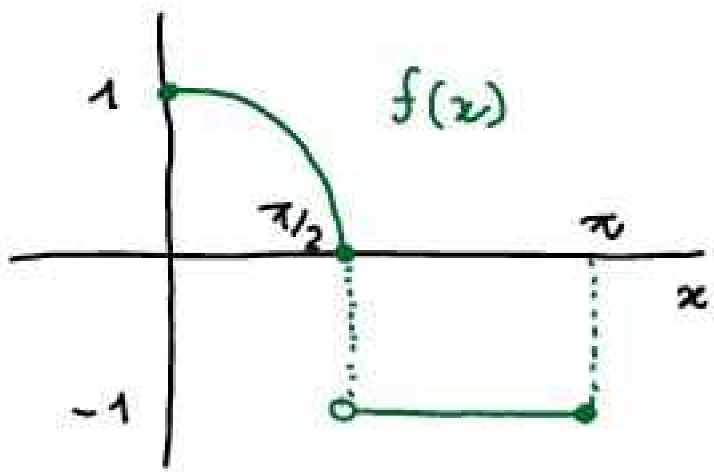
PROBLEMA 10.1

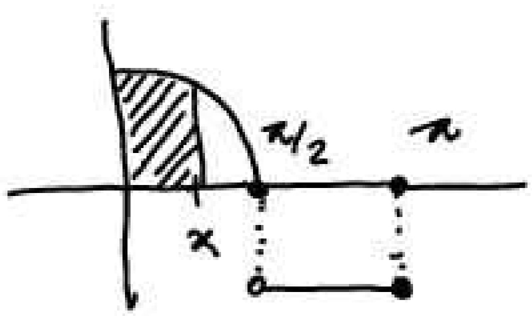
$$f(x) = \begin{cases} \cos(x) & \text{si } x \in [0, \pi/2] \\ -1 & \text{si } x \in (\pi/2, \pi] \end{cases}$$

Calcula $F(x) = \int_0^x f(t)\,dt$; $x \in [0, \pi]$ y compara $F'(x)$ con $f(x)$.



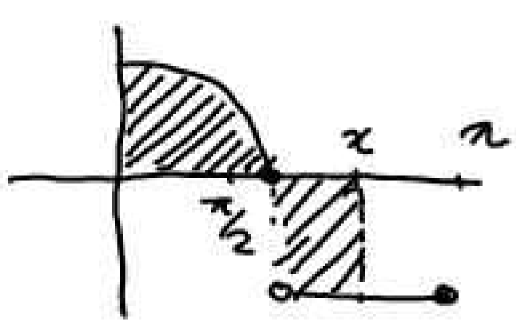
f es DISCONTINUA en $x = \pi/2$

- Si $x \in [0, \pi/2]$:



$$F(x) = \int_0^x f(t)\,dt = \int_0^x \cos(t)\,dt = \operatorname{sen}(x)$$

- Si $x \in [\pi/2, \pi]$:



$$F(x) = \int_0^x f(t)\,dt = \int_0^{\pi/2} \cos(t)\,dt + \int_{\pi/2}^x (-1)\,dt$$

$$= 1 - \int_{\pi/2}^x dt = 1 + \pi/2 - x$$

$$\Rightarrow F(x) = \int_0^x f(t)\,dt = \begin{cases} \operatorname{sen}(x) & \text{si } 0 \leq x \leq \pi/2 \\ 1 + \frac{\pi}{2} - x & \text{si } \pi/2 < x \leq \pi \end{cases}$$

- F es CONTINUA en $[0, \pi]$ y DERIVABLE en $[0, \pi/2) \cup (\pi/2, \pi]$.

En particular, si $x \neq \pi/2$ se cumple que (ver TFC) :

$$F'(x) = f(x) = \begin{cases} \cos(x) & \text{si } 0 \leq x < \pi/2 \\ -1 & \text{si } \pi/2 < x \leq \pi \end{cases}$$