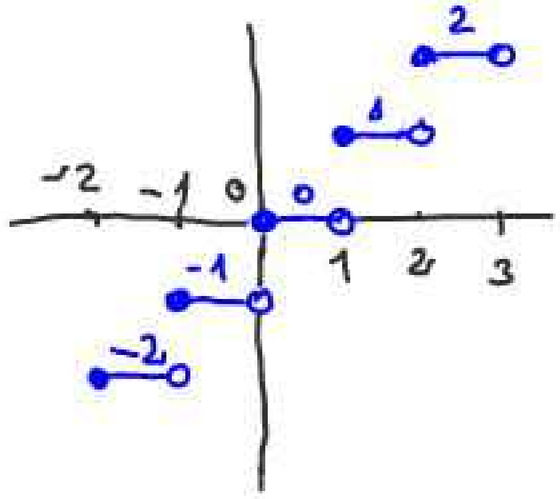
Problema 4.1

1) $f(x) = \lfloor x \rfloor$

Dom $f = \mathbb{R}$

Im $f = \mathbb{Z} = \{0, \pm 1, \pm 2, \dots\}$

f es continua en $\mathbb{R} \setminus \mathbb{Z} = \mathbb{R} \setminus \{0, \pm 1, \pm 2, \dots\}$



2) $f(x) = x - \lfloor x \rfloor$

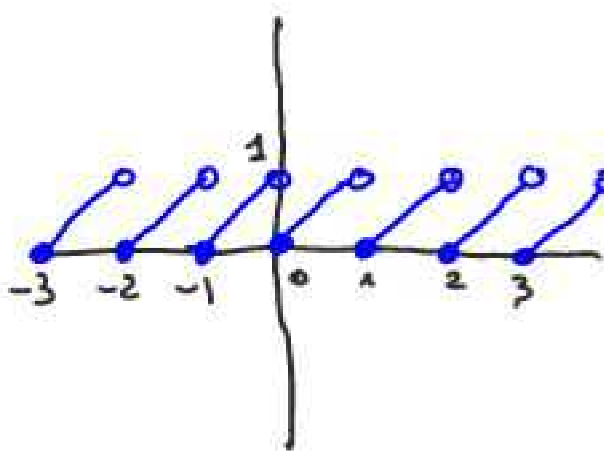
Dom $f = \mathbb{R}$

Im $f = [0, 1)$

f es continua en $\mathbb{R} \setminus \mathbb{Z} = \mathbb{R} \setminus \{0, \pm 1, \dots\}$

f es PERIÓDICA de periodo 1 :

$$f(x+n) = x + n - \lfloor x + n \rfloor =$$
$$= x + \lfloor x \rfloor = f(x)$$
$$\forall x \in \mathbb{R} \; ; \; n \in \mathbb{Z}$$



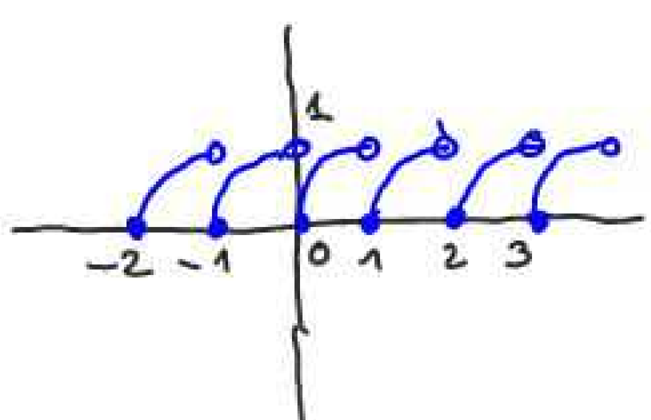
3) $f(x) = \sqrt{x - \lfloor x \rfloor}$

Dom $f = \mathbb{R}$

Im $f = [0, 1)$

f periódica de periodo 1

f es continua en $\mathbb{R} \setminus \mathbb{Z}$.



4) $f(x) = \lfloor x \rfloor + \sqrt{x - \lfloor x \rfloor}$; $\quad \text{Dom} f = \mathbb{R}$

$\text{Im} f = \mathbb{R}$

f es continua en todo $\mathbb{R}$:

* Si $x_0 = n = 0, \pm 1, \pm 2, \ldots$

$$\lim_{x \to n^+} f(x) = \lim_{x \to n^+} \lfloor x \rfloor + \lim_{x \to n^+} \sqrt{x - \lfloor x \rfloor}$$

$$= n + \sqrt{n - n} = n$$

$$\lim_{x \to n^-} f(x) = \lim_{x \to n^-} \lfloor x \rfloor + \lim_{x \to n^-} \sqrt{x - \lfloor x \rfloor}$$

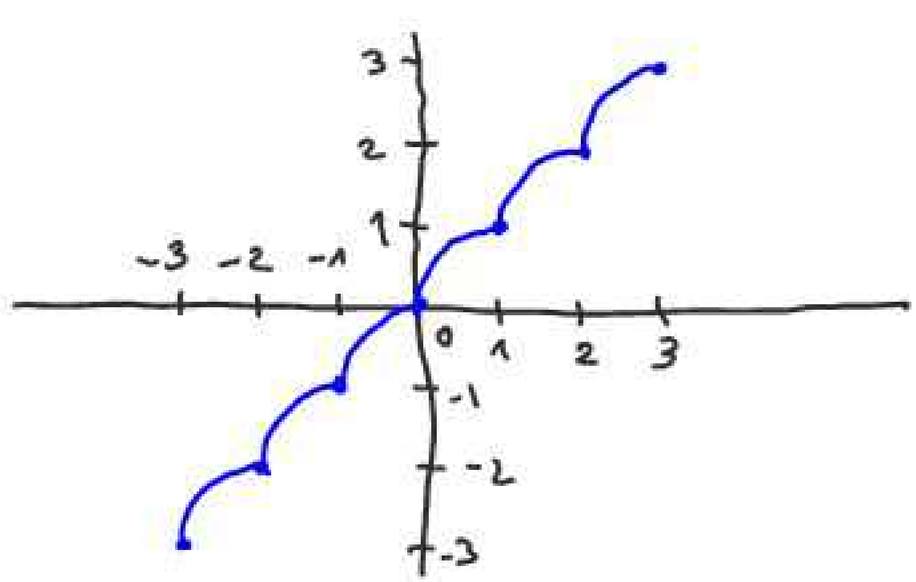
$$= n - 1 + \sqrt{n - (n-1)} =$$

$$= n - 1 + \sqrt{1} = n$$

$$f(n) = \lfloor n \rfloor + \sqrt{n - \lfloor n \rfloor} = n + \sqrt{n - n} = n$$

$\Rightarrow \lim_{x \to n} f(x) = n = f(n)$: f es CONTINUA en $x_0 = n$

* Si $x_0 \notin \mathbb{N}$, f es trivialmente continua en x_0 ya que $\lfloor x \rfloor$; x ; $\sqrt{x - \lfloor x \rfloor}$ son continuas en x_0



5) $f(x) = \lfloor 1/x \rfloor$ $\qquad$ $\mathrm{Dom} f = \mathbb{R} \setminus \{0\}$

$\mathrm{Im} f = \mathbb{Z}$

Sea $x \in \mathrm{Dom} f = \mathbb{R} \setminus \{0\}$:

- Si $\frac{1}{x} \neq \pm 1, \pm 2, \pm 3, \dots$

  entonces f es continua en x [al ser composición de funciones continuas]

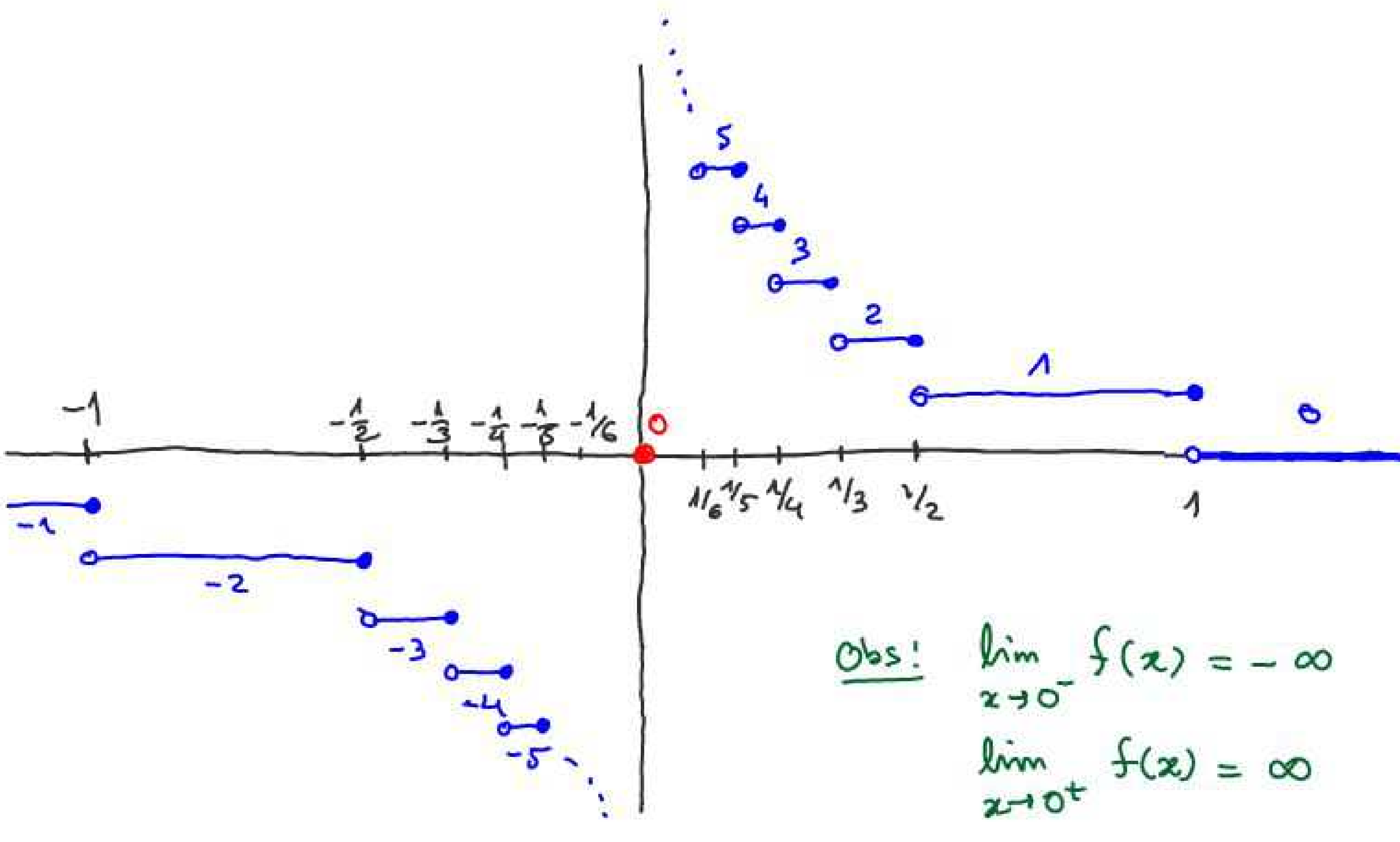
- Si $\frac{1}{x} = \pm 1, \pm 2, \dots \iff x = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \dots$

  f es discontinua en x :

$$\lim_{x \to 1/n^+} f(x) = \lim_{\substack{\varepsilon \to 0 \\ \varepsilon > 0}} f\left(\frac{1}{n} + \varepsilon\right) = n - 1$$

$$\lim_{x \to 1/n^-} f(x) = \lim_{\substack{\varepsilon \to 0 \\ \varepsilon > 0}} f\left(\frac{1}{n} - \varepsilon\right) = n$$

$$f(1/n) = n$$



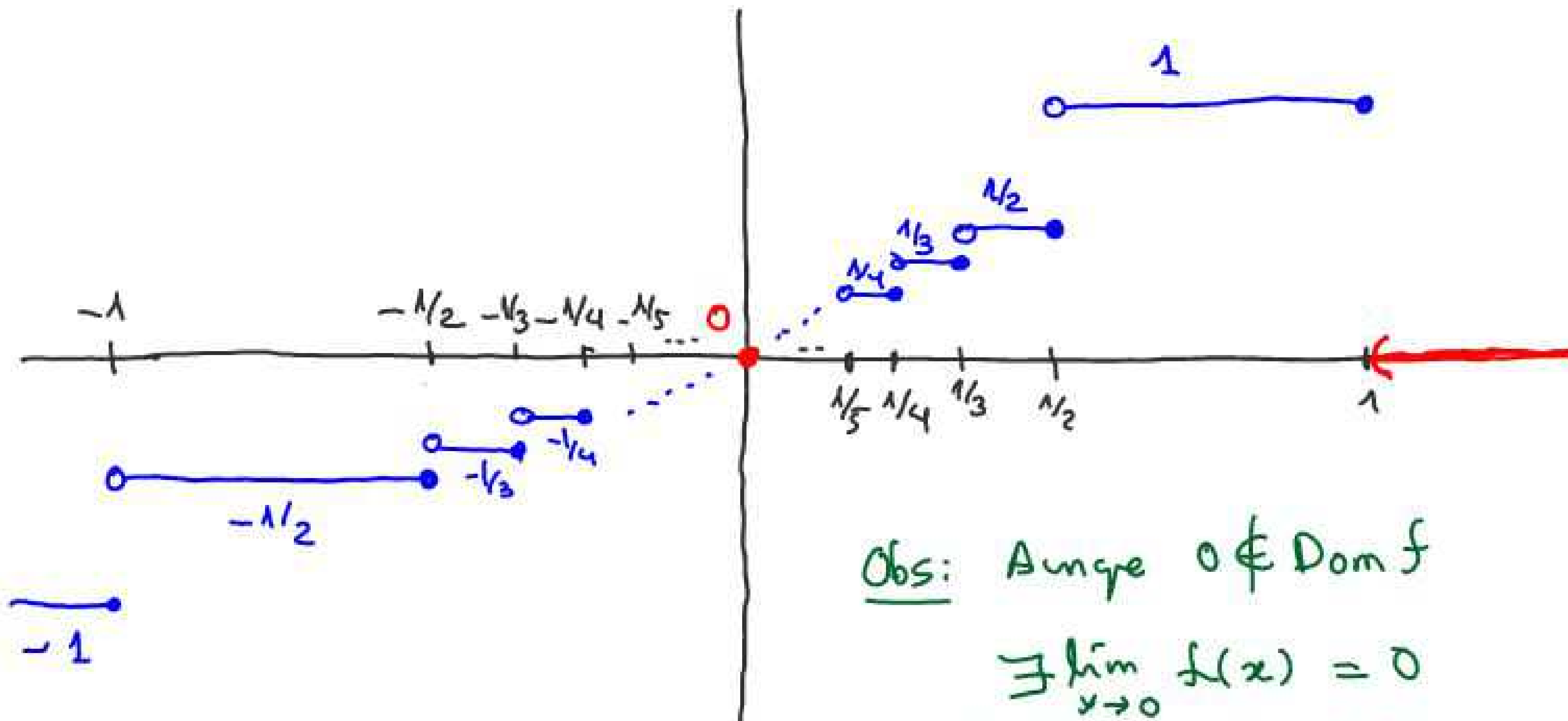
<u>Obs!</u> $\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to 0^+} f(x) = \infty$

6) $f(x) = \dfrac{1}{\lfloor 1/x \rfloor}$ $\qquad$ $\text{Dom} f = (-\infty, 0) \cup (0, 1]$

$\text{Im} f = \left\{ \dfrac{1}{n} : n = \pm 1, \pm 2, \dots \right\}$

f es continua en $x_0 \Longleftrightarrow x_0 \neq \dfrac{1}{n}$ con $n \neq -1, \pm 2, \pm 3, \dots$



Obs: Aunque $0 \notin \text{Dom} f$

$\exists \lim_{x \to 0} f(x) = 0$

Por tanto, la función:

$$F(x) = \begin{cases} \dfrac{1}{\lfloor 1/x \rfloor} & \text{si } x \in \text{Dom} f \\ 0 & \text{si } x = 0 \end{cases}$$

es CONTINUA en $x_0 = 0$.