Soluciones # 8

Valores y vectores propios

Problema 8.1

a) $\sigma(A_{T^{x,\theta}}) = \{1, \cos(\theta) + i\ \text{sen}(\theta), \cos(\theta) - i\ \text{sen}(\theta)\}$.

b) $\sigma(A_{T^{x,\theta}}) = \{1, -1\}, \quad m_a(1) = m_g(1) = 1, \quad m_a(-1) = m_g(-1) = 2.$

c) $B = (e_1, e_2, e_3)$.

Problema 8.2 $a = 6, \quad b = -11$ y $\quad c = 6$.

Problema 8.3 $\sigma(A) = \{a + b, d - b\}$.

Problema 8.4 *Indicación:* Evaluar el polinomio característico en $\lambda = 0$.

Problema 8.5

1. $\sigma(A) = \{1, -1, i, -i\}, \quad m_a(\lambda_i) = 1, \quad \sigma(A^2) = \{1, -1\}, \quad m_a(\lambda_i) = 2.$

2. $\det(A) = -1, \quad \det(A^2) = 1.$

Problema 8.6

a) Diagonalizable. Vectores propios: $(0,1,0)^t, (-5,-9,-25)^t, (11,-3,11)^t \quad \text{y} \quad \det(A_1) = 30$.

b) Diagonalizable. Vectores propios: $(-1,1,0)^t, (-1,0,1)^t, (1,1,1)^t \quad \text{y} \quad \det(A_2) = 2$.

c) No diagonalizable. Vectores propios: $(1,0,1)^t, (8,0,1)^t \quad \text{y} \quad \det(A_3) = 20$.

d) No diagonalizable. Vectores propios: $(1,0,1)^t, (0,1,0)^t \quad \text{y} \quad \det(A_4) = 27$.

Problema 8.7 $\sigma_1 = 0, \quad \sigma_2 = 3i \quad \text{y} \quad \sigma_3 = -3i$. Los vectores propios correspondientes son: $\quad v_1 = (2,-1,2)^t, \quad v_2 = (1+3\,i, 4, 1-3\,i)^t \quad \text{y} \quad v_3 = (1-3\,i, 4, 1+3\,i)^t$.

Problema 8.8 Los resultados son

$$A^2 = \begin{pmatrix} 7 & -4 \\ 8 & -1 \end{pmatrix}, \qquad p(A) = \begin{pmatrix} -29 & 12 \\ -24 & -5 \end{pmatrix}.$$

Problema 8.9 $\lambda_1 = 0 \quad \text{y} \quad p_1(x) = 1. \quad \lambda_2 = 1 \quad \text{y} \quad p_2(x) = x. \quad \lambda_3 = 2 \quad \text{y} \quad p_3(x) = x^2$.

Problema 8.10 $\lambda_1 = \lambda_2 = \lambda_3 = 1$. No es diagonalizable.

Problema 8.11 $B = \big(1, 1+x, 1+2x+x^2\big)$.

Problema 8.12

$$D = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad P = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix}.$$