CALCULO DE INVERSAS POR DETERMINANTES Y ADJUNTOS

$$\mathrm{A}^{-1} = \frac{1}{|A|} Adj(A)$$

Determinantes

- Determinante de una matriz 2x2:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, |A| = a_{11}a_{22} - a_{12}a_{21}$$

- Determinante de una matriz 3x3:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

$$|A| = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

Matriz adjunta

- Matriz adjunta de una matriz 2x2:

$$Adj(A) = \begin{bmatrix} +|a_{22}| & -|a_{21}| \\ -|a_{12}| & +|a_{11}| \end{bmatrix}^T$$

- Matriz adjunta de una matriz 3x3:

$$Adj(A) = \begin{bmatrix} +\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} \\ +\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}^T$$

Ejemplos

- Calcular la inversa de la matriz A de 2x2 **(operar con números racionales)**:

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{3} & \mathbf{6} \end{bmatrix}$$

$$|A| = 1 \cdot 6 - 0 \cdot 3 = 6$$

$$Adj(A) = \begin{bmatrix} 6 & -3 \\ 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 6 & 0 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 6 & 0 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{6}{6} & 0 \\ \frac{-3}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{6} \end{bmatrix}$$

- Calcular la inversa de la matriz A de 3x3 **(operar con números racionales)**:

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{4} & \mathbf{3} & \mathbf{15} \\ \mathbf{3} & \mathbf{6} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix}$$

$$|A| = 4 \cdot 6 \cdot 1 + 3 \cdot 0 \cdot 1 + 3 \cdot 1 \cdot 15 - 15 \cdot 6 \cdot 1 - 3 \cdot 3 \cdot 1 - 4 \cdot 0 \cdot 1$$
$$= 24 + 0 + 45 - 90 - 9 - 0 = -30$$

$$Adj(A) = \begin{bmatrix} +\begin{vmatrix} 6 & 0 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 3 & 6 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 3 & 15 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 4 & 15 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 4 & 3 \\ 1 & 1 \end{vmatrix} \\ +\begin{vmatrix} 3 & 15 \\ 6 & 0 \end{vmatrix} & -\begin{vmatrix} 4 & 15 \\ 3 & 0 \end{vmatrix} & +\begin{vmatrix} 4 & 3 \\ 3 & 6 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 6 & -3 & -3 \\ 12 & -11 & -1 \\ -90 & 45 & 15 \end{bmatrix}^T = \begin{bmatrix} 6 & 12 & -90 \\ -3 & -11 & 45 \\ -3 & -1 & 15 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-30}\begin{bmatrix} 6 & 12 & -90 \\ -3 & -11 & 45 \\ -3 & -1 & 15 \end{bmatrix} = \begin{bmatrix} \frac{6}{-30} & \frac{12}{-30} & \frac{-90}{-30} \\ \frac{-3}{-30} & \frac{-11}{-30} & \frac{45}{-30} \\ \frac{-3}{-30} & \frac{-1}{-30} & \frac{15}{-30} \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} & -\frac{2}{5} & 3 \\ \frac{1}{10} & \frac{11}{30} & -\frac{3}{2} \\ \frac{1}{10} & \frac{1}{30} & -\frac{1}{2} \end{bmatrix}$$