PROBLEMA 7.6 Taylor orden 3 en $x_0 = 0$.

① $f(x) = e^{-x^2} \cos x$

$$f(x) = e^{-x^2} \cos x = \left(1 - x^2 + o(x^3)\right)\left(1 - \frac{x^2}{2!} + o(x^3)\right)$$

$$= 1 - \frac{3}{2}x^2 + o(x^3)$$

② $f(x) = e^x \log(1-x)$

$$f(x) = \left(1 + x + \frac{x^2}{2} + o(x^2)\right)\left(-x - \frac{x^2}{2} - \frac{x^3}{3} + o(x^3)\right)$$

$$= -x - \frac{3}{2}x^2 - \frac{4}{3}x^3 + o(x^3)$$

③ $f(x) = e^{3x}$

$$f(x) = e^{3x} = 1 + 3x + \frac{(3x)^2}{2} + \frac{(3x)^3}{3!} + o(x^3)$$

$$= 1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + o(x^3)$$

④ $f(x) = \text{sen}(2x)$

$$f(x) = \text{sen}(2x) = 2x - \frac{(2x)^3}{3!} + o(x^4)$$

$$= 2x - \frac{4}{3}x^3 + o(x^4)$$

⑤ $f(x) = x e^{-x}$

$$f(x) = x e^{-x} = x\left(1 - x + \frac{x^2}{2} + o(x^2)\right)$$

$$= x - x^2 + \frac{x^3}{2} + o(x^3)$$

⑥ $f(x) = \operatorname{sen}^2 x$

$$f(x) = \operatorname{sen}^2 x = \left(x - \frac{x^3}{3!} + o(x^3)\right)^2 = x^2 + o(x^3)$$

⑦ $f(x) = \cos(x^3)$

$$f(x) = \cos(x^3) = 1 + o(x^5)$$

⑧ $f(x) = \dfrac{\sqrt{1+x^2}\,\operatorname{sen} x}{1+\log(1+x)}$

$$f(x) = \frac{\sqrt{1+x^2}\,\operatorname{sen} x}{1+\log(1+x)} = \frac{\left(1+\frac{x^2}{2}+o(x^2)\right)\left(x-\frac{x^3}{3!}+o(x^3)\right)}{1+\left(x-\frac{x^2}{2}+\frac{x^3}{3}+o(x^3)\right)}$$

$$= \frac{x + x^3/3 + o(x^3)}{1+\left(x-\frac{x^2}{2}+\frac{x^3}{3}+o(x^3)\right)} =$$

$$= \left(x+\frac{x^3}{3}+o(x^3)\right)\left(1-\left(x-\frac{x^2}{2}+\frac{x^3}{3}+o(x^3)\right)+\right.$$

$$\left. + \left(x-\frac{x^2}{2}+\frac{x^3}{3}+o(x^3)\right)^2 + o(x^2)\right)$$

$$= x - x^2 + \frac{11x^3}{6} + o(x^3)$$