PROBLEMA 10.9

- $\lim\limits_{x\to 0^+} \dfrac{1}{x^{3/2}} \displaystyle\int_0^{x^2} \operatorname{sen}(t^{1/4})\,dt$

Indeterminación $\frac{0}{0}$:

$$\operatorname{sen}(t^{1/4}) \underset{\substack{\uparrow \\ \text{serie de Taylor}}}{=} t^{1/4} - \frac{t^{3/4}}{3!} + \frac{t^{5/4}}{5!} + \cdots$$

$$\int_0^{x^2} \operatorname{sen}(t^{1/4})\,dt = \frac{4}{5}x^{5/2} - \frac{1}{3!}\frac{4}{7}x^{7/2} + \frac{1}{5!}\frac{4}{9}x^{9/2} + \cdots$$

$$\Rightarrow \frac{1}{x^{3/2}}\int_0^{x^2} \operatorname{sen}(t^{1/4})\,dt = \frac{4}{5}x - \frac{2}{21}x^2 + \frac{1}{270}x^3 + \cdots$$

$$\lim_{x\to 0^+} \frac{1}{x^{3/2}}\int_0^{x^2} \operatorname{sen}(t^{1/2})\,dt = 0$$

- $\lim\limits_{x\to 0} \left(\dfrac{1}{x^2} - \dfrac{1}{x^3}\displaystyle\int_0^x e^{t^2}\,dt \right)$

$$\lim_{x\to 0} \frac{x - \int_0^x e^{t^2}\,dt}{x^3} \qquad \text{Indeterminación } \frac{0}{0}$$

$$\text{Usamos } e^{t^2} \underset{\substack{\uparrow \\ \text{Serie de Taylor (radio } \infty)}}{=} 1 + t^2 + \frac{t^4}{2} + \cdots$$

$$\int_0^x e^{t^2}\,dt = x + \frac{x^3}{3} + \frac{x^5}{10} + \cdots$$

$$\Rightarrow \frac{x - \int_0^x e^{t^2}\,dt}{x^3} = \frac{x - \left(x + \frac{x^3}{3} + \frac{x^5}{10} + \cdots\right)}{x^3}$$

$$= -\frac{1}{3} - \frac{x^2}{10} + \cdots \xrightarrow[x\to 0]{} -1/3$$