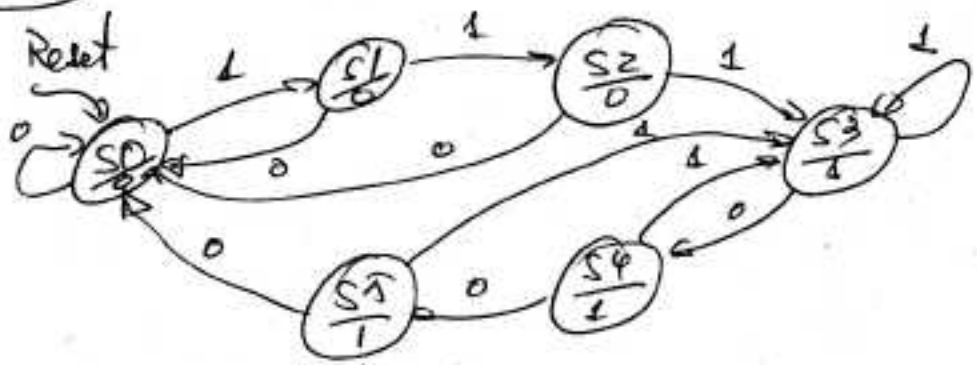
$$\begin{array}{rl} & {\scriptstyle 1\ 1} \\ A & 1110\ 0011 = -29 \\ B & 0110\ 0000 = +96 \\ \hline & 0100\ 0011 = +67 \end{array}$$

No overflow (operands have different sign)

$$\begin{array}{rl} & {\scriptstyle 1\ 1} \\ A & 1110\ 0011 = -29 \\ -B & 1010\ 0000 = -96 \\ \hline & 1000\ 0011 = -125 \end{array}$$

No overflow (operands and result are negative.

1.3

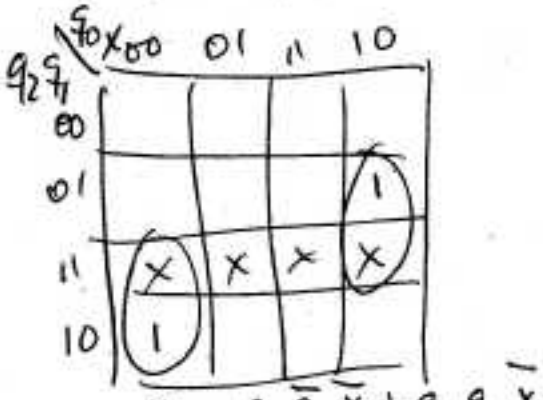


	$q_2q_1q_0$
S0: Initial	000
S1: x=1	001
S2: 11	010
S3: 111	011
S4: 0	100
S5: 00	101
S6: 000	111

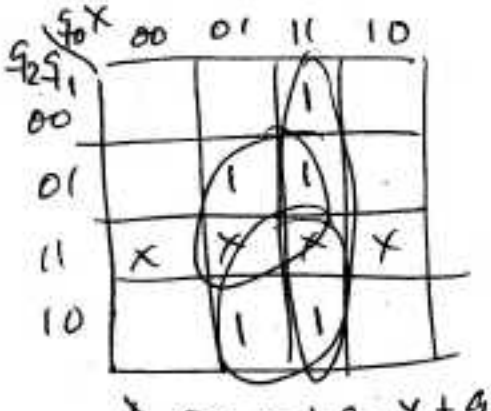
3 FFs.

$q_2q_1q_0$	x	$D_2D_1D_0$ = $q_2^+q_1^+q_0^+$	z
000	0	000	
000	1	001	
001	0	000	
001	1	010	
010	0	000	
010	1	011	
011	0	100	
011	1	100	
100	0	101	
100	1	101	
101	0	000	
101	1	011	
110	0	xxx	
110	1	xxx	
111	0	xxx	
111	1	xxx	

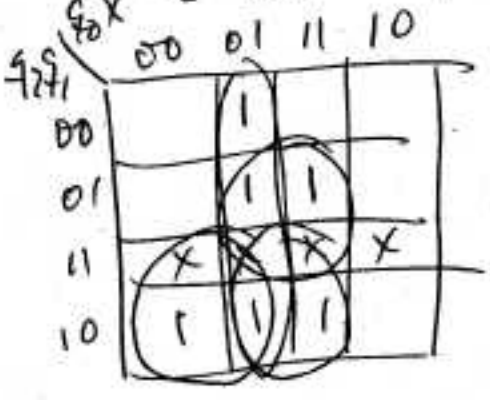
$q_2q_1q_0$	z
000	0
001	0
010	0
011	1
100	1
101	1
110	x
111	x



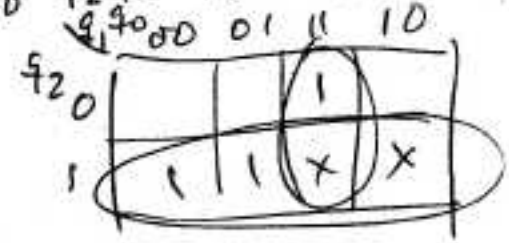
$D_2 = q_2\bar{q}_0\bar{x} + q_1q_0\bar{x}$



$D_1 = q_1x + q_2x + q_0x = x(q_2+q_1+q_0)$

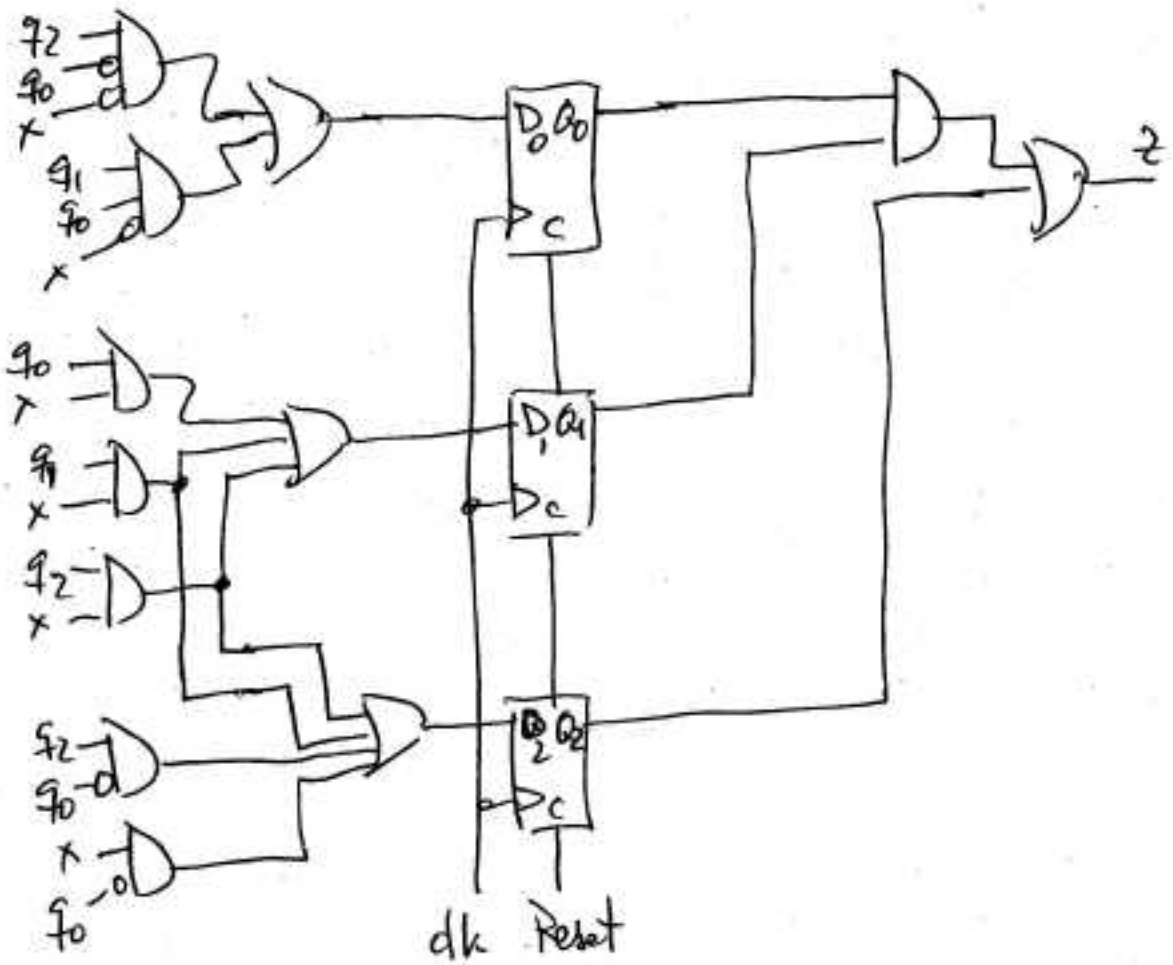


$D_0 = q_2\bar{q}_0 + q_2x + q_1x + \bar{q}_0x =$



$z = q_2 + q_1q_0$

e)


q2
q0
x
q1
q0
x
q0
x
q1
x
q2
x
q2
q0
x
q0
D0 Q0
C
D1 Q1
C
D2 Q2
C
z
clk
Reset