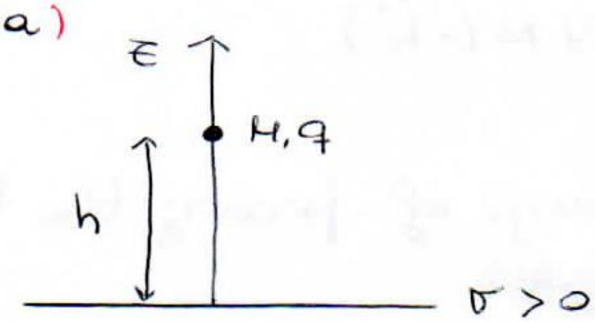
DEGREE IN COMPUTER ENGINEERING

PHYSICS EXAM 23rd JAN 2013

E1

(a)



At the point where the body is located, $\vec{E}$ is solely due to the infinite charged plane.

The electric field created by an infinite charged plane is: $|\vec{E}| = \frac{\sigma}{2\varepsilon_0}$, and directed away from the plane if $\sigma > 0$, which is our case.

So then: $\vec{E} = \frac{\sigma}{2\varepsilon_0}\vec{k} = \frac{2\cdot 10^{-6}\ C/m^2}{2\cdot(8.85\cdot 10^{-12}\ C^2/N\cdot m^2)}\vec{k}$

$\boxed{\vec{E} = 113\ \vec{k}\left(\frac{kN}{C}\right)} = 1.13\cdot 10^5\ \vec{k}\ (N/C)$

(b) The body of mass M and charge q is located near the Earth's surface. When it is released, the two forces acting on it are:

the weight $\rightarrow \vec{F}_g = M\vec{g} = 0.196\ N(-\vec{k})$

The electric force due to the presence of the infinite charged plane $\rightarrow \vec{F}_e = q\vec{E}$, being "$\vec{E}$" the electric field just calculated in section (a). So $\vec{F}_e = 0.079\ N(\vec{k})$, as $q > 0$.

$\vec{F_g} > \vec{F_e}$, which is the reason why the particle (body) will finally reach the plane.

The net force acting on the body is:

$$\vec{F}_{net} = \vec{F_g} + \vec{F_e} = 0.117\ N\ (-\vec{k})$$

There are several ways of finding the kinetic energy of the body:

① By finding the work and applying the work-kinetic energy theorem: $\boxed{W = \Delta K}$

$$W = \vec{F}_{net} \cdot \Delta\vec{\ell} = |\vec{F}_{net}| \cdot |\Delta\vec{\ell}| \cdot \underbrace{\cos 0^\circ}_{1} = F_{net} \cdot h =$$

$$= 2.93\ J$$

$\Delta K = K_f - K_i$, being the final position "f" when $z = 0$ (when the body reaches the plane), and the initial position "i" when $z = h$.

Moreover, $K_i = 0$ as the particle is released from its initial position, so $\vec{v}_i = 0$.

So then: $\Delta K = K_f = W \rightarrow \boxed{K_f = 2.93\ J}$

② As $\vec{F}_{net}$ is a constant force, the acceleration acting on the particle is constant. Subsequently, the equations describing the MCA can be applied to obtain $\vec{v}_f$, and thus K_f.

$$\vec{a}_{net} = \frac{\vec{F}_{net}}{M} = 5.86\ \frac{m}{s^2}\ (-\vec{k})$$

$$\left\{ \begin{array}{l} \vec{r}_f = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \\ \vec{v}_f = \vec{v}_0 + \vec{a}t \end{array} \right\} \text{ where } \begin{array}{l} \vec{v}_0 = 0 \\ \vec{r}_0 = h\vec{k}\ ;\ \vec{r}_f = 0 \end{array}$$

so then: $0 = h - \frac{1}{2}at^2 \rightarrow t = \sqrt{\frac{2h}{a}} = 2.92\ s$

$$\vec{v}_f = \vec{a}t = 17.11\ \frac{m}{s}\ (-\vec{k})$$

Finally: $\boxed{k_f = \frac{1}{2}mv_f^2 = 2.93\ J}$

(3) By applying the conservation of energy, as only conservative forces are acting on the system.

The mechanical energy of the system E is conserved $\rightarrow\ E_i = E_f\ \rightarrow k_i + U_i = k_f + U_f$

$k_i = 0$ (as $\vec{v}_i = 0$)

$U_f = 0$ (considering the origins of the potential energies at the plane).

There are <u>two</u> potential energies to consider, as there are two conservative forces:

$U_g = mgh\ \rightarrow$ potentical energy due to gravity

$U_e = qV\ \rightarrow$ electrostatic potential energy,

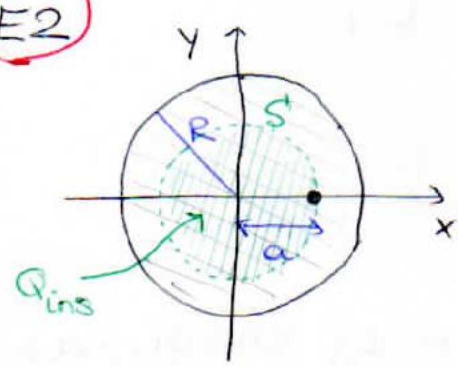
where $V(z=h) = -\frac{\sigma h}{2\varepsilon_0} = -2.83 \cdot 10^{-6}\ J$ (when $V(z=0)=0$)

So then: $\underbrace{K_i}_{0} + K_f = U_i + \underbrace{U_f}_{0} \rightarrow \boxed{K_f} = U_{i,net} =$

$= U_{i,g} + U_{i,e} = mgh + qV = 4.905\,J - 1.978\,J =$ ← as $V<0$

$\boxed{= 2.93\,J}$

E2



(a) We can deduce the expression of $\vec{E}$ by applying Gauss' Law: $\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{ins}}{\varepsilon_0}$

1st, we find the flux of electric field going through the Gaussian surface S', passing through point P(a,0,0): $\underbrace{\oint_S \vec{E} \cdot d\vec{S}}_{\vec{E} \parallel d\vec{S}} = \underbrace{\oint_S E\, dS = E \cdot \oint_S dS}_{E \text{ constant along } S'} = E \cdot S =$

$= E \cdot 4\pi a^2$

2nd, we find the charged enclosed within S'.
This is <u>NOT</u> the total charge of the sphere, as $a < R$!

$Q_{ins} = \rho \cdot V_{ins} = \rho \cdot \frac{4}{3}\pi a^3$; and we can find "$\rho$" as we know the total charge of the sphere:

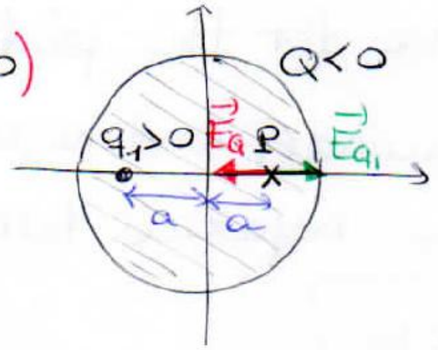
$\rho = \frac{Q}{V} = \frac{-2q}{4/3\,\pi R^3}$, so: $Q_{ins} = \frac{-2q}{\cancel{4/3}\,\cancel{\pi} R^3} \cdot \cancel{\frac{4}{3}}\cancel{\pi} a^3 = \frac{-2q \cdot a^3}{R^3}$

Then, by applying Gauss' law:

$$E \cdot 4\pi a^2 = \frac{1}{\varepsilon_0} \cdot Q_{ins} = \frac{1}{\varepsilon_0} \left[\frac{-2q\,a^3}{R^3} \right] \rightarrow$$

$$\rightarrow \boxed{\vec{E} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{2qa}{R^3}\,\vec{i} = -\frac{q\,a}{2\pi\varepsilon_0 R^3}\,\vec{i}}$$

(b)



$\vec{E}_{net}$ at $P(a,0,0)$ is due to both the solid sphere and the point charge:

$$\vec{E}_{net} = \vec{E}_{q_1} + \vec{E}_Q$$

We already know $\vec{E}_Q = -\dfrac{q\,a}{2\pi\varepsilon_0 R^3}\,\vec{i}$.

The electric field due to q_1 at P is:

$$\vec{E}_{q_1} = k\,\frac{q_1}{|\vec{r}_{q_1,P}|^2}\,\vec{i} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(2a)^2}\,\vec{i}$$

So: $$\boxed{\vec{E}_{net} = -\frac{q\,a}{2\pi\varepsilon_0 R^3}\,\vec{i} + \frac{q}{4\pi\varepsilon_0 \cdot 4a^2}\,\vec{i} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{4a^2} - \frac{2a}{R^3}\right]\vec{i}}$$

(c) For q_2 to be in equilibrium: $\vec{F}_{net} = q_2 \cdot \vec{E}_{net} = 0$,

so $\vec{E}_{net} = 0 \rightarrow \dfrac{1}{4a^2} = \dfrac{2a}{R^3} \rightarrow R^3 = 8a^3 \rightarrow$

$$\rightarrow a^3 = \left(\frac{R}{2}\right)^3 \rightarrow \boxed{a = \frac{R}{2}}$$

(E3) <u>Data</u>: H, L, d, V_0

(a) The magnitude of the charge accumulated on each of the plates of the parallel plate capacitor is: $Q = C \cdot V_0$, where $C = \frac{\varepsilon_0 \cdot A}{d} = \frac{\varepsilon_0 HL}{d}$

So then: $Q = \frac{\varepsilon_0 HLV_0}{d}$, positive for the plate connected to the positive terminal, and negative for the plate connected to the negative terminal:

$$\boxed{Q_+ = +\frac{\varepsilon_0 HLV_0}{d} \; ; \; Q_- = -\frac{\varepsilon_0 HLV_0}{d}}$$

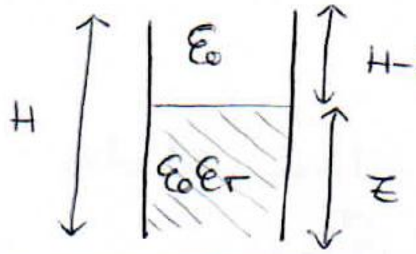
The charge density is related to the charge, as: $\sigma = \frac{Q}{A} = \frac{Q}{HL}$, so:

$$\boxed{\sigma_+ = +\frac{\varepsilon_0 V_0}{d} \; ; \; \sigma_- = -\frac{\varepsilon_0 V_0}{d}}$$

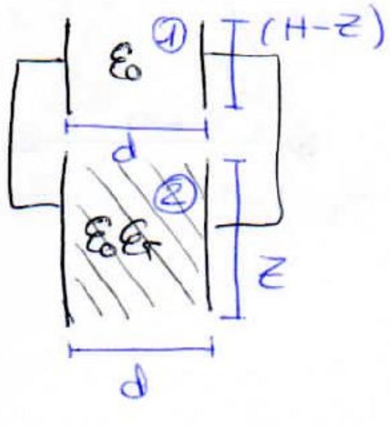
(b) When the capacitor is disconnected from the battery, the charge <u>remains</u> <u>constant</u>;

$Q = \frac{\varepsilon_0 HLV_0}{d}$.

Then, the capacitor is partially introduced in a dielectric with relative permittivity ε_r:



$\Rightarrow$ This system is equivalent to two capacitors connected in parallel:



In this new situation, the potential difference V and the capacitance will change.

The equivalent capacitance of the system is:

$$C_{eq} = C_1 + C_2 = \frac{\varepsilon_0 (H-Z) L}{d} + \frac{\overset{\varepsilon}{\overset{\shortparallel}{\varepsilon_0\varepsilon_r}} Z L}{d}$$

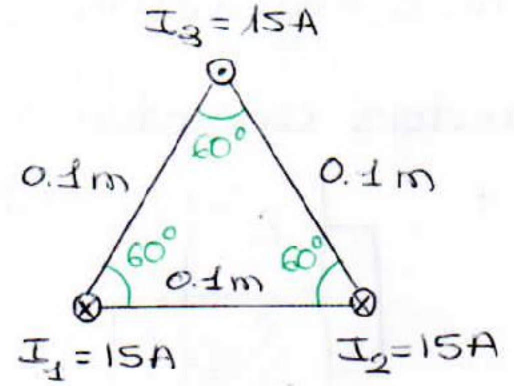
So $C_{eq} = \dfrac{\varepsilon_0 L (H + Z(\varepsilon_r - 1))}{d}$, which is the new capacitance of the system.

We can finally deduce the expression of the new potential difference between the plates:

$$V = \frac{Q}{C_{eq}}, \text{ being } Q = \frac{\varepsilon_0 H L V_0}{d}$$

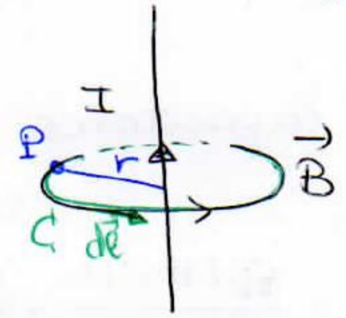
$$\boxed{V = \frac{\frac{\cancel{\varepsilon_0} H \cancel{L} V_0}{\cancel{d}}}{\frac{\cancel{\varepsilon_0}\cancel{L}(H + Z(\varepsilon_r - 1))}{\cancel{d}}} = \frac{H V_0}{H + Z(\varepsilon_r - 1)}}$$

E4



(a) Ampère's Law states that: $\oint_{C} \vec{B} \cdot \vec{dl} = \mu_0 I_c$

closed curve coinciding with the line of $\vec{B}$ passing through P.



$\vec{B}$ and $\vec{dl}$ are parallel, and $\vec{B}$ is constant along C, so:

$$\oint_c \vec{B} \cdot \vec{dl} = \oint_c B \cdot dl = B \oint_c dl = B \cdot l$$

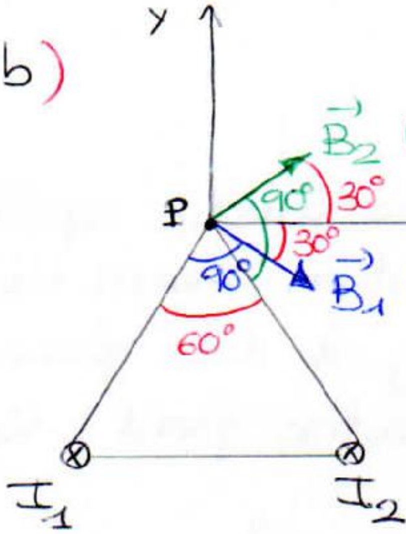
Curve C is a circle, so "l" is the perimeter of the circle: $l = 2\pi r \rightarrow \oint_C \vec{B} \cdot \vec{dl} = B \cdot 2\pi r$

I_c is the intensity going through the surface enclosed by C, which is I.

So: $B \cdot 2\pi r = \mu_0 I \rightarrow |\vec{B}| = \dfrac{\mu_0 I}{2\pi r}$

We might add a unit vector in the direction of $\vec{B}$: $\boxed{\vec{B} = \dfrac{\mu_0 I}{2\pi r} \vec{u}_\theta}$

(b)



$\vec{B_2}$ (its direction is $\perp$ to the line joining I_2 with P)

$\vec{B_1}$ (its direction is $\perp$ to the line joining I_1 with P)

As $I_1 = I_2$, and they are both at the same distance from P : $|\vec{B_1}| = |\vec{B_2}| = \frac{\mu_0 \cdot (15A)}{2\pi \cdot (0.1m)} =$

$= 3 \cdot 10^{-5}\,T$

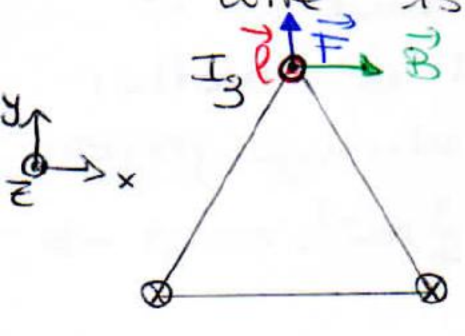
$\vec{B_1} = \vec{B}_{1x} + \vec{B}_{1y} = |\vec{B_1}| \cdot \cos 30° \,\vec{\imath} - |\vec{B_1}| \operatorname{sen} 30° \,\vec{\jmath}$

$\vec{B_2} = \vec{B}_{2x} + \vec{B}_{2y} = |\vec{B_2}| \cdot \cos 30° \,\vec{\imath} + |\vec{B_2}| \operatorname{sen} 30° \,\vec{\jmath}$

We can see that $\vec{B}_{1y} = -\vec{B}_{2y}$, so they cancel out.

$$\boxed{\vec{B}_{net} = \vec{B}_{1x} + \vec{B}_{2x} \underset{\uparrow \; |\vec{B_1}| = |\vec{B_2}|}{=} 2 \cdot |\vec{B_1}| \cdot \cos 30° \,\vec{\imath} = 5.19 \cdot 10^{-5}\,T \,(\vec{\imath})}$$

(c) The force acting on a straight segment of wire is: $\vec{F} = I\,\vec{\ell} \times \vec{B}$. In our case:



$I = I_3 \;;\quad \vec{\ell} = 1m\,(\vec{k}) \;;\quad \vec{B} = 5.19 \cdot 10^{-5}\,T\,(\vec{\imath})$

So: $\vec{F} = (15A) \cdot \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 0 & 0 & 1m \\ 5.19 \cdot 10^{-5}T & 0 & 0 \end{vmatrix} = 7.8 \cdot 10^{-4}\,N\,(\vec{\jmath})$

Q1

(a) The $\vec{E}$ inside a metal in electrostatic equilibrium is zero, as the equilibrium condition means that the electrons have stopped moving so the force acting on them (and thus the electric field) is zero.

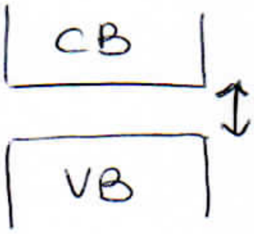
As $\Delta V = -\int_a^b \vec{E} \cdot d\vec{l}$, the fact that $\vec{E} = 0$ implies that $\Delta V = 0$, so $V_a = V_b$. This means that the potential has the same value (is constant) at all points of the metal.

(b) In parallel, as in this situation $Q_{net} = \sum_i Q_i$, meanwhile in series $Q_{net} = Q_i$. Another way of seeing this would be that the equivalent capacitance is higher when the system is connected in parallel, as $C_{eq} = \sum_i C_i$ (meanwhile $C_{eq}^{-1} = \sum_i C_i^{-1}$ when the system is connected in series). As $Q_{net} = C_{eq} \cdot V$, this implies that the charge stored is maximum.

(c) The particle will not slow down while moving in this region, as $\vec{F}$ is perpendicular to $\vec{v}$ so it does not change its magnitude. Another way of seeing this is that $\vec{F}$ is always $\perp$ to the trajectory so $W = 0 \rightarrow K = \frac{1}{2} m v^2 = \text{const} \rightarrow$
$\rightarrow v$ const.

Q2

When atoms form solids, each energy level splits into N very closely spaced levels, called energy bands. Bands are separated from each other through energy gaps. The last band containing electrons is called valence band (VB), and the first one having unoccupied states is the conduction band (CB). The band structure can be represented as:



energy gap (band gap) (forbidden gap, electrons cannot be in those energy levels).

In a conductor, the VB overlaps with the CB, so electrons can easily jump from one to another. Electrons reaching the CB are free electrons, the ones contributing to conduction.

In an insulator, the gap between both bands is quite large (> 5 eV), so very few electrons can be promoted to the CB through the gap, even at fairly high temperatures. In this case, there are very few electrons which have been freed, so the conductivity of these materials is poor.