Problema 1.2

(1) $A = \{1/n : n \in \mathbb{N}\} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$

$0 \leq a \leq 1 \quad \forall a \in A \Rightarrow$ A es un conjunto ACOTADO

$\sup A = 1 \in A \Rightarrow \max A = 1$

$\inf A = 0 \notin A \Rightarrow \nexists \min A$

(2) $A = \{1/n : n \in \mathbb{Z} \setminus \{0\}\} = \{\pm 1, \pm 1/2, \pm 1/3, \dots\}$

$-1 \leq a \leq 1 \quad \forall a \in A \Rightarrow$ A es ACOTADO

$\sup A = 1 \in A \Rightarrow \max A = 1$

$\inf A = -1 \in A \Rightarrow \min A = -1$

(3) $A = \{a \in \mathbb{Q} : 0 \leq a \leq \sqrt{2}\}$

$0 \leq a \leq \sqrt{2} \quad \forall a \in A \Rightarrow$ A es ACOTADO

$\sup A = \sqrt{2} \notin A \Rightarrow \nexists \text{ máx } A$

$\inf A = 0 \in A \Rightarrow \min A = 0$

(4) $A = \{a \in \mathbb{R} : a^2 + a + 1 \geq 0\}$

Como $a^2 + a + 1 = 0 \Leftrightarrow a = \dfrac{-1 \pm \sqrt{1-4}}{2} \notin \mathbb{R}$

&

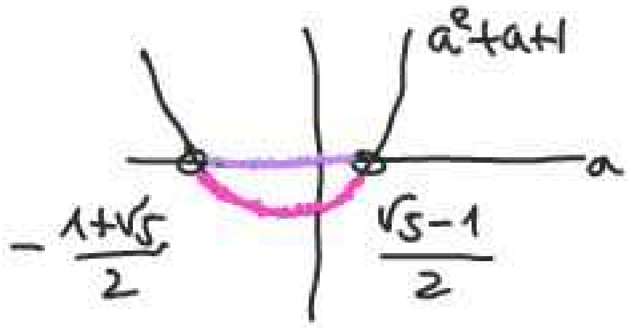
$0^2 + 0 + 1 \geq 0$

$\Rightarrow A = \{a \in \mathbb{R} : a^2 + a + 1 \geq 0\} = \mathbb{R}$

A no es ACOTADO $\nexists$ sup, inf, máx, min

(5) $A = \{a \in \mathbb{R} : a^2 + a - 1 < 0\}$

Como $a^2 + a - 1 = 0 \Leftrightarrow a = \dfrac{-1 \pm \sqrt{1+4}}{2}$



$a = \dfrac{-1 \pm \sqrt{5}}{2}$

$\Rightarrow A = \left(-\dfrac{1+\sqrt{5}}{2}, \dfrac{\sqrt{5}-1}{2}\right)$ (intervalo abierto)

A es ACOTADO $\quad \sup A = \dfrac{\sqrt{5}-1}{2} \notin A \Rightarrow \nexists \max A$

$\inf A = -\dfrac{1+\sqrt{5}}{2} \notin A \Rightarrow \nexists \min A$

(6) $A = \{a : a < 0, a^2 + a + 1 < 0\}$

Usando el apartado anterior:

$A = \left(-\dfrac{1+\sqrt{5}}{2}, 0\right)$ ACOTADO

$\sup A = 0 \notin A \Rightarrow \nexists \text{ máx } A$

$\inf A = -\dfrac{1+\sqrt{5}}{2} \notin A \Rightarrow \nexists \min A$

(7) $A = \{1/n + (-1)^n : n \in \mathbb{N}\}$

$= \{0, -1 + 1/3, -1 + \frac{1}{5}, \dots\} \cup \{1 + \frac{1}{2}, 1 + \frac{1}{4}, \dots\}$

$\Rightarrow -1 \leq a \leq 1 + 1/2 \quad \forall a \in A$

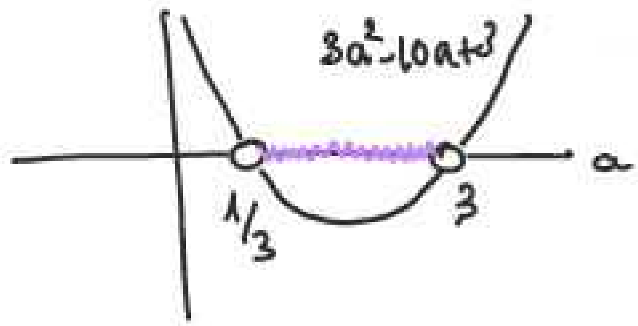
A es ACOTADO

$\sup A = 1 + 1/2 \in A \Rightarrow \text{máx } A = 1 + 1/2$

$\inf A = -1 \notin A \Rightarrow \nexists \min A$

(8) $A = \{a \in \mathbb{R} : 3a^2 - 10a + 3 < 0\}$

$3a^2 - 10a + 3 = 0 \Leftrightarrow a = \dfrac{10 \pm \sqrt{100-36}}{6}$



$a = \dfrac{10 \pm 8}{6} \begin{cases} 3 \\ \frac{1}{3} \end{cases}$

$\Rightarrow A = (1/3, 3)$ ACOTADO

$\sup A = 3 \notin A \Rightarrow \nexists \max A$

$\inf A = 1/3 \notin A \Rightarrow \nexists \min A$

(9) $A = \{x \in \mathbb{R} : (x-a)(x-b)(x-c)(x-d) < 0\}$

donde : $a < b < c < d$

Un cálculo directo muestra que:

$A = (a,b) \cup (c,d) \Rightarrow a \leq x \leq d \quad \forall x \in A$

A es acotado.

Además:

$\sup A = d \notin A \Rightarrow \nexists \max A$

$\inf A = a \notin A \Rightarrow \nexists \min A$

(10) $A = \{a = 2^{-p} + 5^{-q} : p, q \in \mathbb{N}\}$

$= \left\{ \frac{1}{2} + \frac{1}{5}, \frac{1}{2^2} + \frac{1}{5}, \frac{1}{2} + \frac{1}{5^2}, \dots \right\}$

$0 \leq a \leq \frac{1}{2} + \frac{1}{5} \quad \forall a \in A \Rightarrow$ A es acotado

$\sup A = \frac{1}{2} + \frac{1}{5} \in A \Rightarrow \max A = \frac{7}{10}$

$\inf A = 0 \notin A \Rightarrow \nexists \min A$.

(11) $A = \{a = (-1)^n + \frac{1}{m} : n, m \in \mathbb{N}\} =$

$= \{1 + \frac{1}{m} : m \in \mathbb{N}\} \cup \{-1 + \frac{1}{m} : m \in \mathbb{N}\}$

$= \{2, 1+\frac{1}{2}, 1+\frac{1}{3}, \dots\} \cup \{0, -1+\frac{1}{2}, -1+\frac{1}{3}, \dots\}$

$\Rightarrow -1 \leq a \leq 2 \quad \forall a \in A$; ACOTADO

$\sup A = 2 \in A \Rightarrow \max A = 2$

$\inf A = -1 \notin A \Rightarrow \nexists \min A$