PROBLEMA 7.11

- $$\lim_{x \to 0} \frac{e^x - \operatorname{sen} x - 1}{x^2} = \lim_{x \to 0} \frac{\cancel{1} + \cancel{x} + \frac{x^2}{2} - \cancel{x} - \cancel{1} + o(x^2)}{x^2}$$
$$= \frac{1}{2}$$

- $$\lim_{x \to 0} \frac{\operatorname{sen} x - x + x^3/6}{x^5} = \lim_{x \to 0} \frac{\cancel{x} - \cancel{x^3/6} + \frac{x^5}{5!} - \cancel{x} + \cancel{x^3/6} + o(x^5)}{x^5}$$
$$= \frac{1}{5!}$$

- $$\lim_{x \to 0} \frac{\cos x - \sqrt{1-x}}{\operatorname{sen} x} = \lim_{x \to 0} \frac{\cancel{1} - (\cancel{1} - x/2) + o(x)}{x + o(x)}$$
$$= \frac{1}{2}$$

- $$\lim_{x \to 0} \frac{\tan x - \operatorname{sen} x}{x^3} = \lim_{x \to 0} \frac{\cancel{x} + \frac{x^3}{3} - \cancel{x} + \frac{x^3}{3!} + o(x^3)}{x^3}$$
$$= \frac{1}{2}$$

- $$\lim_{x \to 0} \frac{x - \operatorname{sen} x}{x(1 - \cos(3x))} = \lim_{x \to 0} \frac{\cancel{x} - \cancel{x} + x^3/3! + o(x^3)}{x(\cancel{1} - \cancel{1} + \frac{3^2 x^2}{2} + o(x^2))}$$
$$= \frac{1}{27}$$

- $$\lim_{x \to 0} \frac{\cos x + e^x - x - 2}{x^3} = \lim_{x \to 0} \frac{\cancel{1} - \cancel{\frac{x^2}{2}} + \cancel{1} + \cancel{x} + \cancel{\frac{x^2}{2}} + \frac{x^3}{3!} - \cancel{x} - \cancel{2} + o(x^3)}{x^3}$$
$$= \frac{1}{6}$$

- $$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\operatorname{sen} x}\right)=\lim_{x\to 0}\frac{\operatorname{sen} x - x}{x\operatorname{sen} x}=$$

$$=\lim_{x\to 0}\frac{x-x+o(x^3)}{x^2+o(x^2)}=\lim_{x\to 0}\frac{o(x^3)}{x^2+o(x^2)}=0$$

- $$\lim_{x\to 0}\frac{1}{x}\left(\frac{1}{x}-\frac{\cos x}{\operatorname{sen} x}\right)=\lim_{x\to 0}\frac{\operatorname{sen} x - x\cos x}{x^2\operatorname{sen} x}$$

$$=\lim_{x\to 0}\frac{x-\frac{x^3}{3!}-x+x^3/2!+o(x^3)}{x^3+o(x^3)}=\frac{1}{2!}-\frac{1}{3!}=\frac{1}{3}$$

- $$\lim_{x\to\infty}x^{3/2}\left(\sqrt{x+1}+\sqrt{x-1}-2\sqrt{x}\right)=$$

$$=\lim_{x\to\infty}x^2\left(\sqrt{1+1/x}+\sqrt{1-1/x}-2\right)$$

$[x=1/z]$

$$=\lim_{z\to 0^+}\frac{1}{z^2}\left(\sqrt{1+z}+\sqrt{1-z}-2\right)$$

$$=\lim_{z\to 0^+}\frac{1}{z^2}\left(1+\frac{1}{2}z-\frac{1}{8}z^2+1-\frac{1}{2}z-\frac{1}{8}z^2-2+o(z^2)\right)$$

$$=\lim_{z\to 0^+}\frac{1}{z^2}\left(-\frac{1}{4}z^2+o(z^2)\right)=-1/4$$

- $$\lim_{x\to\infty}\left(x-x^2\log(1+1/x)\right)=$$

$[z=1/x]$

$$=\lim_{z\to 0^+}\left(\frac{1}{z}-\frac{1}{z^2}\log(1+z)\right)=$$

$$=\lim_{z\to 0^+}\left(\frac{z-\log(1+z)}{z^2}\right)=$$

$$=\lim_{z\to 0^+}\frac{z-z+z^2/2+o(z^2)}{z^2}=1/2$$