PROBLEMA 6.7

- Aproximar $26^{2/3}$ mediante el teorema del valor medio

Consideremos $f(x) = x^{2/3}$: continua y derivable en $(0,\infty)$

$$f'(x) = \frac{2}{3} x^{-1/3}$$

$$f(26) = 26^{2/3}$$

$$f(27) = 27^{2/3} = 3^2 = 9$$

Aplicando el teorema del valor medio en $[26,27]$:

$$f'(x_0) = \frac{f(27)-f(26)}{27-26} \quad \text{para } x_0 \in (26,27)$$

$$\frac{2}{3x_0^{1/3}} = \frac{9-26^{2/3}}{1} \quad \text{para } x_0 \in (26,27)$$

$$\Rightarrow 26^{2/3} = 9 - \frac{2}{3x_0^{1/3}} \quad \text{para } x_0 \in (26,27)$$

$$\Rightarrow \underbrace{9-\frac{2}{3\cdot 26^{1/3}}}_{> 9-\frac{2}{3\cdot 8^{1/3}} = 9-\frac{1}{3} = \frac{26}{3}} \underset{x_0=26}{<} 26^{2/3} \underset{x_0=27}{<} 9-\frac{2}{3\cdot 27^{1/3}} = 9-\frac{2}{9} = \frac{79}{9}$$

Por tanto: $\frac{26}{3} < 26^{2/3} < \frac{79}{9}$

$$\boxed{8.\widehat{6} < 26^{2/3} < 8.\widehat{7}}$$

<u>Obs:</u> $26^{2/3} = 8.77638295\ldots$

- Aproximar $\log(3/2)$ mediante el teorema del valor medio:

Consideremos $f(x) = \log(1+x)$ continua y derivable en $(-1, \infty)$

$$f'(x) = \frac{1}{1+x}$$

$$f(0) = \log(1) = 0$$

$$f(1/2) = \log(3/2)$$

Usando el teorema del valor medio:

$$f'(x_0) = \frac{f(1/2) - f(0)}{1/2 - 0} \iff \frac{1}{1+x_0} = \frac{\log(3/2) - 0}{1/2}$$

$x_0 \in (0, 1/2)$ para algún $x_0 \in (0, 1/2)$

$$\Rightarrow \log\left(\frac{3}{2}\right) = \frac{1}{2}\,\frac{1}{1+x_0} \quad \text{con} \quad x_0 \in (0, 1/2)$$

Por tanto:

$$\underbrace{\frac{1}{2}\,\frac{1}{1+1/2}}_{1/3} < \log(3/2) < \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{3} < \log(3/2) < \frac{1}{2}$$

$$\boxed{0.\widehat{3} < \log(3/2) < 0.5}$$

Obs: $\log(3/2) = 0.40546\ldots$