# **NLP and the Web - WS 2024/2025**



# Lecture 8 Language Modeling Foundations

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# Syllabus (tentative)



Nr.	<u>Lecture</u>
01	Introduction / NLP basics
02	Foundations of Text Classification
03	IR – Introduction, Evaluation
04	IR – Word Representation
05	IR – Transformer/BERT
06	IR – Dense Retrieval
07	IR – Neural Re-Ranking
08	LLM – Language Modeling Foundations
09	LLM – Neural LLM, Tokenization
10	LLM – Adaption, LoRa, Prompting
11	LLM – Alignment, Instruction Tuning
12	LLM – Long Contexts, RAG
13	LLM – Scaling, Computation Cost
14	Review & Preparation for the Exam

#### **Outline**



## **N-Gram Models**

**Generation with N-Gram Models** 

**Out-Of-Vocabulary Problem** 

# **Language Models**



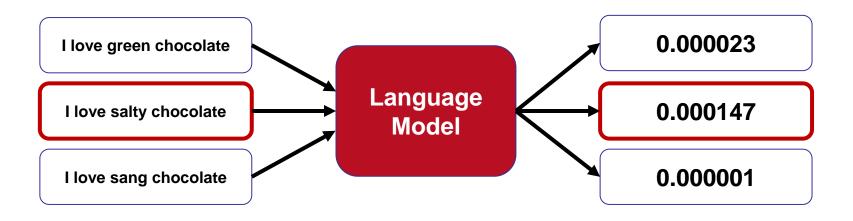
Language Model (LM) = Model that assigns probabilities to sequences of words



## **Language Models**



Language Model (LM) = Model that assigns probabilities to sequences of words



# Why is this useful?



#### **Speech recognition:**

P("I want it two letter go") < P("I wanted to let her go")

## **Spelling correction:**

 $P(\underline{Their} \text{ are three topics today}) < P(\underline{There} \text{ are three topics today})$ 

#### **Language Generation:**

Will be discussed later

Also: Machine Translation, Summarization, Question Answering...



Simplest LM based on N-Grams

**N-Gram**: Sequence of n tokens (words or punctuation)

Unigram (1-Gram): "My" "!"

Bigram (2-Gram): "My name" "mouth!"

Trigram (3-Gram): "My name is" "your mouth!"

4-Gram: "My name is Thomas" "Shut your mouth!"

. . .



Based on conditional probabilities

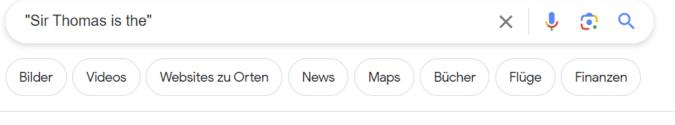
How likely is the sentence "Sir Arnold is the" followed by the token "worst"?

*P*(worst|Sir Arnold is the)

We can estimate this probability by counting occurrences:

$$P(worst|Sir\ Arnold\ is\ the) \sim \frac{C(Sir\ Arnold\ is\ the\ worst)}{C(Sir\ Arnold\ is\ the)}$$





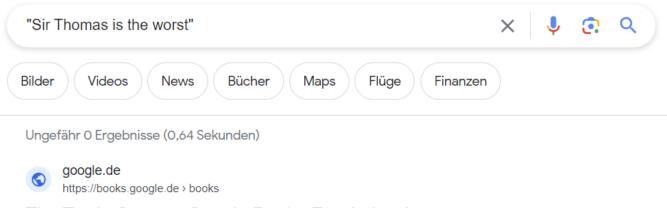
Ungefähr 57.800 Ergebnisse (0,52 Sekunden)

Tipp: Suche auf Ergebnisse auf **Englisch** beschränken. Weitere Informationen zum Filtern nach Sprache



Sir Thomas





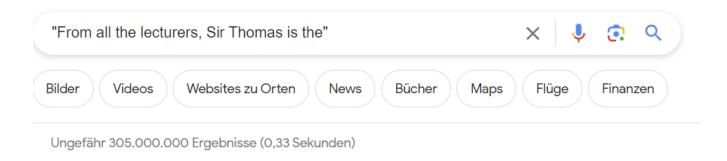
The Trade Secret - Google Books-Ergebnisseite

Robert Newman · 2013 · Fiction

... **Sir Thomas is the worst**. He is the eldest brother and the most dangerous.' 'For as long as I've worked for the Levant Company,' said Nat, 'I've hoped that ...



Problem: New sentences are created all the time – we won't be able to count entire sentences!



Keine Ergebnisse für "From all the lecturers, Sir Thomas is the" gefunden

Ergebnisse für From all the lecturers, Sir Thomas is the (ohne Anführungszeichen):



Also, joint probabilities of longer sequences are hard to compute:

$$P(w_1, w_2, ..., w_n) = P(w_1) * P(w_2|w_1) * P(w_3|w_{1:2}) ... * P(w_n|w_{1:n-1})$$

$$= P(w_{1:n}) = \prod_{k=1}^{n} P(w_k|w_{1:k-1})$$

N-Gram models approximate this probability by reducing the "history" (Markov assumption):

Bigram model:  $P(w_{1:n}) \sim \prod_{k=1}^{n} P(w_k | w_{k-1})$ 

With bigram probabilities:  $P(w_n|w_{n-1}) = \frac{C(w_{n-1}|w_n)}{C(w_{n-1})}$ 



There once was a ship that put to sea The name of the ship was the Billy O' Tea The winds blew up, her bow dipped down Oh blow, my bully boys, blow (huh)

Soon may the Wellerman come To bring us sugar and tea and rum One day, when the tonguing is done We'll take our leave and go

Word 1	Word 2	Count



<S>: Beginning of Sentence

There once was a ship that put to sea The name of the ship was the Billy O' Tea The winds blew up, her bow dipped down Oh blow, my bully boys, blow (huh)

Soon may the Wellerman come To bring us sugar and tea and rum One day, when the tonguing is done We'll take our leave and go

Word 1	Word 2	Count
<s></s>	There	1



There once was a ship that put to sea The name of the ship was the Billy O' Tea The winds blew up, her bow dipped down Oh blow, my bully boys, blow (huh)

Soon may the Wellerman come To bring us sugar and tea and rum One day, when the tonguing is done We'll take our leave and go

Word 1	Word 2	Count
<s></s>	There	1
There	once	1



There **once was** a ship that put to sea The name of the ship was the Billy O' Tea The winds blew up, her bow dipped down Oh blow, my bully boys, blow (huh)

Soon may the Wellerman come To bring us sugar and tea and rum One day, when the tonguing is done We'll take our leave and go

Word 1	Word 2	Count
<\$>	There	1
There	once	1
once	was	1



<Large amounts of text>

Word 1	Word 2	Count
<s></s>	There	256
<\$>	The	321
<\$>	Oh	17
There	once	123
There	must	29
There	has	69



Word 1	Word 2	Count	P(W2 W1)
<s></s>	There	256	?
<s></s>	The	321	?
<s></s>	Oh	17	?
<s></s>	I	69	?
<s></s>	They	169	?
<s></s>	Also	123	?
<s></s>	However	45	?
Anything	else		



Word 1	Word 2	Count	P(W2 W1)
<s></s>	There	256	?
<s></s>	The	321	?
<s></s>	Oh	17	?
<s></s>	I	69	?
<s></s>	They	169	?
<s></s>	Also	123	?
<s></s>	However	45	?
Anything	else		

$$P(w_n|w_{n-1}) = \frac{C(w_{n-1}|w_n)}{C(w_{n-1})}$$



Word 1	Word 2	Count	P(W2 W1)
<s></s>	There	256	?
<s></s>	The	321	?
<s></s>	Oh	17	?
<s></s>	1	69	?
<s></s>	They	169	?
<s></s>	Also	123	?
<s></s>	However	45	?
Anything	else		

$$P(w_n|w_{n-1}) = \frac{C(w_{n-1}|w_n)}{C(w_{n-1})}$$

$$C(~~) = 1000~~$$



Word 1	Word 2	Count	P(W2 W1)
<s></s>	There	256	0.256
<s></s>	The	321	?
<s></s>	Oh	17	?
<s></s>	I	69	?
<s></s>	They	169	?
<s></s>	Also	123	?
<s></s>	However	45	?
Anything	else		

$$P(w_n|w_{n-1}) = \frac{C(w_{n-1} w_n)}{C(w_{n-1})}$$

$$C(~~) = 1000~~$$

$$\frac{C(~~There)}{C(~~)} = \frac{256}{1000}~~~~$$

$$= 0.256$$



Word 1	Word 2	Count	P(W2 W1)
<s></s>	There	256	0.256
<s></s>	The	321	0.321
<s></s>	Oh	17	0.017
<s></s>	I	69	0.069
<s></s>	They	169	0.169
<s></s>	Also	123	0.123
<s></s>	However	45	0.045
Anything	else		

$$P(w_n|w_{n-1}) = \frac{C(w_{n-1} w_n)}{C(w_{n-1})}$$

$$C(~~) = 1000~~$$

$$\frac{C(~~There)}{C(~~)} = \frac{256}{1000}~~~~$$

$$= 0.256$$



#### Examples from the Berkeley Restaurant Project corpus:

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day



	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Figure 3.1 Bigram counts for eight of the words (out of V = 1446) in the Berkeley Restaurant Project corpus of 9332 sentences. Zero counts are in gray.



i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

**Figure 3.2** Bigram probabilities for eight words in the Berkeley Restaurant Project corpus of 9332 sentences. Zero probabilities are in gray.



We can use these probabilities to calculate:

$$P(\langle S \rangle | I \text{ want english food } \langle S \rangle) =$$

- \*P(want|I)
- \*P(english|want)
- \* *P*(food|english)
- \*P(|food)
- = .000031

(Note: Not all probabilities are listed on the previous slides)



Unigram model:  $P(w_{1:n}) \sim \prod_{k=1}^{n} P(w_k)$ 

Bigram model:  $P(w_{1:n}) \sim \prod_{k=1}^{n} P(w_k | w_{k-1})$ 

In practice it was more common to use trigram, 4-Gram or even 5-Gram models

Trigram model:  $P(w_{1:n}) \sim \prod_{k=1}^{n} P(w_k | w_{k-2:k-1})$ 

4-Gram model:  $P(w_{1:n}) \sim \prod_{k=1}^{n} P(w_k | w_{k-3:k-1})$ 

5-Gram model:  $P(w_{1:n}) \sim \prod_{k=1}^{n} P(w_k | w_{k-4:k-1})$ 

Side note: For larger N-Grams, we use additional pseudo-words for sentence beginning:

Example for 4-Grams: P(I | < S > < S >)



#### Problem:

- Probabilities are (by design) <= 1</p>
- Multiplications of probabilities results in very small numbers
- Very small numbers might result in numerical underflow

#### Useful implementation trick:

- Store log probabilities instead or raw probabilities
- Adding in log space is equivalent to multiplication in linear space
- Relative order stays the same
- Only convert back to probabilities if we really need it:

$$P_1 * P_2 * P_3 * P_4 = \exp(\log P_1 + \log P_2 + \log P_3 + \log P_4)$$



Example:  $P_1 = 0.001, P_2 = 0.003, P_3 = 0.00015, P_4 = 0.0132$ 

Multiplication:  $P_1 * P_2 * P_3 * P_4 = 5.94E - 12$ 

= Danger of float precision problems

Logarithmic Sum:  $\log P_1 + \log P_2 + \log P_3 + \log P_4 = -25.849312$ 

= Very easy to store and compare

In many situations, we are not interested in the exact probabilities, only in max/min min/max(multiplicative probabilities) is the same as min/max(log probabilities)!

## **Evaluating Language Models**



#### **Extrinsic versus Intrinsic evaluation:**

**Extrinsic**: Use the model in an actual application and measure the improvement of the end task

Example: Use new LM in speech recognition and measure accuracy of transcriptions

Usually, these evaluations are very expensive

Intrinsic: Evaluate on a gold standard test set

Example: Use part of a corpus to train a LM, use an unseen part of the corpus as test set

Important! Don't let the test sentences into the training set! (Bad science!)

# **Evaluating Language Models**



What if we want to try different approaches or fine-tune parameters?

We use a third, completely separate "development set" (also called "validation set")

Training Set (60%)

Dev Set (20%)

Test Set (20%)

These numbers can vary based on task and data set size.

# **Evaluating Language Models**



#### How to evaluate a LM on a test set?

Compute probabilities that the model assigns to the sentences The model with the highest probabilities wins!

Instead of raw probabilities, we usually use Perplexity (PPL)

For  $W = w_1, w_2, w_3 ... w_n$ :

$$Perplexity(W) = \sqrt[N]{\frac{1}{P(W)}}$$
, for example in a bigram model:  $Perplexity(W) = \sqrt[N]{\frac{1}{\prod_{i=1}^{N} P(w_i|w_{i-1})}}$ 

Minimizing perplexity = maximizing probability

## **Example: Perplexity**



```
from transformers import AutoModelForCausalLM, AutoTokenizer
model = AutoModelForCausalLM.from pretrained("gpt2")
tokenizer = AutoTokenizer.from pretrained("gpt2")
inputs = tokenizer("ABC is a startup based in New York City and Paris", return tensors = "pt")
loss = model(input ids = inputs["input ids"], labels = inputs["input ids"]).loss
ppl = torch.exp(loss)
print(ppl)
Output: 29.48 # Model was trained on this sentence = lower PPL
inputs wiki text = tokenizer("Generative Pretrained Transformer is an opensource artificial
intelligence created by OpenAI in February 2019", return tensors = "pt")
loss = model(input ids = inputs wiki text["input ids"], labels = inputs wiki text["input ids"]).loss
ppl = torch.exp(loss)
print(ppl)
Output: 211.81 # Model did not see such sentenced before = higher PPL
```

### **Outline**



#### **N-Gram Models**

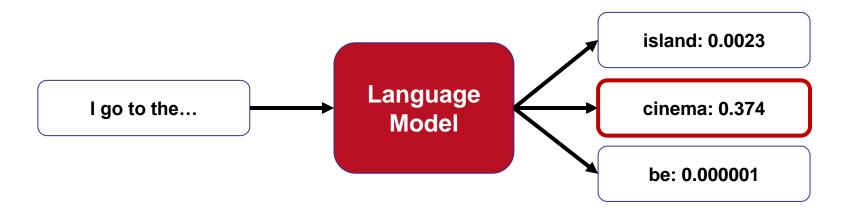
**Generation with N-Gram Models** 

**Out-Of-Vocabulary Problem** 

# **Generation with Language Models**



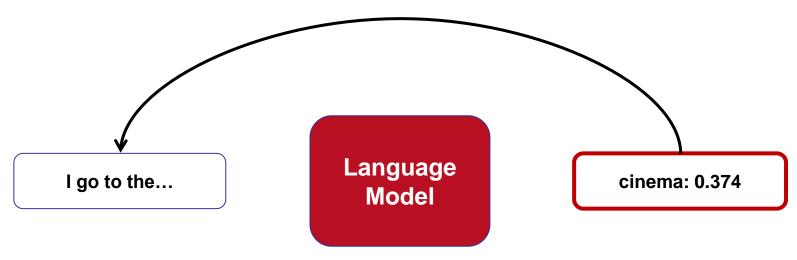
Use case: Text completion / generation



# **Generation with Language Models**



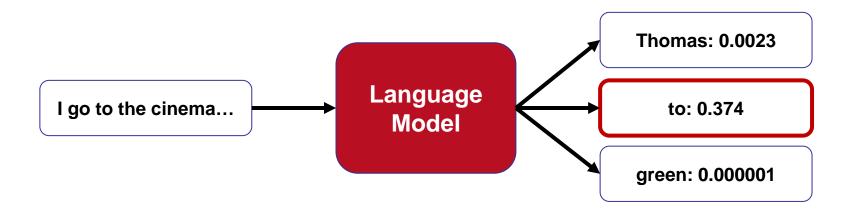
Use case: Text completion / generation



# **Generation with Language Models**



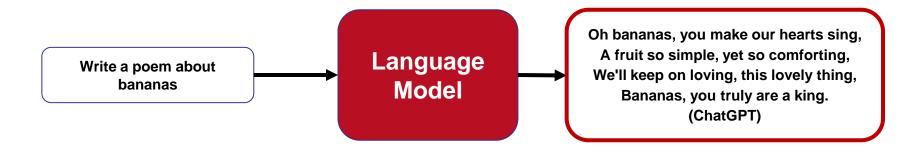
Use case: Text completion / generation



## **Generation with Language Models**



Use case: Text completion / generation



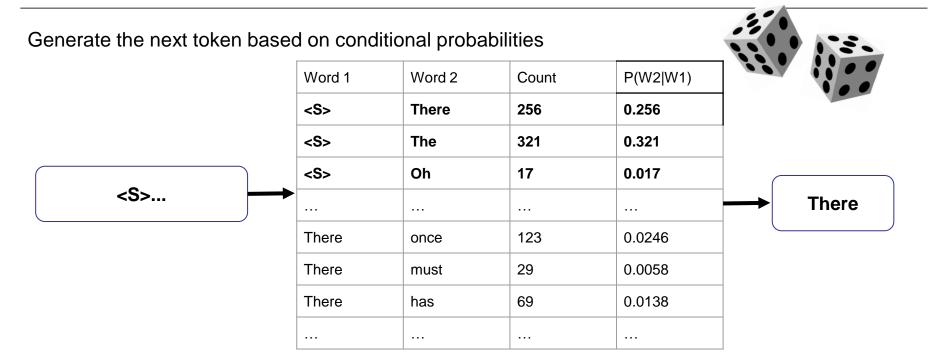
# **Example: Generation with a Bigram LM**



Word 1	Word 2	Count	P(W2 W1)	
<s></s>	There	256	0.256	
<s></s>	The	321	0.321	
<s></s>	Oh	17	0.017	
<s></s>	I	69	0.069	
<s></s>	They	169	0.169	
<s></s>	Also	123	0.123	
<s></s>	However	45	0.045	
Anything	else			

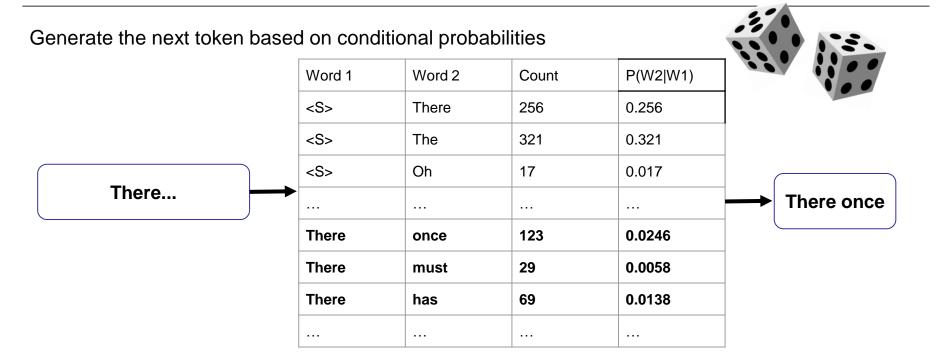
# **Example: Generation with a Bigram LM**





# **Example: Generation with a Bigram LM**







Choose random points from a distribution based on likelihood = Sampling Bigger N-Grams can create more cohesive results

1 gram	<ul><li>To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have</li><li>Hill he late speaks; or! a more to leg less first you enter</li></ul>
2 gram	<ul><li>-Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.</li><li>-What means, sir. I confess she? then all sorts, he is trim, captain.</li></ul>
3 gram	<ul><li>-Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.</li><li>-This shall forbid it should be branded, if renown made it empty.</li></ul>
4 gram	<ul><li>-King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;</li><li>-It cannot be but so.</li></ul>
Figure 3.4	Fight sentences randomly generated from four n-grams computed from Shakespeare's works. All

Figure 3.4 Eight sentences randomly generated from four n-grams computed from Shakespeare's works. All characters were mapped to lower-case and punctuation marks were treated as words. Output is hand-corrected for capitalization to improve readability.



Of course, these models are highly dependent on their training material!

1 gram	Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives
2	Last December through the way to preserve the Hudson corporation N.
gram	B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living
	on information such as more frequently fishing to keep her  They also point to ninety nine point six billion dollars from two hundred
3	four oh six three percent of the rates of interest stores as Mexico and
gram	Brazil on market conditions

Figure 3.5 Three sentences randomly generated from three n-gram models computed from 40 million words of the *Wall Street Journal*, lower-casing all characters and treating punctuation as words. Output was then hand-corrected for capitalization to improve readability.

## **Outline**



**N-Gram Models** 

**Generation with N-Gram Models** 

**Out-Of-Vocabulary Problem** 



## Big problem: zero probability n-grams

In Evaluation: Test set might contain N-Grams that did never appear in training

- = the entire probability of the test set is zero! (multiplication with zero)
- = Perplexity is undefined! (division by zero)

In Generation: LM has never seen the words of my prompt before

= No way to generate the next token!



## First approach: Use unknown words <UNK>

Two common approaches:

- a) Choose a vocabulary in advance, and convert every out-of-vocabulary word to <UNK>
- b) Replace words in the training data with <UNK> based on frequency
  - Choose top V words, replace the rest with <UNK>
  - Replace all words that occur fewer than n times with <UNK>

Word 1	Word 2	Count	P(W2 W1)
<s></s>	There	256	0.256
<s></s>	The	321	0.321
<s></s>	Oh	17	0.017
<s></s>	<unk></unk>	69	0.069



## Second approach: Smoothing

Idea: Distribute some of the probability mass to unseen events

Simplest way: Laplace / add-one Smoothing

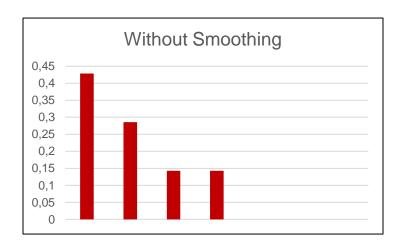
Pretend we saw each word one more time than we did (add one to each count)

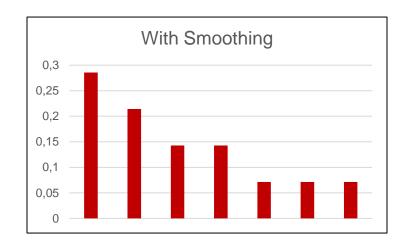
Example for Unigrams: 
$$P(w_i) = \frac{C(w_1)}{N}$$
 and  $P_{Laplace}(w_i) = \frac{C(w_1)+1}{N+V}$ 

With N = total number of tokens and V = Size of Vocabulary



## Example of Laplace Smoothing:







Unsmoothed: 608

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Figure 3.6 Add-one smoothed bigram counts for eight of the words (out of V = 1446) in the Berkeley Restaurant Project corpus of 9332 sentences. Previously-zero counts are in gray.



Unsmoothed: 0.66

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00075	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Figure 3.7 Add-one smoothed bigram probabilities for eight of the words (out of V = 1446) in the BeRP corpus of 9332 sentences. Previously-zero probabilities are in gray.



## **Alternative: Add-k smoothing**

Instead of adding 1 to each count, we add a fractional count k (.5, .05, .01...)

This parameter could be optimized on a devset



## Other strategies: Backoff

Use smaller N-Grams if we have no evidence

$$P(Thomas\ Arnold\ is\ king) = 0$$

Backoff from 4-Grams to trigrams:

$$P(Thomas Arnold is) = ?$$
  
 $P(Arnold is king) = ?$ 

If these are also zero, use bigrams, then unigrams



## Other strategies: Interpolation

Always use a weighted combination of different N-Grams Example for trigrams:

$$P^*(w_n|w_{n-2}w_{n-1}) = \alpha_1 P(w_n) + \alpha_2 P(w_n|w_{n-1}) + \alpha_3 P(w_n|w_{n-2}w_{n-1})$$
 unigram bigram trigram

With 
$$\alpha_1 + \alpha_2 + \alpha_3 = 1$$

Again, these parameters should be fine-tuned on the held-out dev set!

## Word-level vs. Sub-word tokenization



#### The Problem with Word-level Tokenization

- Struggles with out-of-vocabulary (OOV) words.
- Fails to generalize well to unseen or rare terms.
- Requires a larger vocabulary to account for all word variations.

#### The Sub-word Solution

- Breaks words into smaller units, enabling flexible tokenization.
- Guarantees near-complete coverage, even for unseen text.

Example: Unseenword is amazing!

Word-level: [UNK, "is", "amazing", "!"]

Sub-words: ["Un", "seen", "word", "is", "am", "az", "ing", "!"]

# **Recap: Byte Pair Encoding (BPE)**



## Subword tokenization technique

Used for data compression and dealing with unknown words

#### Initialization:

Vocabulary = set of all individual characters

$$V = \{A, B, C, ..., a, b, c, ..., 1, 2, 3, ..., 1, \$, \%, ...\}$$

## Repeat:

- Choose two symbols that appear as a pair most frequently (say "a" and "t")
- Add new merged symbol ("at")
- Replace each occurrence with the new symbol ("t","h","a","t" -> "t","h","at")

Until k merges have been done

# **Recap: Byte Pair Encoding (BPE)**

## see slides IR2 for full example



#### Segments:

#### Vocabulary:

</w>, d, e, I, l, n, o, r, s, t, w

Most frequent symbol pair: er (9 times)

#### Segments:

```
1 o w </w>
2 l o w e s t </w>
6 n e w er </w>
3 w i d er </w>
2 n e w </w>
```

#### **Vocabulary:**

</w>, d, e, I, 1, n, o, r, s, t, w, er

## **WordPiece**



Introduced by Google, used in models like BERT and DistilBERT

#### Similar to BPE:

- Initializes the vocabulary with every character in the training data
- Learns a predefined number of merge rules

#### Difference from BPE:

- Selects symbol pairs that maximize the likelihood of the training data, not the most frequent pair
- Evaluates the impact of merging symbols to avoid reducing overall data probability

## **Example:**

Merges "u" and "g" only if  $\frac{P(ug)}{P(u)*P(g)}$  is greater than for any other pair

## **WordPiece Example**



#### Segments:

- 2 n ##e ##w

Most likely symbol pair: [##s, ##t]:

#### Vocabulary:

l, n, w, ##o, ##w, ##e, ##s, ##t, ##r, ##i, ##d

Note: P(x) != TF(x), but has the same max



 $\frac{TF(\#\#s,\#\#t)}{TF(\#\#s)*TF(\#\#t)} = \frac{2}{2*2} = 0.5$ , which is the max of all pairs

#### Segments:

n ##e ##w

Vocabulary:

l, n, w, ##o, ##w, ##e, ##s, ##t, ##r, ##i, ##d, ##st

## **SentencePiece**



Both BPE and WordPiece assume that words are separated by white spaces Not all languages do that

SentencePiece treats white spaces like any other character

- No need for pre-tokenization
- "\_" (white space) is included in the base vocabulary
- It then uses BPE or similar algorithms to merge the tokens

De-Tokenization does not need any language-specific methods

[Hello] [\_wor] [ld] [.] -> "Hello World."

T. Kudo and J. Richardson (2018). "SentencePiece: A simple and language independent subword tokenizer and detokenizer for Neural Text Processing". In: Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing: System Demonstrations. Brussels, Belgium: Association for Computational Linguistics, pp. 66–71

## **Next Lecture**



# Neural Language Modeling