# NLP and the Web – WS 2024/2025

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# Lecture 2
# Foundations of Text Classification

  

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**Technische Universität Darmstadt**

# Syllabus (tentative)

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| Nr. | Lecture |
|---|---|
| 01 | Introduction / NLP basics |
| **02** | **Foundations of Text Classification** |
| 03 | IR – Introduction, Evaluation |
| 04 | IR – Word Representation, Data Collection |
| 05 | IR – Re-Ranking Methods |
| 06 | IR – Language Domain Shifts, Dense / Sparse Retrieval |
| 07 | LLM – Language Modeling Foundations |
| 08 | LLM – Neural LLM, Tokenization |
| 09 | LLM – Transformers, Self-Attention |
| 10 | LLM – Adaption, LoRa, Prompting |
| 11 | LLM – Alignment, Instruction Tuning |
| 12 | LLM – Long Contexts, RAG |
| 13 | LLM – Scaling, Computation Cost |
| 14 | Review & Preparation for the Exam |

# Last Lecture – Linguistic Analysis Levels

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- Phonetics and Phonology
- Segmentation
- Morphology
- Syntax
- Semantics
- Pragmatics and Discourse

# Today's lecture

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**Text Classification: Introduction**

**Algorithms**

- **Naive Bayes**
- **Hidden Markov Models**

# What is Text Classification?

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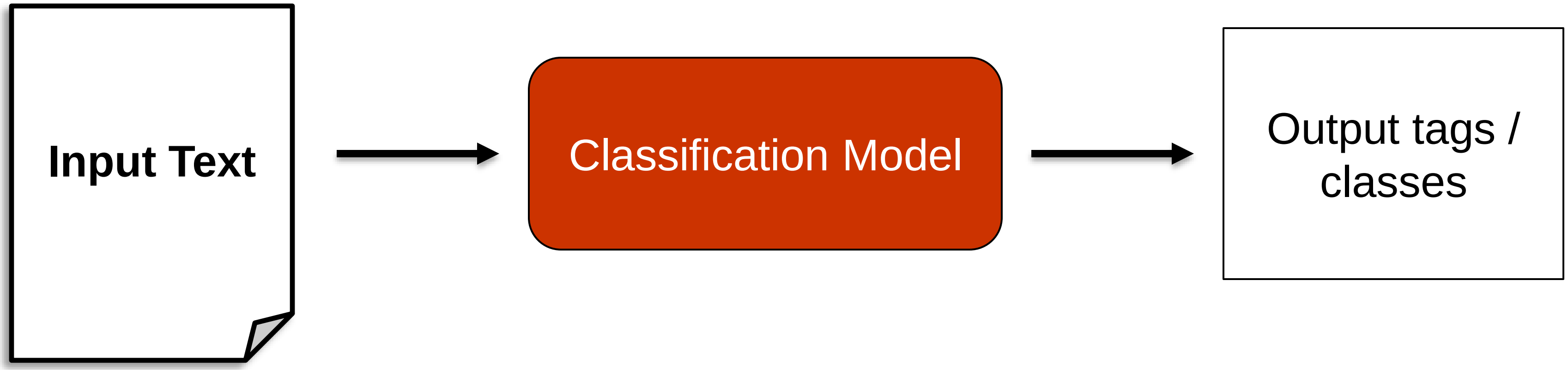

# Examples – Spam Detection

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From: Danke (**archdigest@news.condenast.com)**

Sehr geehrter kunde,

Seit Sie unsere Dienste nutzen, haben Sie **40,259** Treuepunkte gesammelt. Sie erhalten ein Handy als Geschenk für Ihre Treue.

Der Versand erfolgt nach Bestätigung Ihrer Anschrift und Bezahlung der Versandkosten

Dein Geschenk: **Apple iPhone X (64GB) - Space Grey**

* Indem Sie Ihren Gewinn akzeptieren, stimmen Sie zu, dass Ihr Konto mit 35.154 Punkten belastet wird. Ohne Ihre vorherige Zustimmung wird kein Abonnement abonniert.

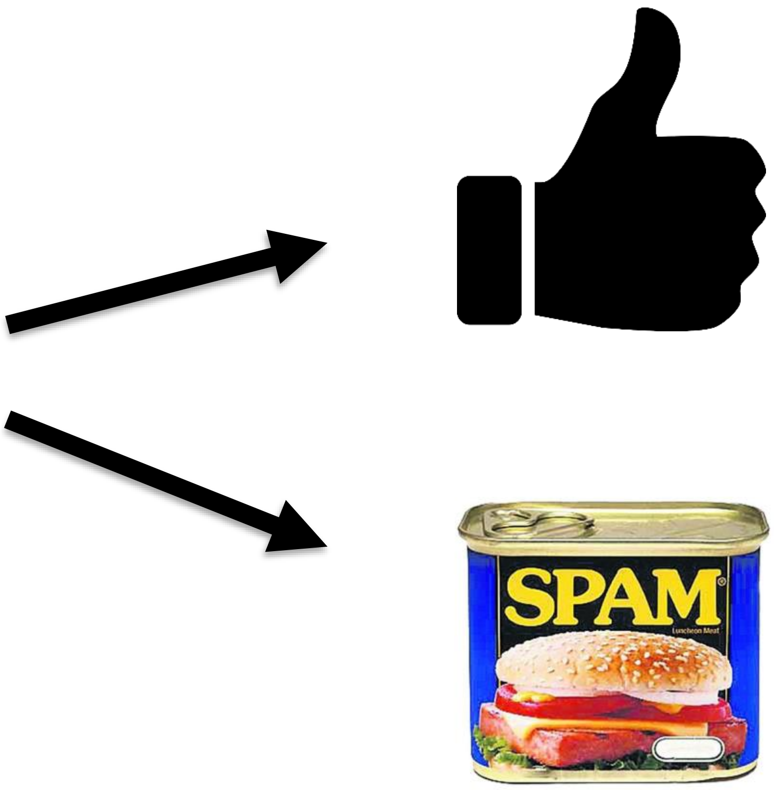
Ihre Treuepunkte verfallen bald. Fordern Sie Ihre Prämien vor Ablauf des Angebots an.

Klicken Sie auf die Schaltfläche unten, um zu bestätigen und Ihr Handy zu erhalten.

KLICK HIER

Grüße,

Julien I Produktmanager



# Examples – Sentiment Analysis

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Positive or negative movie review?

- This is it. This is the one. This is the worst movie ever made. Ever. It beats everything. I have never seen worse.
- Expertly scripted and perfectly delivered, this searing parody leaves you literally rolling with laughter.
- While watching this film I started to come up with things I would rather be doing, including drinking bleach, rubbing sand in my eyes, and tax returns.
- Just finished watching this movie for maybe the 7th or 8th time

# More Examples

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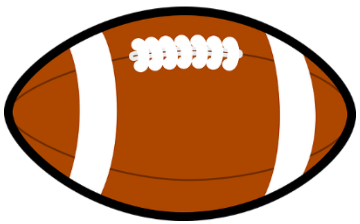
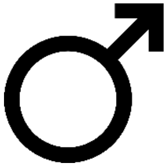
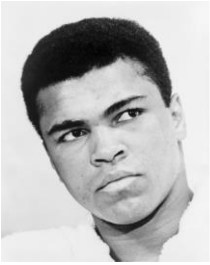
Topic Labeling



Age/Gender Identification



**I've seen George Foreman shadow boxing and the shadow won.**



Authorship Identification

Language Identification

# Approaches
# Rule-based

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Handcrafted linguistic rules

Human comprehensible

Pro: Precision can be high

Con: Very expensive to build and maintain

```
IF "basketball" THEN
            return top_sports
ELSEIF…
```

# Approaches
# Supervised Machine Learning

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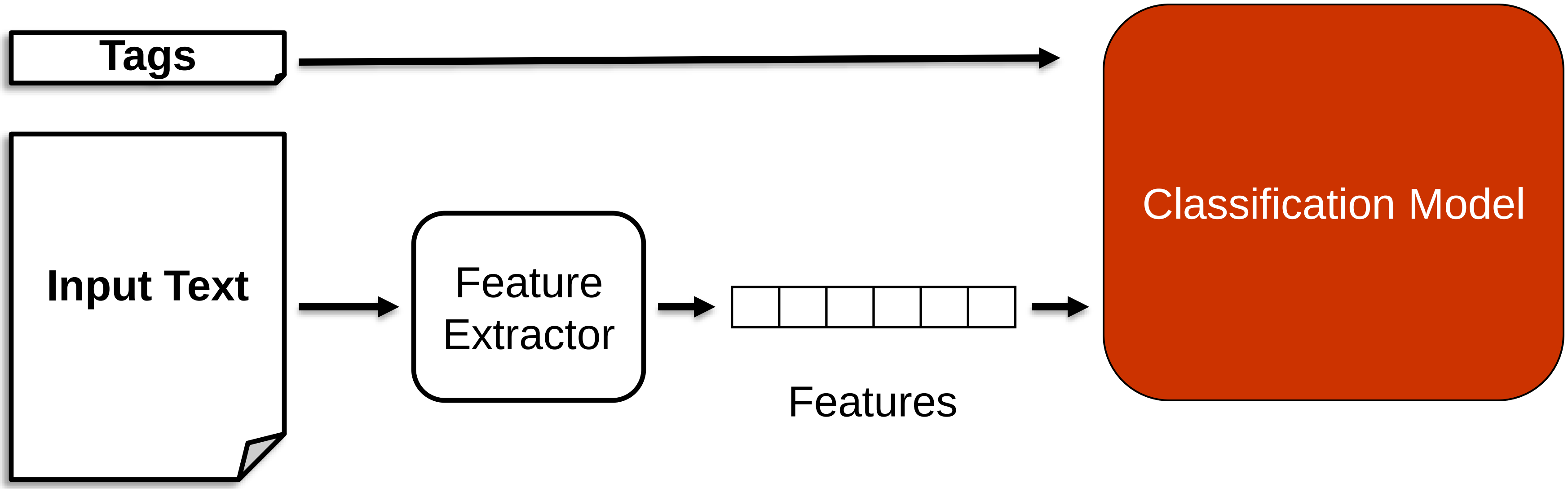
## Step 1: Training



# Approaches
# Supervised Machine Learning

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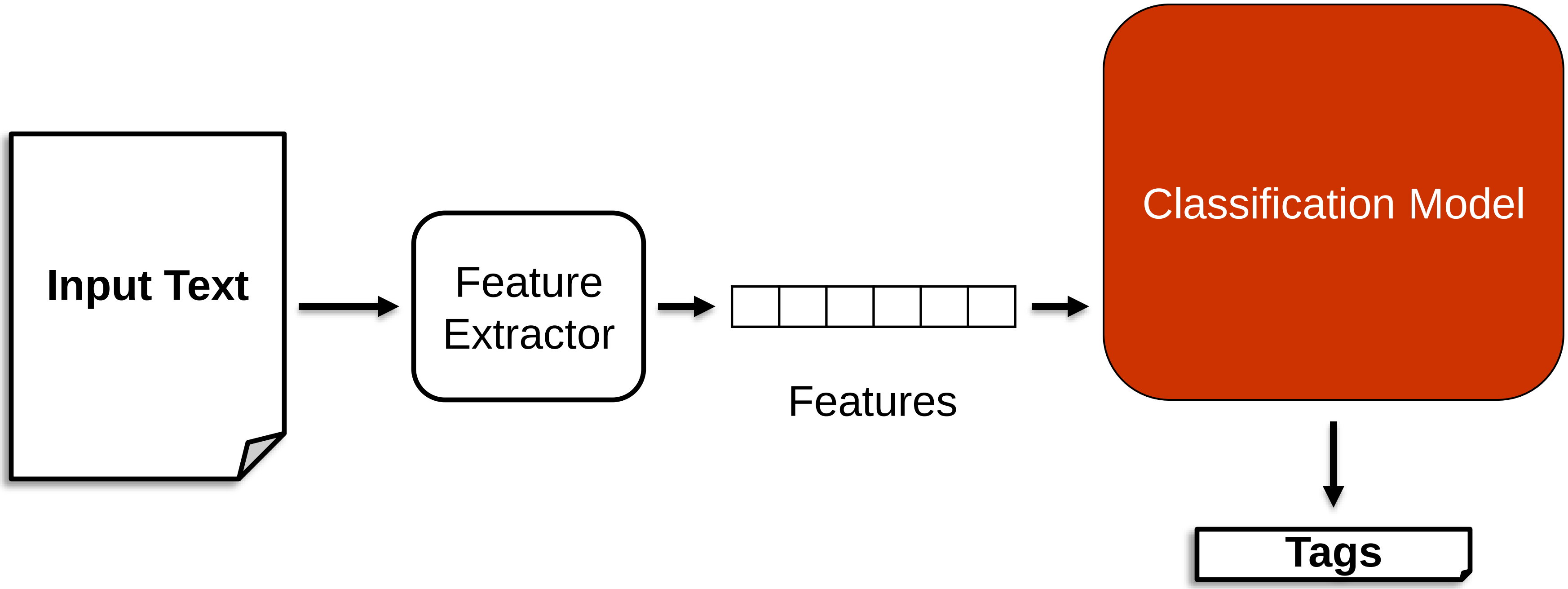
## Step 2: Prediction



# Approaches
# Supervised Machine Learning

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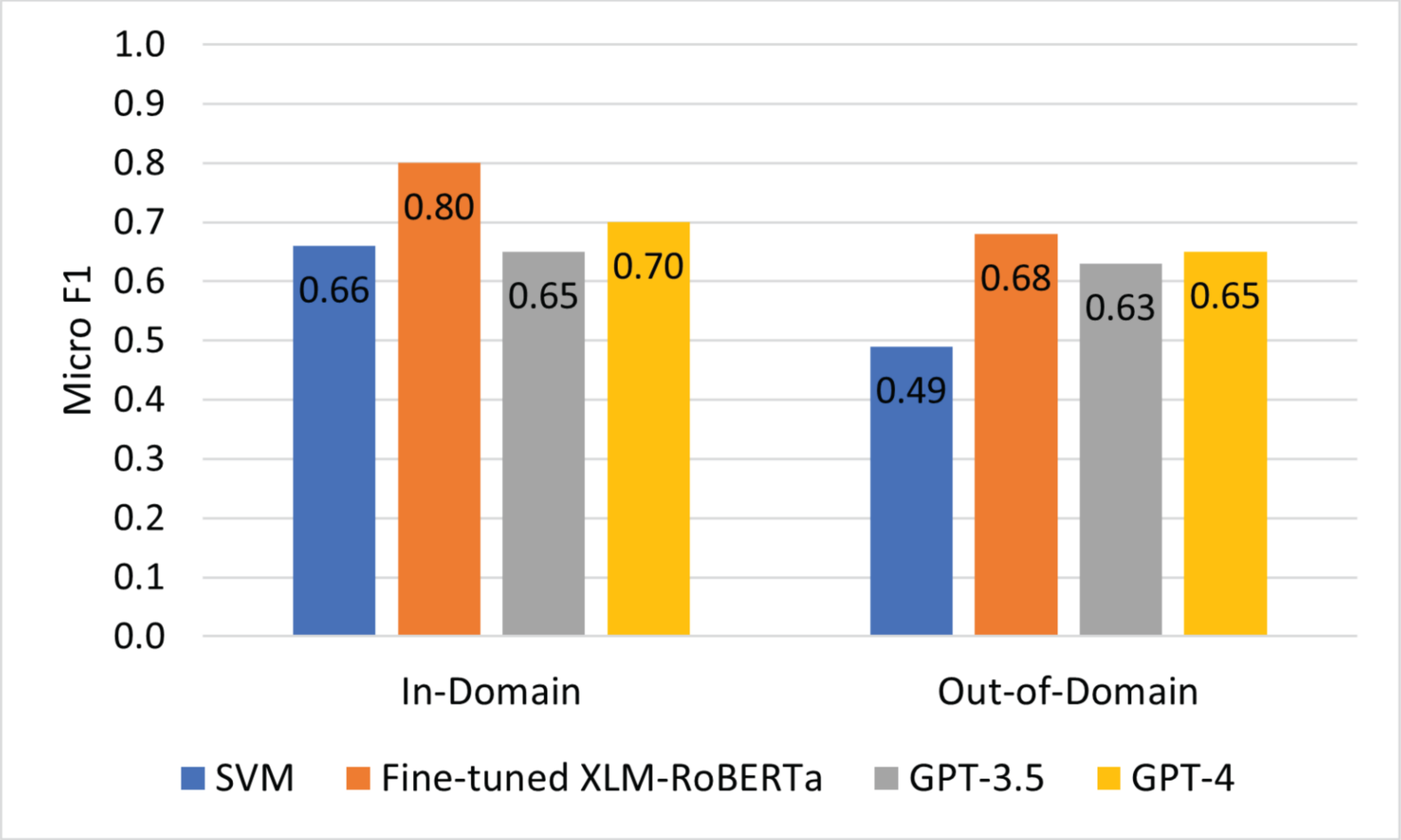
Classification model based on Training Data

Human comprehensible? Dependant on model!

Pro: Easier to maintain, usually more accurate

Con: Needs training data

# Comparison to Neural Models


Kuzman T, Mozetič I, Ljubešić N. Automatic Genre Identification for Robust Enrichment of Massive Text Collections: Investigation of Classification Methods in the Era of Large Language Models. Machine Learning and Knowledge Extraction. 2023; 5(3):1149-1175

# Today's lecture

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**Text Classification: Introduction**

**Algorithms**

- **Naive Bayes**
- **Hidden Markov Models**

# Naïve Bayes Classifier: Background

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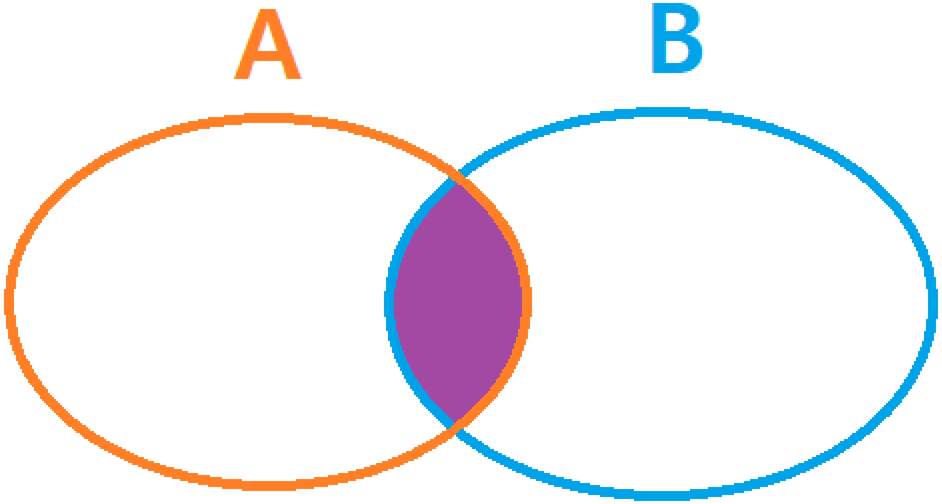
- Before starting let's review related concepts:
  - Conditional Probability
  - Bayes' Rule.



# Conditional Probability

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- **Conditional Probability** in plain English:
  - What is the probability that something will happen, *given that something else* has already happened.
- Assume we have some **Outcome** O and some **Evidence** E.
  - $P(O, E)$: the Probability of having *both* the Outcome O and Evidence E is the multiplication of two probabilities:
    - $P(O)$: Probability of O occurring
    - $P(E|O)$: Probability of E given that O happened

$$P(O, E) = P(O) \times P(E|O)$$

# Conditional Probability: Example

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- Let say we have a group of students
  - Students could be either awake or asleep
  - Students are also either bachelor or master students
- If we select one student randomly: *what is the probability that this person is a sleeping bachelor student?*
- Conditional Probability can help us answer that:
  - $P(Asleep\,, Bachelor) = P(Asleep) * P(Bachelor|Asleep)$
- We could compute the exact same thing, the reverse way
  - $P(Asleep\,, Bachelor) = P(Bachelor) * P(Asleep\,|Bachelor)$

# Bayes Rule

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- Conceptually, this is a way to go from $P(Evidence \mid Outcome)$ to $P(Outcome \mid Evidence)$
- Let's see how to do that
  - We have:
    - $P(O,E) = P(O) \times P(E|O)$ and $P(O,E) = P(E) \times P(O|E)$
    - $\rightarrow P(O) \times P(E|O) = P(E) \times P(O|E)$

$$P(O|E) = \frac{P(E|O) \times P(O)}{P(E)}$$

Bayes Rule

# Getting to Naïve Bayes'

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- So far, we have talked only about one piece of evidence.
- In reality, we have to predict an outcome given **multiple evidence**
  - math gets very complicated :(
- Naive Bayes‘ solution:
  - treat each of piece of evidence as independent -> simpler math :)
  - This approach is why this is called ***naive*** Bayes
- Suppose we have multiple evidences $E_1, \dots, E_n$ and an outcome $O$

$$P(O|E_1, \dots, E_n) = \frac{P(E_1|O) \times P(E_2|O) \times \dots \times P(E_n|O) \times P(O)}{P(E_1, E_2, \dots, E_n)}$$

# Getting to Naïve Bayes'

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$$P(O|E_1, \ldots, E_n) = \frac{P(E_1|O) \times \ P(E_2|O) \times \ \ldots \times \ P(E_n|O) \times P(O)}{P(E_1, E_2, \ldots, E_n)}$$

Many people choose to remember this as:

$$P(outcome|evidence) = \frac{P(Likelihood\ of\ Evidence) \ * \ Prior\ prob\ of\ outcome}{P(Evidence)}$$

- Notes:
- If the $P(evidence|outcome)$ is 1, then we are just multiplying by 1.
- If the $P(some\ particular\ evidence|outcome)$ is 0, then the whole probability becomes 0
- Since we divide everything by $P(Evidence)$,
  - we can even get away without calculating it.
- The intuition behind multiplying by the *prior* is
  - to give high probability to more common outcomes, and low probabilities to unlikely outcomes.
  - These are also called **base rates** and they are a way to scale our predicted probabilities

# Naïve Bayes: Example

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- Let's say that we have data on 1000 pieces of fruit: **Banana**, **Orange** or some **Other Fruit**
- We know 3 characteristics about each fruit
  - Whether it is **Long**
  - Whether it is **Sweet** and
  - If its color is **Yellow**
- Our training set

| Type | Long | Not Long | Sweet | Not Sweet | Yellow | Not Yellow | Total |
|---|---|---|---|---|---|---|---|
| Banana | 400 | 100 | 350 | 150 | 450 | 50 | 500 |
| Orange | 0 | 300 | 150 | 150 | 300 | 0 | 300 |
| Other Fruit | 100 | 100 | 150 | 50 | 50 | 150 | 200 |
| Total | 500 | 500 | 650 | 350 | 800 | 200 | 1000 |

# Naïve Bayes: Example cont.

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| Type | Long | Not Long | Sweet | Not Sweet | Yellow | Not Yellow | Total |
|---|---|---|---|---|---|---|---|
| Banana | 400 | 100 | 350 | 150 | 450 | 50 | 500 |
| Orange | 0 | 300 | 150 | 150 | 300 | 0 | 300 |
| Other Fruit | 100 | 100 | 150 | 50 | 50 | 150 | 200 |
| Total | 500 | 500 | 650 | 350 | 800 | 200 | 1000 |

- “Prior” probabilities
  - P(Banana) = 500/1000 = 0.5, P(Orange) = 0.3, P(Other Fruit) = 0.2
- Probability of "Evidence“
  - p(Long) = 500/1000 = 0.5, P(Sweet) = 0.65, P(Yellow) = 0.8
- Probability of "Likelihood“
  - P(Long|Banana) = 0.8, P(Long|Orange) = 0
  - P(Yellow|Other Fruit) = 50/200 = 0.25, P(Not Yellow|Other Fruit) = 0.75

# Naïve Bayes: Example cont.

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- "Prior" probabilities
  - P(Banana) = 0.5 (500/1000), P(Orange) = 0.3, P(Other Fruit) = 0.2
- Probability of "Evidence"
  - p(Long) = 500/100 = 0.5, P(Sweet) = 0.65, P(Yellow) = 0.8
- Probability of "Likelihood"
  - P(Long|Banana) = 0.8, P(Long|Orange) = 0
  - P(Yellow|Other Fruit) = 50/200 = 0.25, P(Not Yellow|Other Fruit) = 0.75

Given an unknown fruit which is long, sweet and yellow, is it Banana, Orange or Other Fruit?

- $\mathrm{P(Banana|Long, Sweet\ and\ Yellow)} = \frac{\mathrm{P(Long|Banana)} * \mathrm{P(Sweet|Banana)} * \mathrm{P(Yellow|Banana)} * \mathrm{P(Banana)}}{\mathrm{P(Long)} * \mathrm{P(Sweet)} * \mathrm{P(Yellow)}}$
  $= 0.8 * 0.7 * 0.9 * 0.5 / \mathrm{P(evidence)}$= 0.252 / P(evidence)
- $\mathrm{P(Orange|Long, Sweet\ and\ Yellow)} = 0$ , why?
- $\mathrm{P(Other\ Fruit|Long, Sweet\ and\ Yellow)} = \frac{\mathrm{P(Long|Other\ Fruit)} * \mathrm{P(Sweet|Other\ Fruit)} * \mathrm{P(Yellow|Other\ Fruit)} * \mathrm{P(Other\ Fruit)}}{\mathrm{P(Long)} * \mathrm{P(Sweet)} * \mathrm{P(Yellow)}}$
  $= (100/200 * 150/200 * 50/150 * 200/1000) \,/\, P(evidence) = 0.01875 \,/\, P(evidence)$

0.252 >> 0.01875 => the unknown fruit is most likely a banana

Is it clear now why we don‘t need to calculated the P(evidence)?

# Naïve Bayes for Text Classification based on Words as Features

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- Multinomial Naïve Bayes Model: The probability of document *d* belonging to a class *c* is proportional to the product of the probabilities of terms *t* belonging to a class *c*, and to the class prior *P(c)*:

$$P(c|d) \propto P(c) \prod_{1 \leq k \leq n_d} P(t_k|c)$$

- The best class *c* for a document *d* is found by selecting the class, for which the maximum a posteriori (map) probability is maximal:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} \hat{P}(c|d) = \arg\max_{c \in \mathbb{C}} \hat{P}(c) \prod_{1 \leq k \leq n_d} \hat{P}(t_k|c).$$

# Summary on Naïve Bayes

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- Bayesian methods provide the basis for probabilistic learning methods
- Bayesian methods can be used to determine the most probable hypothesis given the data
- No training, just probability calculation
- Binary, numeric and nominal features can be mixed
- Naïve Bayes fails if the independence assumption is violated too much
  - Especially identical or highly overlapping features pose a problem that has to be addressed with proper feature selection

# I didn't expect a kind of menti quiz.

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# Today's lecture

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**Text Classification: Introduction**

**Algorithms**

- **Naive Bayes**
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# Limitations of Standard classification

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- Standard classification problem assumes
  - individual cases are disconnected and independent
- Many NLP problems do not satisfy this assumption
  - involve making many connected decisions, each resolving a different ambiguity, but which are mutually dependent
- More sophisticated learning and inference techniques are needed

# Sequence Labeling

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- Many NLP problems can viewed as sequence labeling
- Each token in a sequence is assigned a label
- Labels of tokens are dependent on the labels of other tokens in the sequence, particularly their neighbors
- Examples:
  - Part Of Speech Tagging

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| In: | John | saw | the | saw | and | decided | to | take | it | to | the | table. |
| Out: | PN | V | Det | N | Con | V | Part | V | Pro | Prep | Det | N |

  - Named entity recognition: people organizations places

Michael Dell is the CEO of Dell Computer Corporation and lives in Austin Texas.

# Use case: Part of Speech (POS) Tagging

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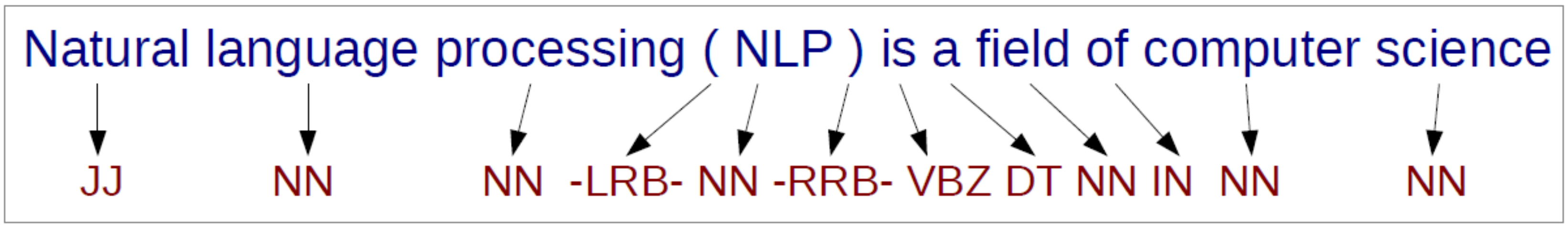
- Given a sentence X, predict its part of speech sequence Y




- A type of “structured” prediction
- How can we do this?

# Use case: Part of Speech (POS) Tagging

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**Possible Answers**

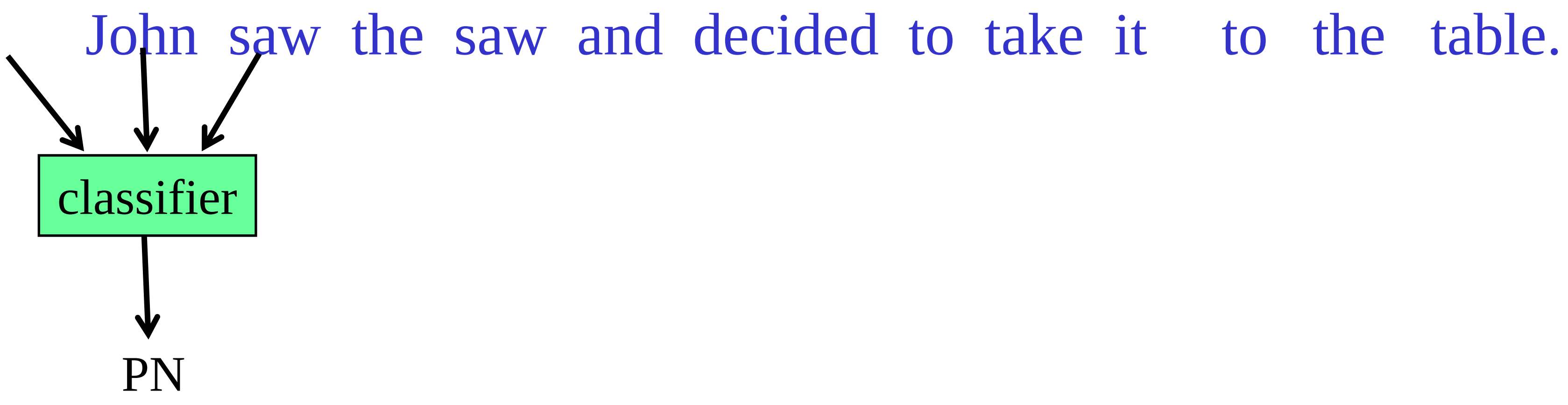
- Sequence labeling as classification:
  - Pointwise prediction: predict each word individually with a classifier
- Generative sequence models: e.g. Hidden Markov Model (HMM)

- Later in the lecture: Neural Sequence Models (RNN / LSTM)

# Sequence Labeling as Classification



- Classify each token independently but use as input features, information about the surrounding tokens (sliding window).




Slides from: Raymond J. Mooney

# Sequence Labeling as Classification

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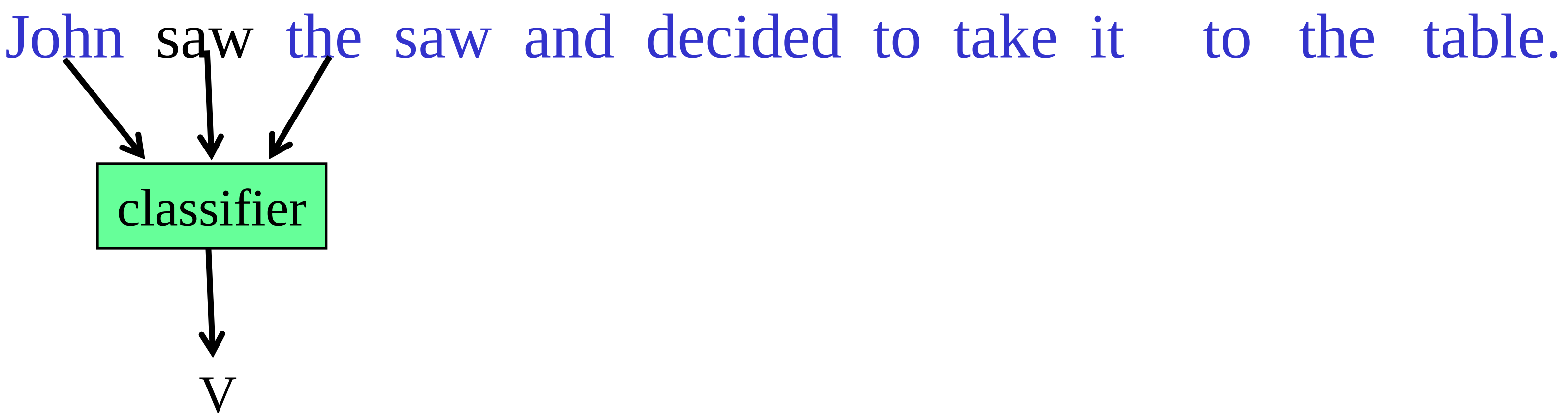

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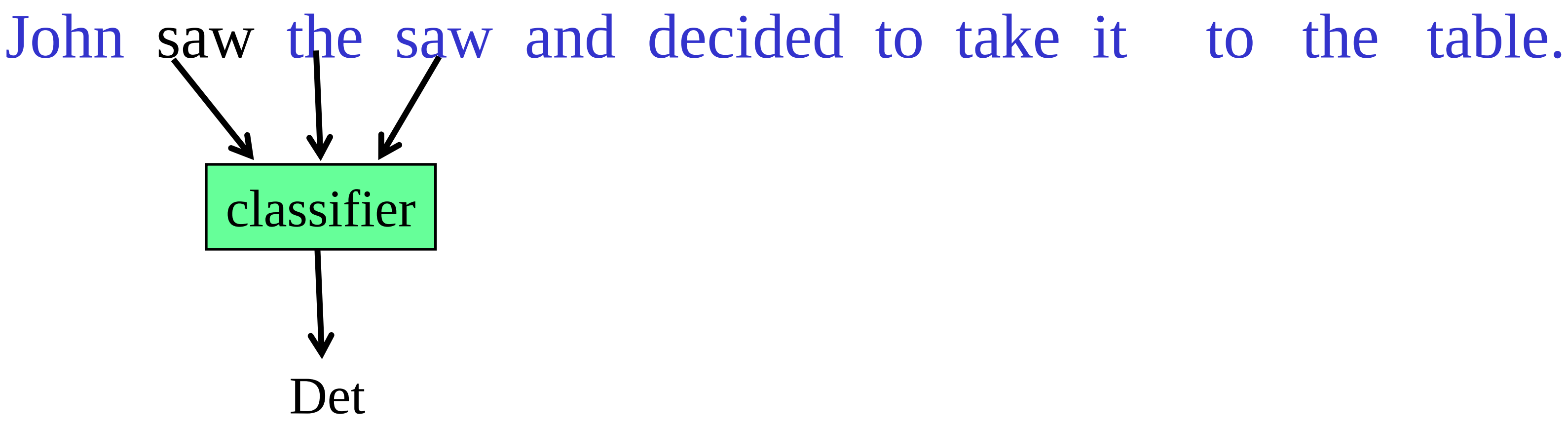

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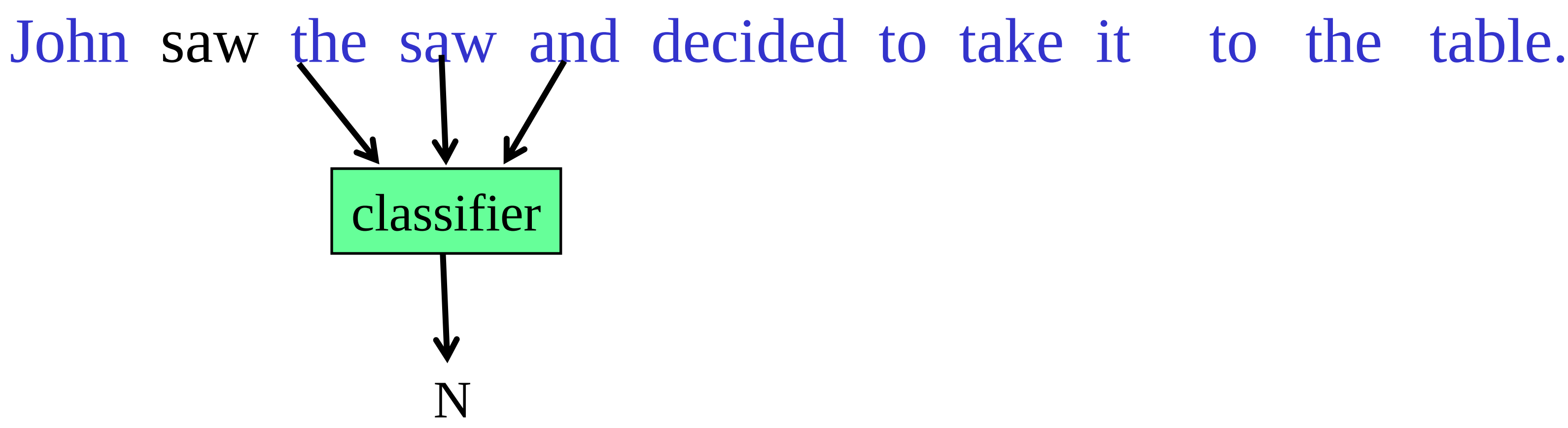

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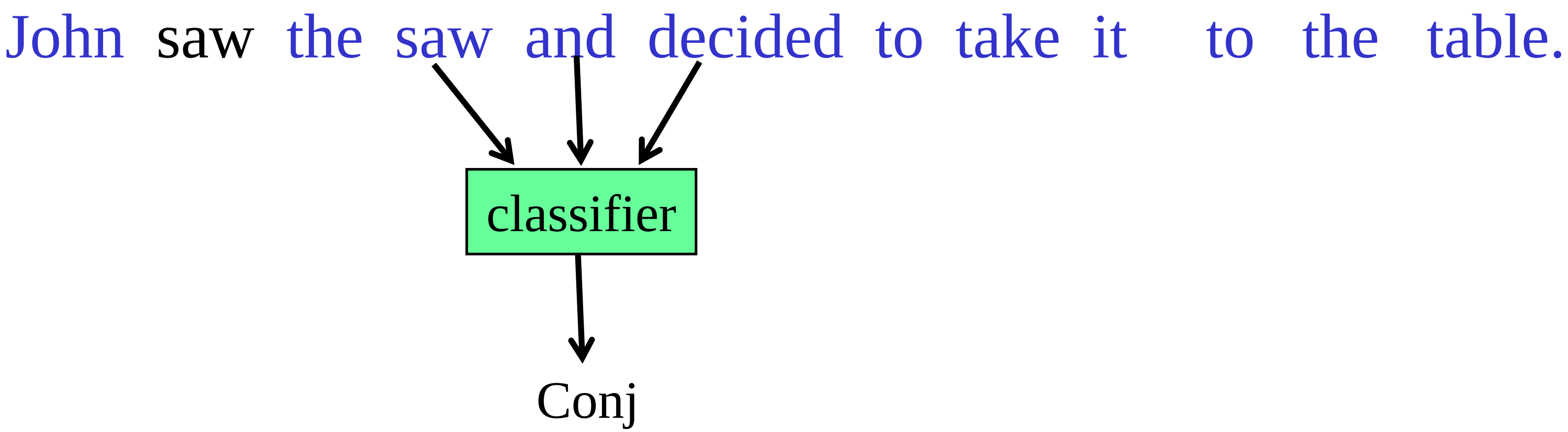

## Sequence Labeling as Classification

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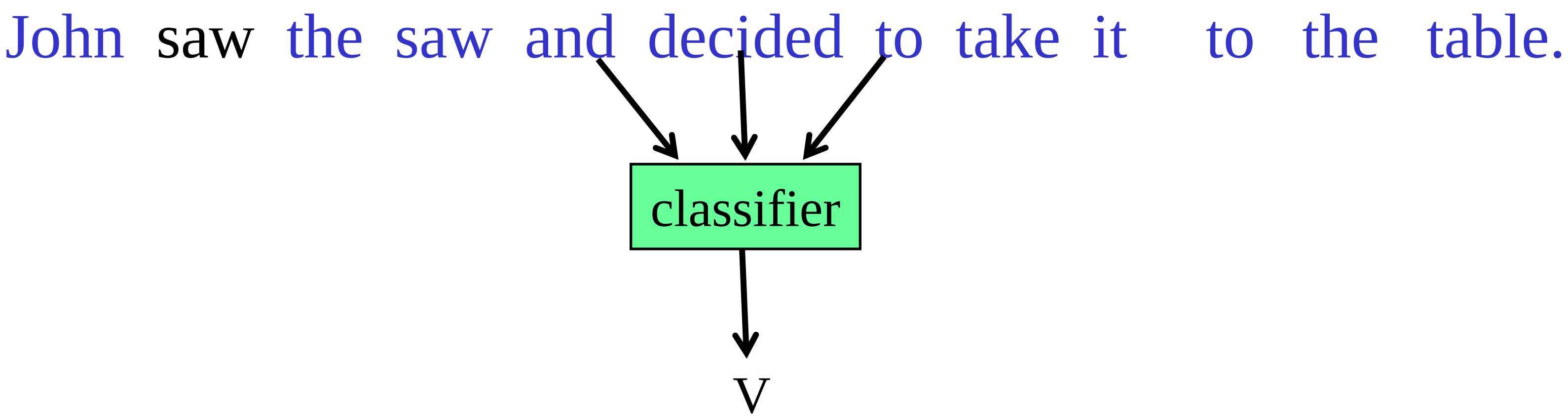

## Sequence Labeling as Classification

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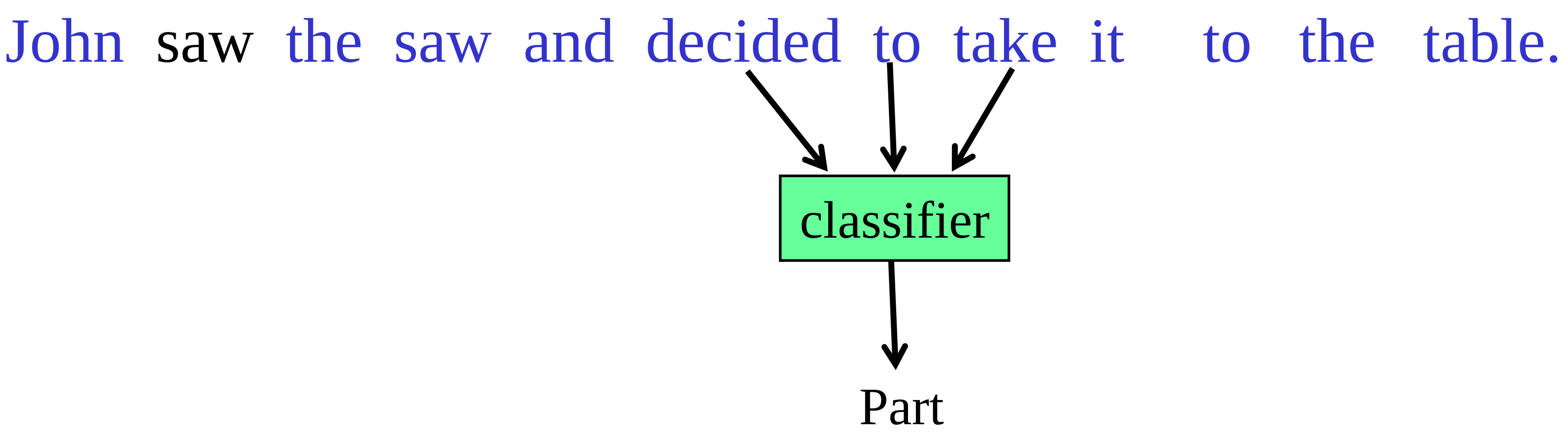

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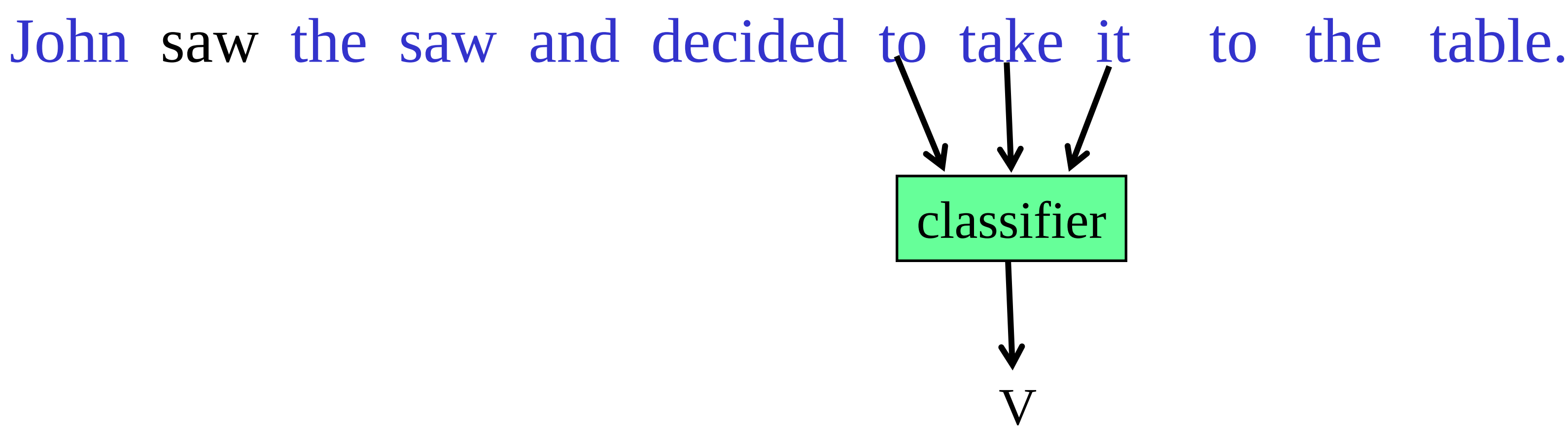

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# Sequence Labeling as Classification

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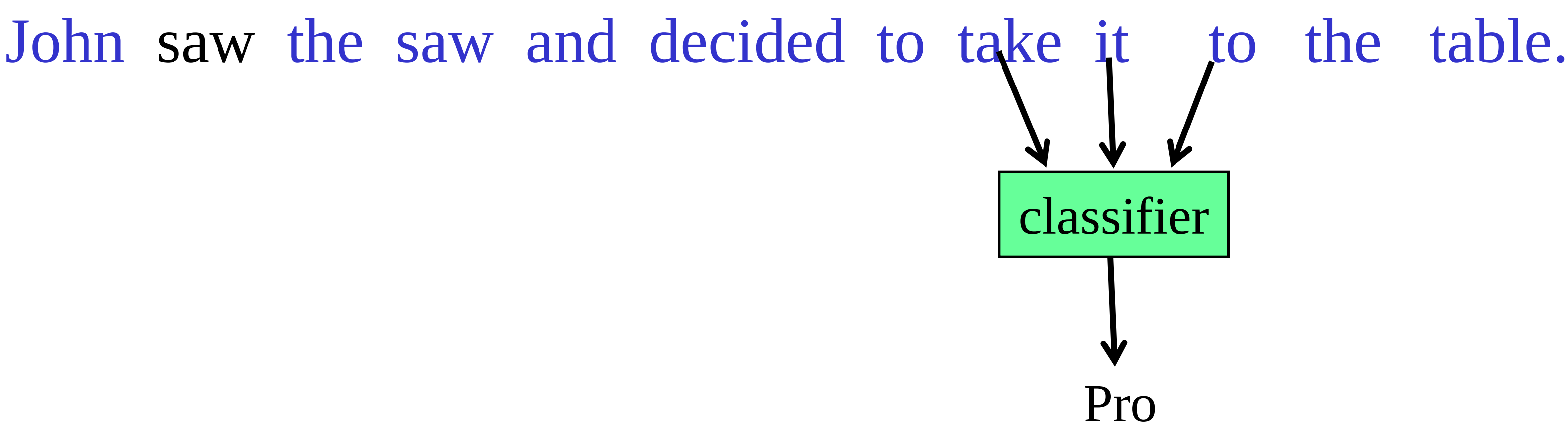

# Sequence Labeling as Classification



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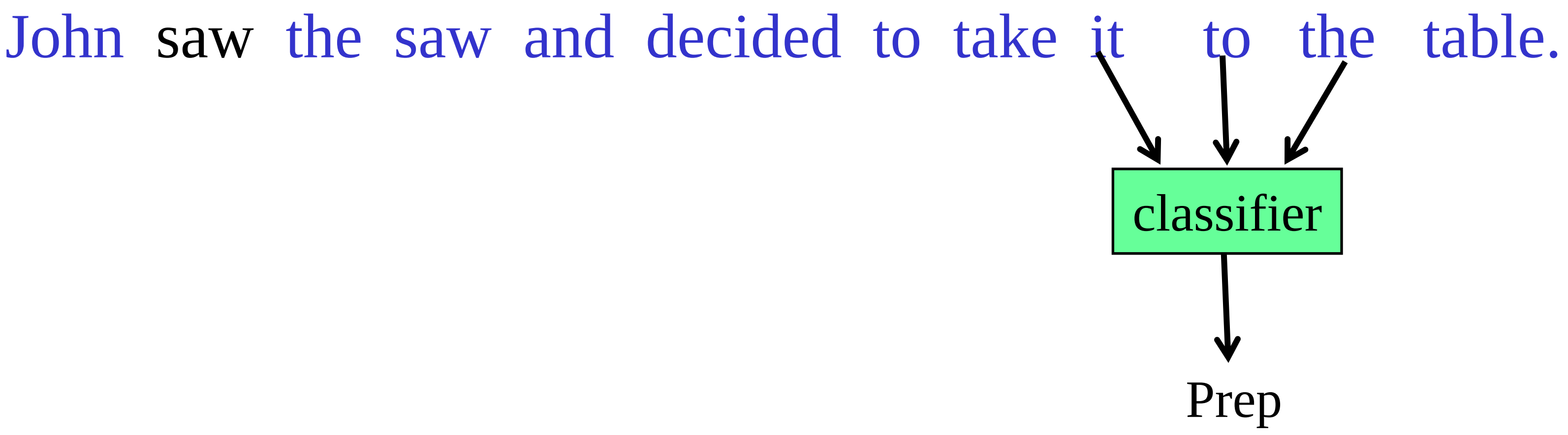

# Sequence Labeling as Classification

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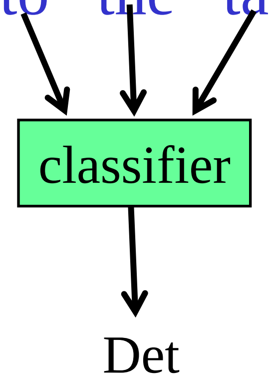
- Classify each token independently but use as input features, information about the surrounding tokens (sliding window).

John saw the saw and decided to take it to the table.



## Sequence Labeling as Classification

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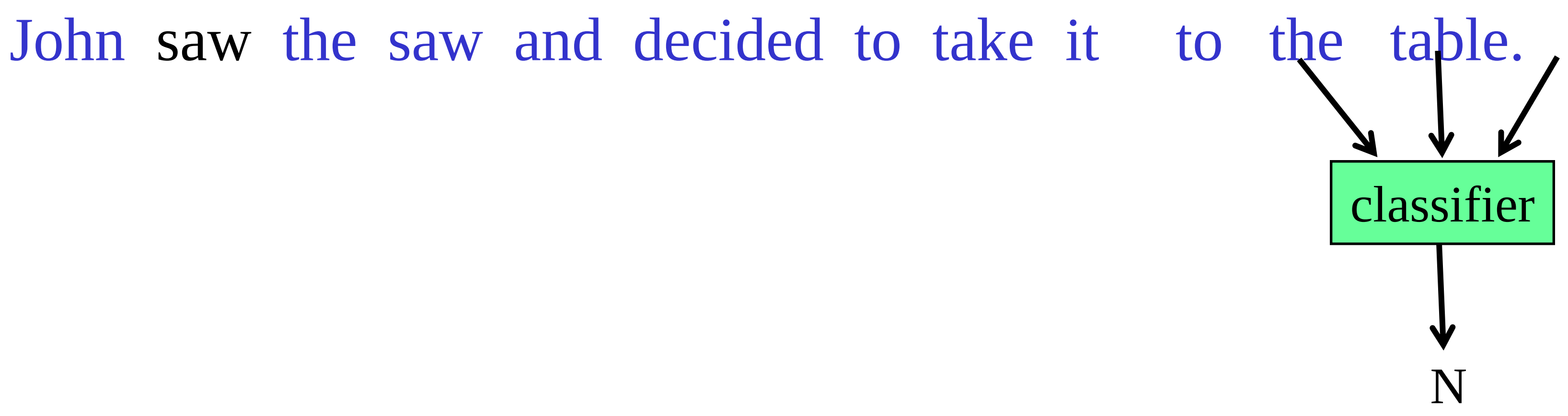

# Sequence Labeling as Classification

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**Using Outputs as Inputs**

- Better input features are usually the categories of the surrounding tokens, but these are not available yet.
- Can use category of either the preceding or succeeding tokens by going forward or back and using previous output.

# Forward Classification

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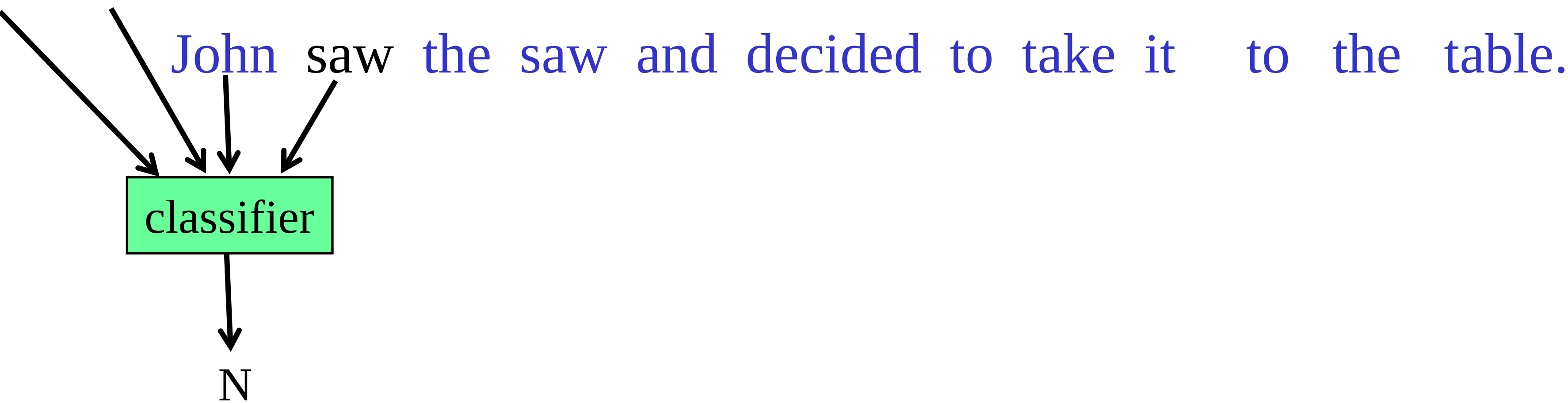

# Forward Classification

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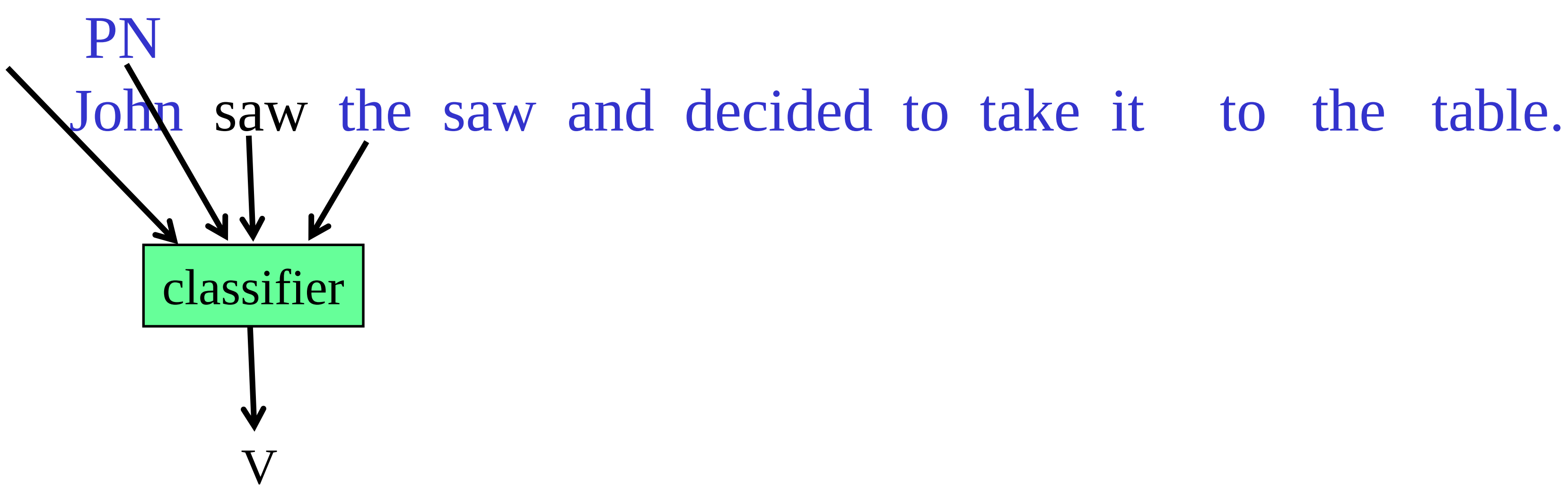

# Forward Classification

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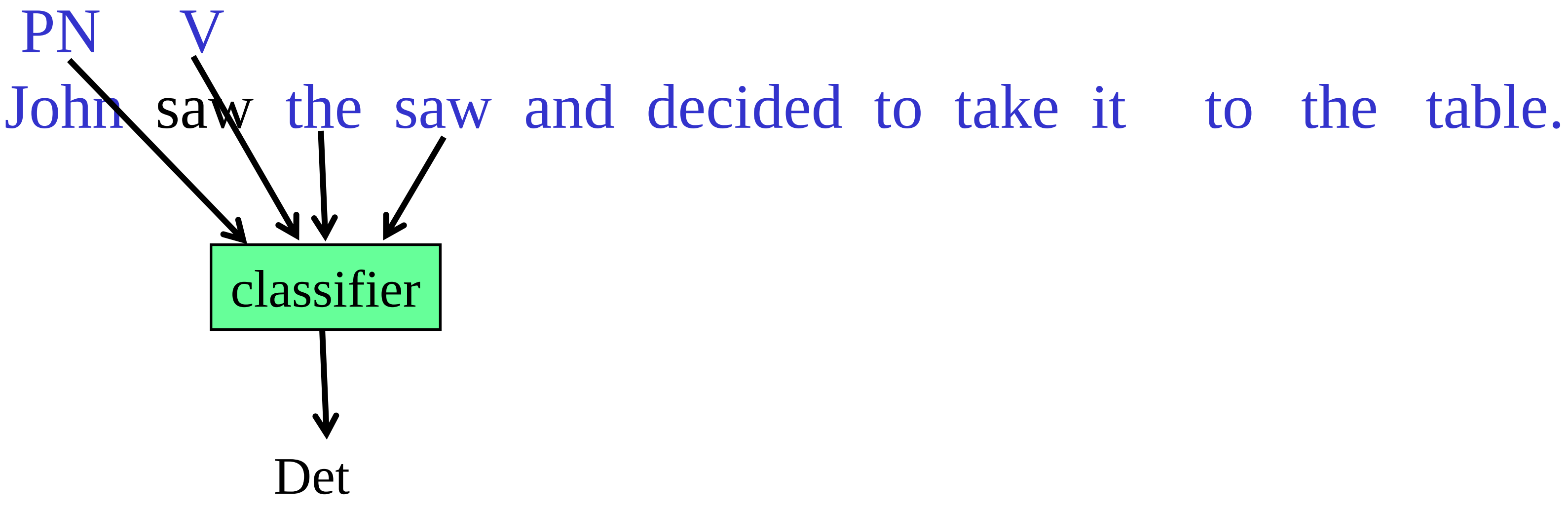

# Forward Classification

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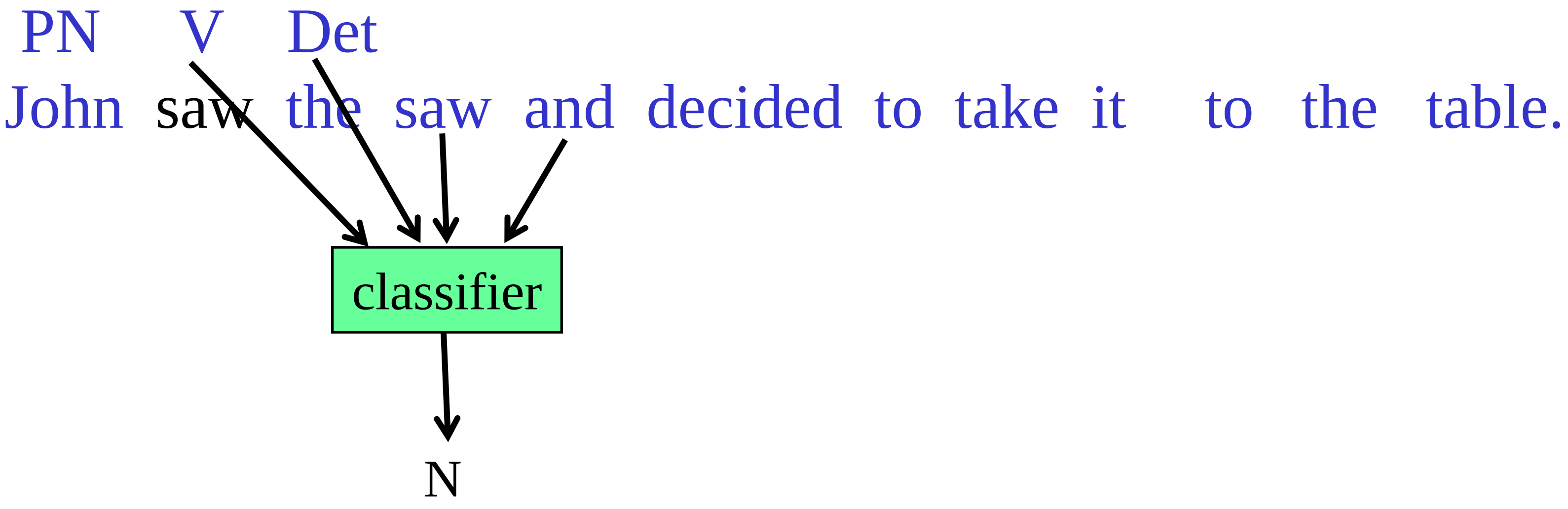

# Forward Classification

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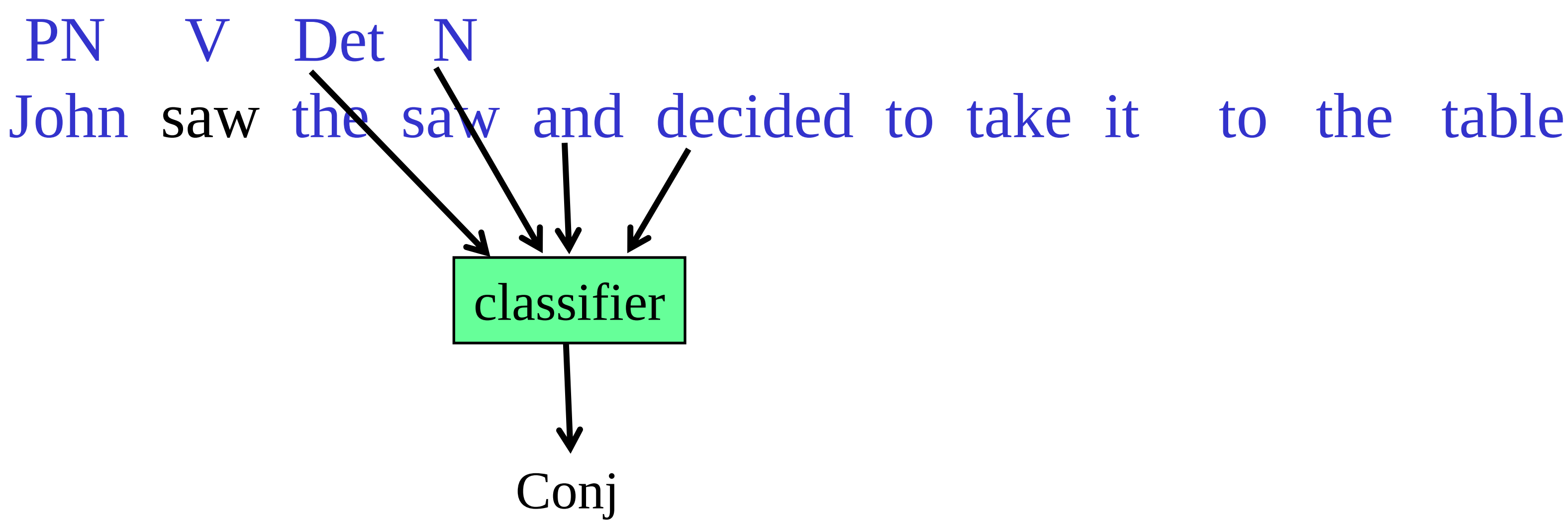

# Forward Classification

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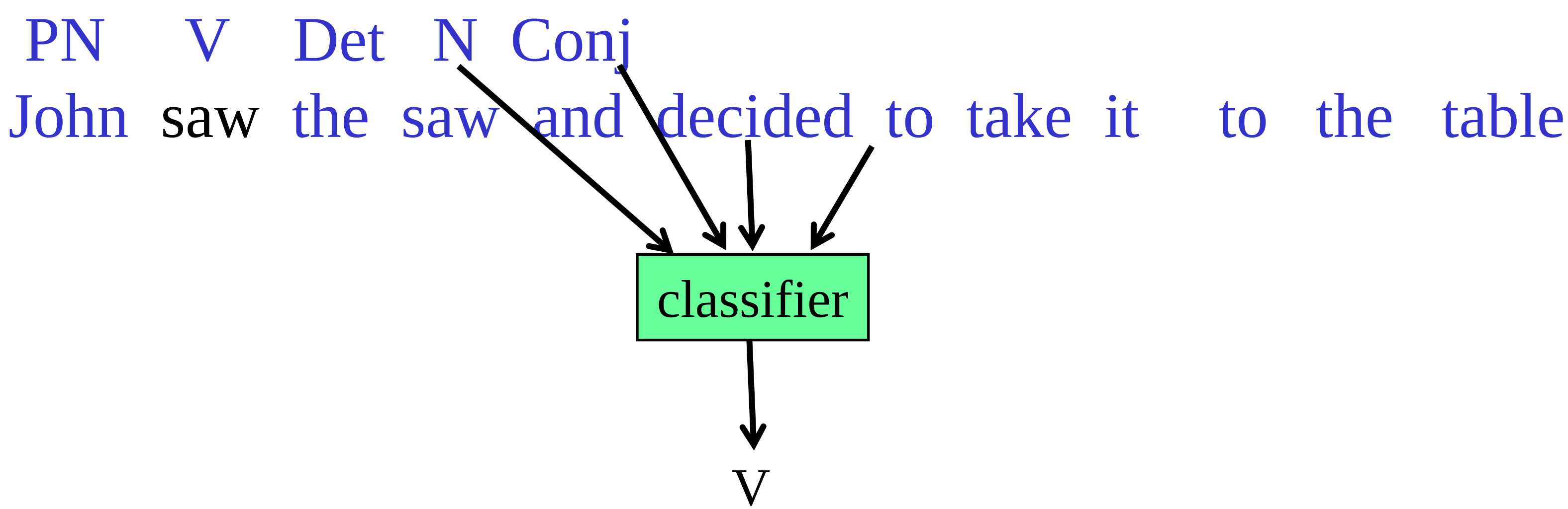
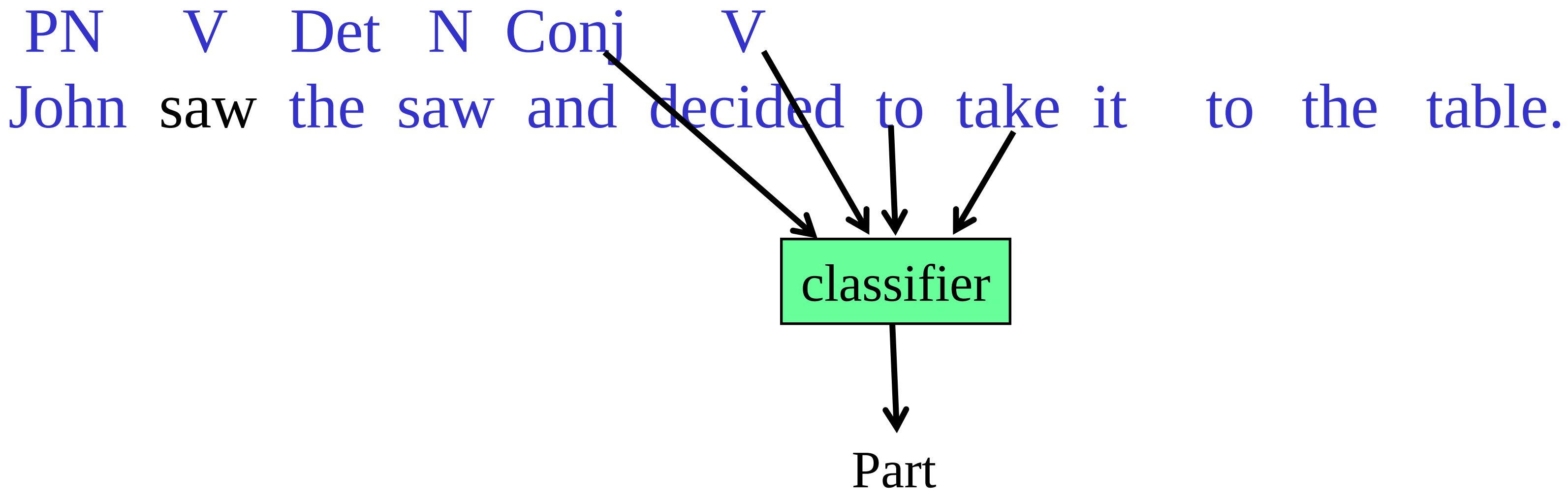


## Forward Classification





# Forward Classification

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PN V Det N Conj V Part

John saw the saw and decided to take it to the table.

classifier

V

# Forward Classification

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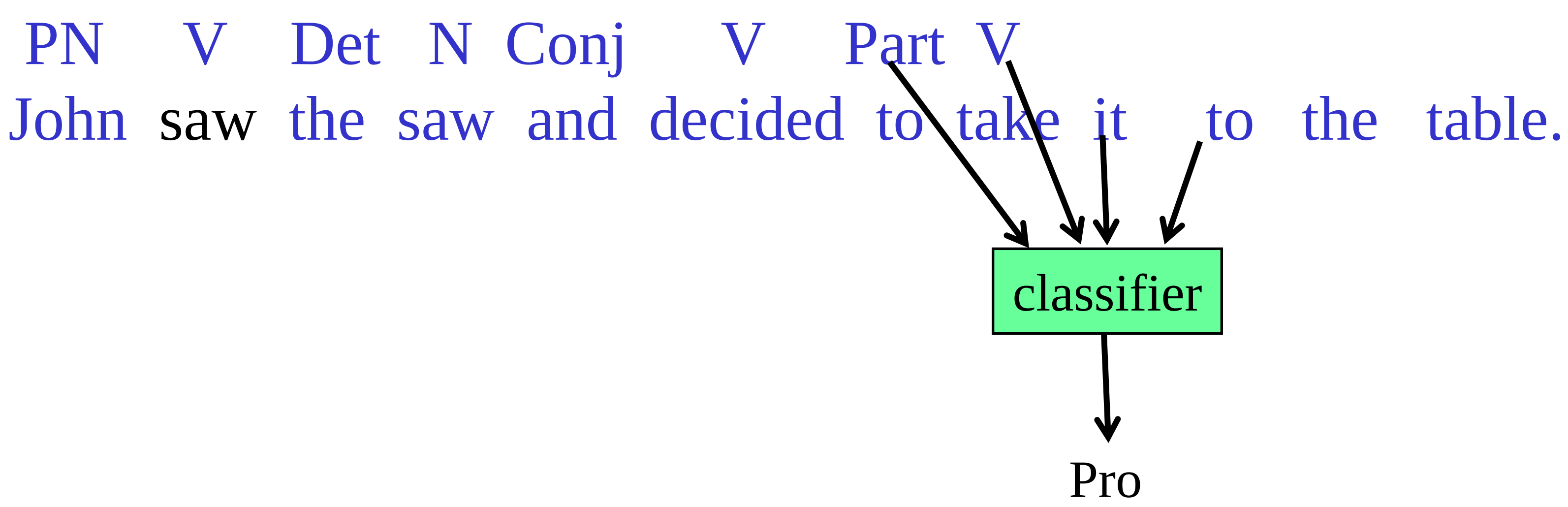

# Forward Classification

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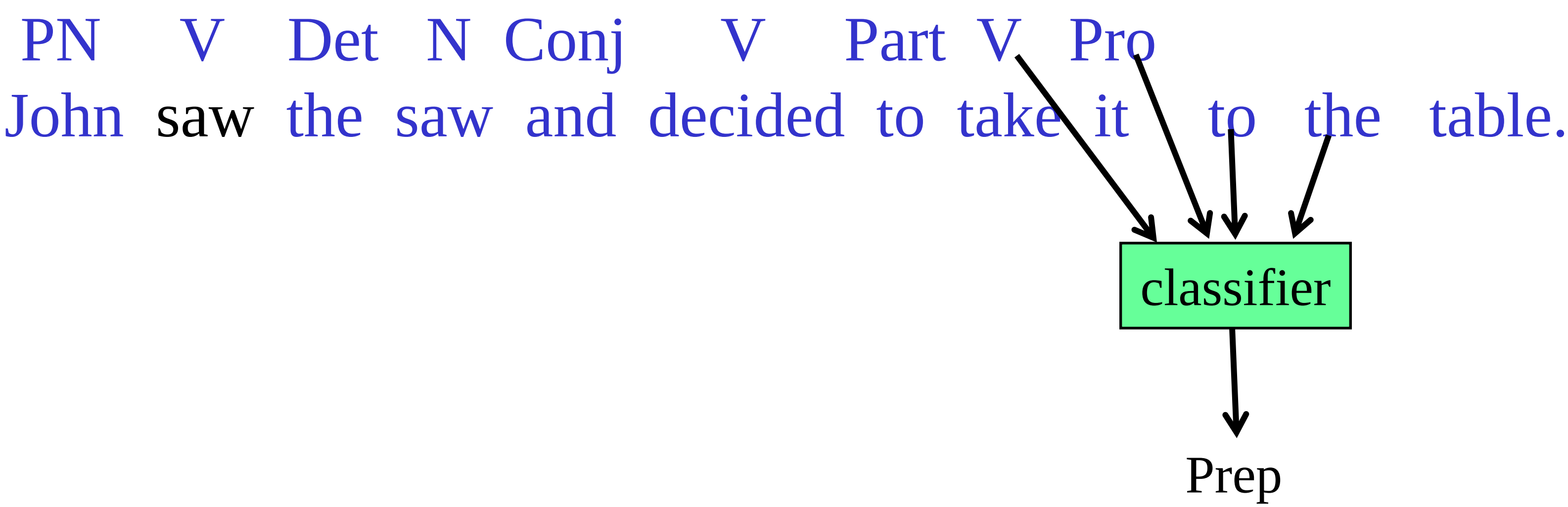
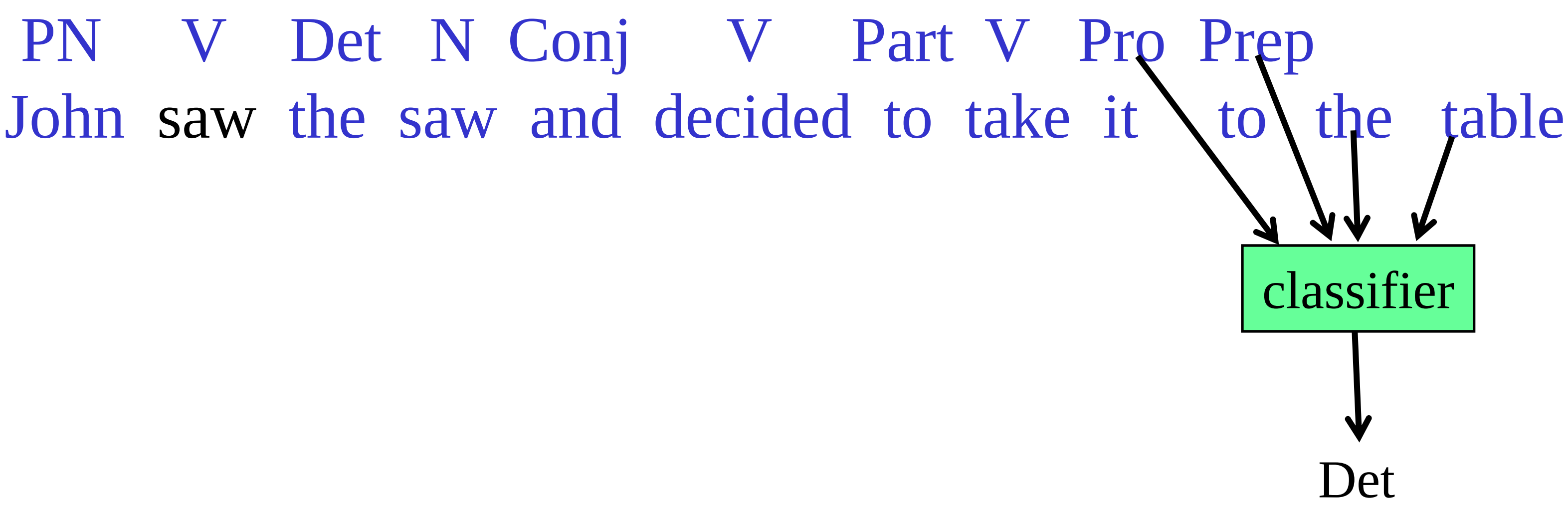


# Forward Classification





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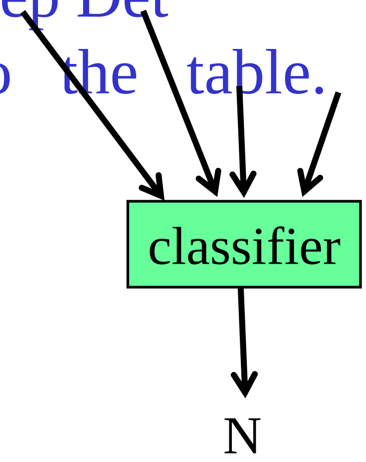
# Forward Classification

PN V Det N Conj V Part V Pro Prep Det

John saw the saw and decided to take it to the table.



# Backward Classification

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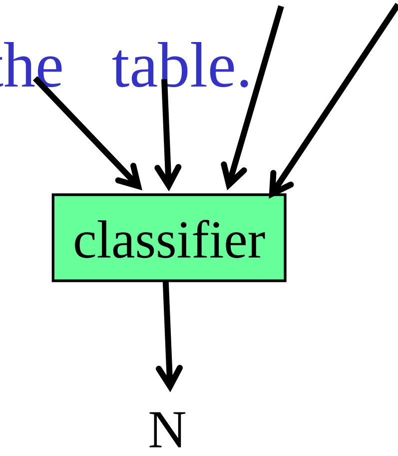
- Disambiguating “to” in this case would be even easier backward.

John saw the saw and decided to take it to the table.



# Backward Classification

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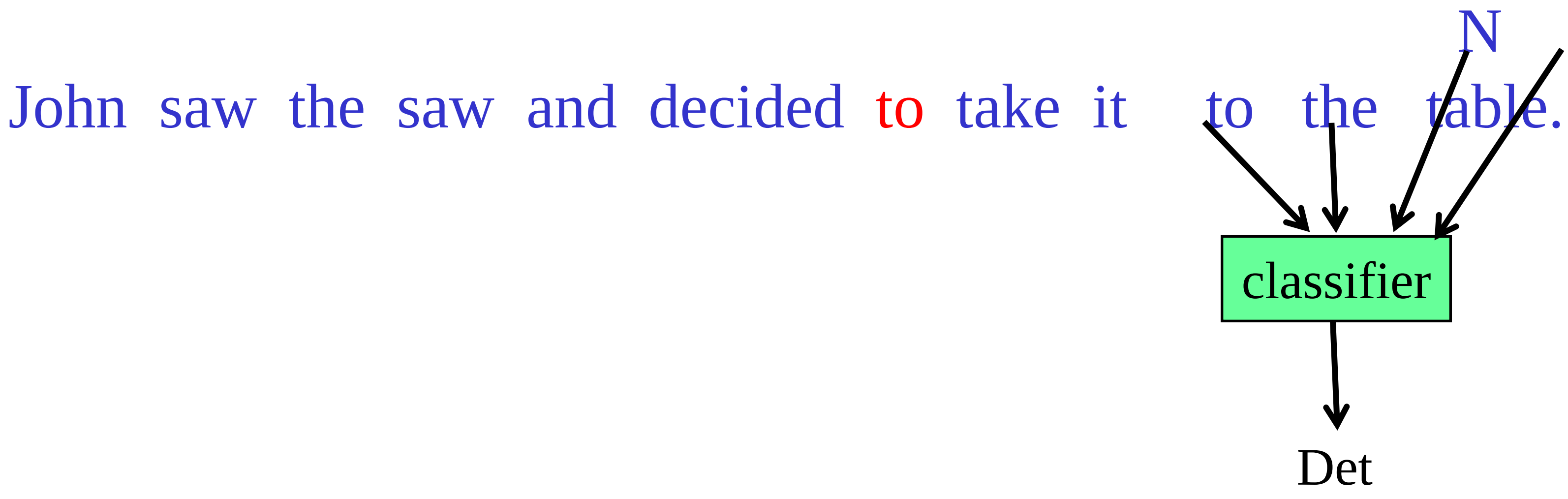
- Disambiguating “to” in this case would be even easier backward.



## Backward Classification

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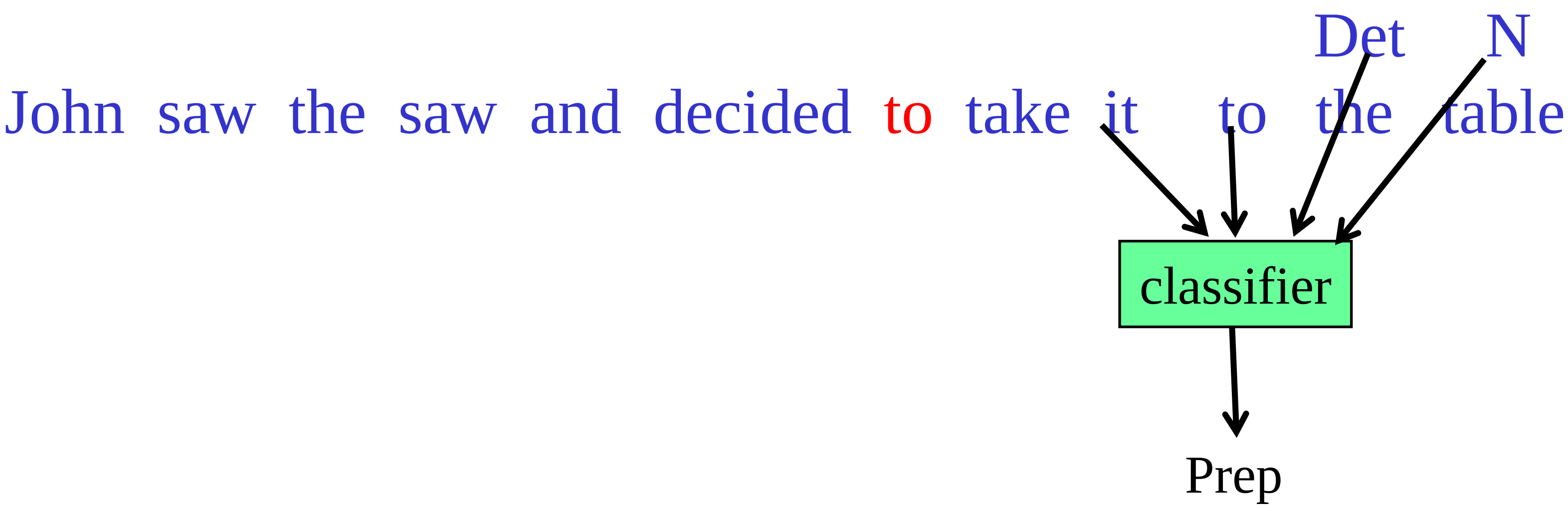

## Backward Classification

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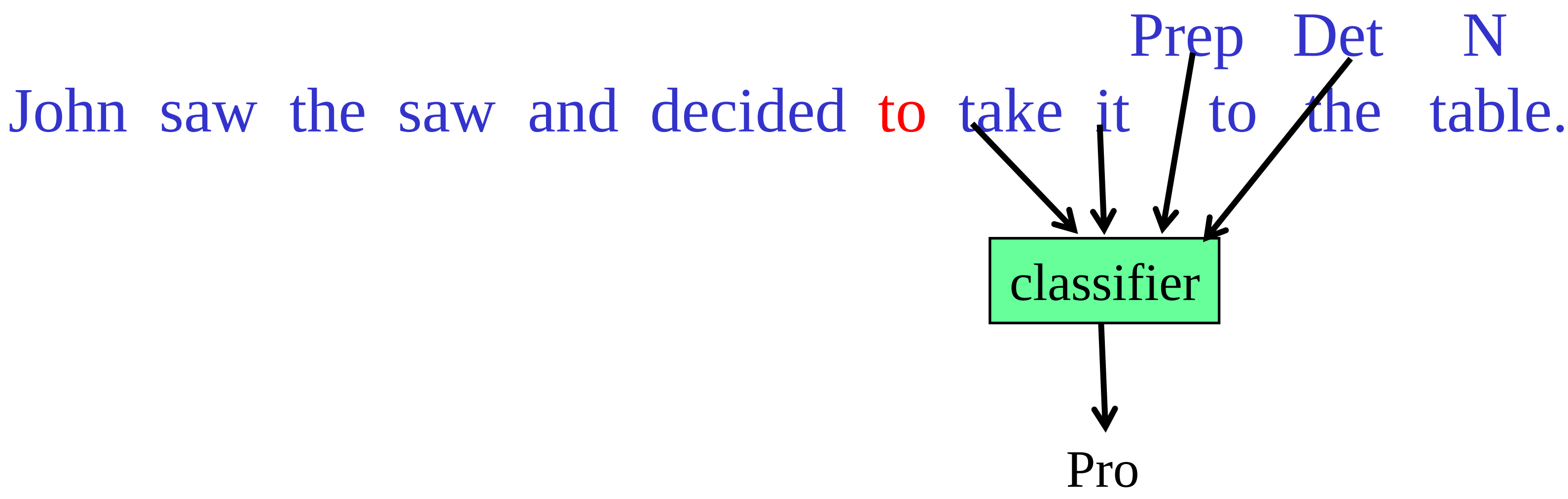

# Backward Classification

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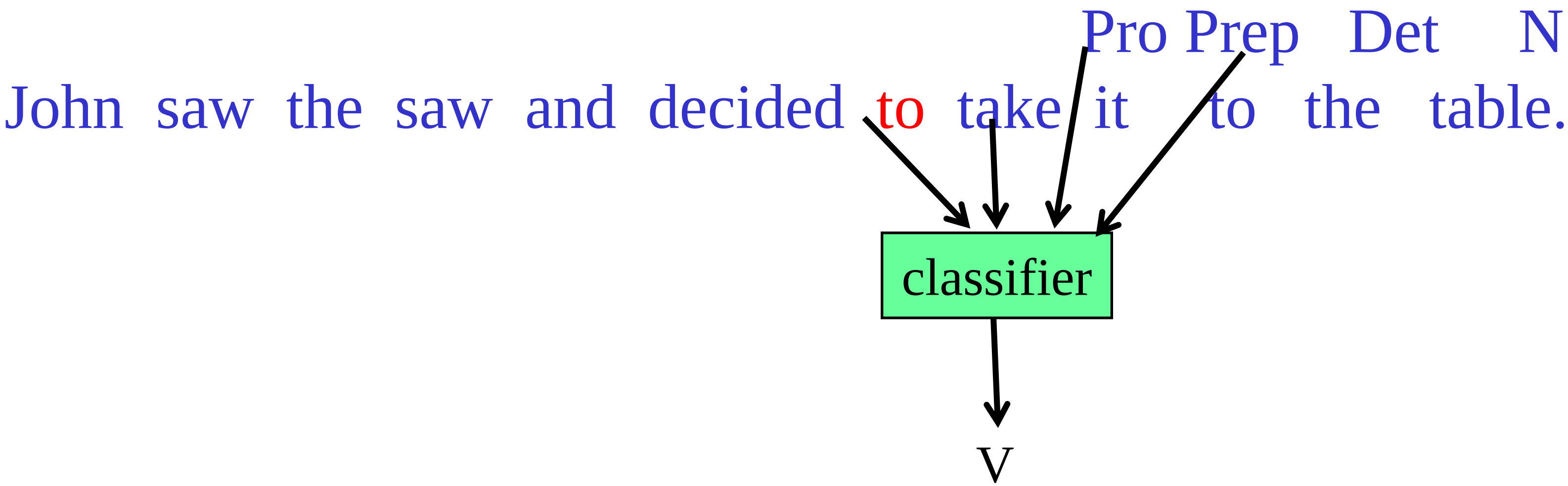
- Disambiguating “to” in this case would be even easier backward.



# Backward Classification

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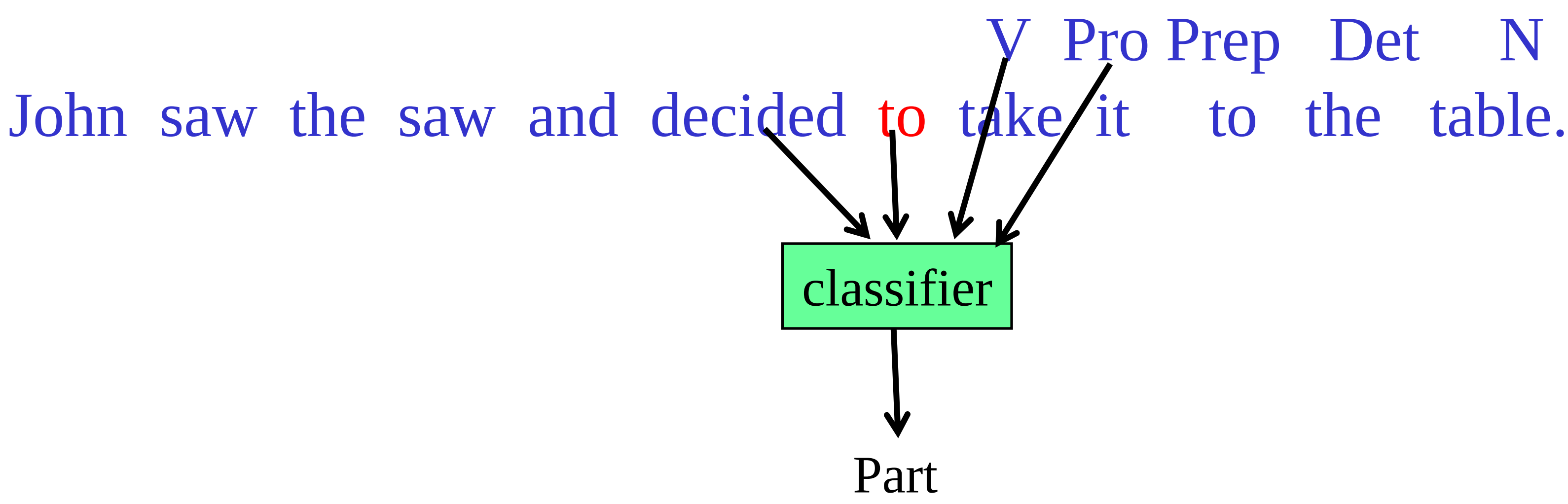
- Disambiguating “to” in this case would be even easier backward.
  - Preposition? (“I am heading to the store”)
  - Particle? (in front if infinitive verbs, like “I want to eat. I am going to leave.”)



## Backward Classification

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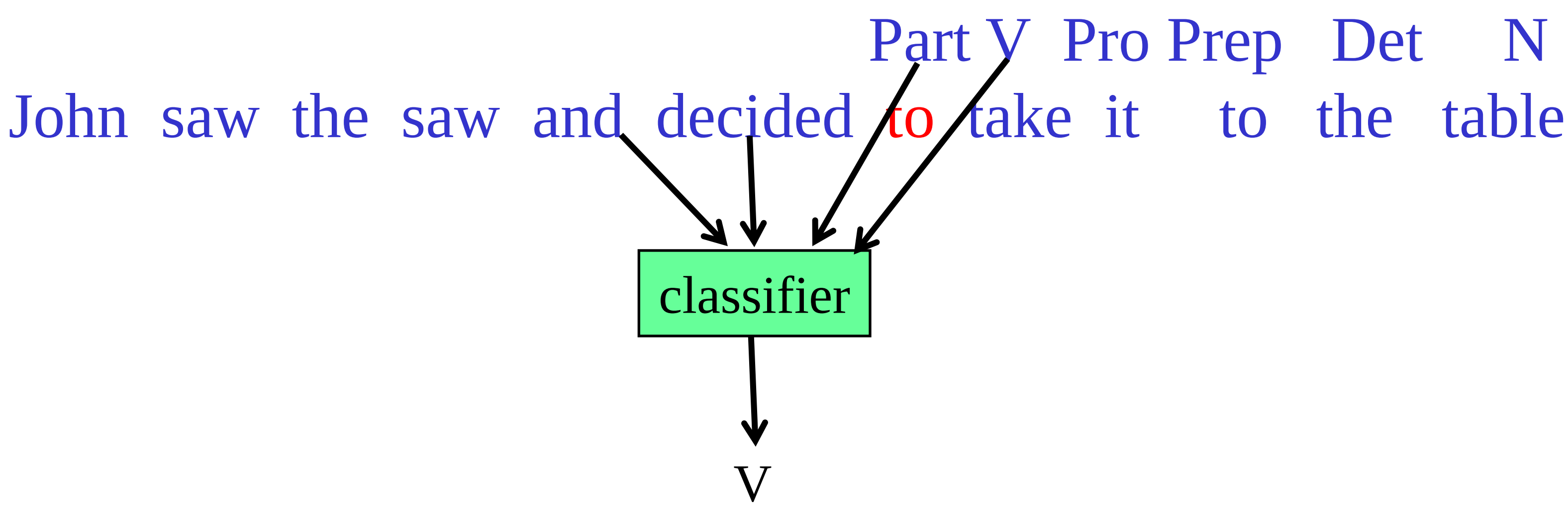
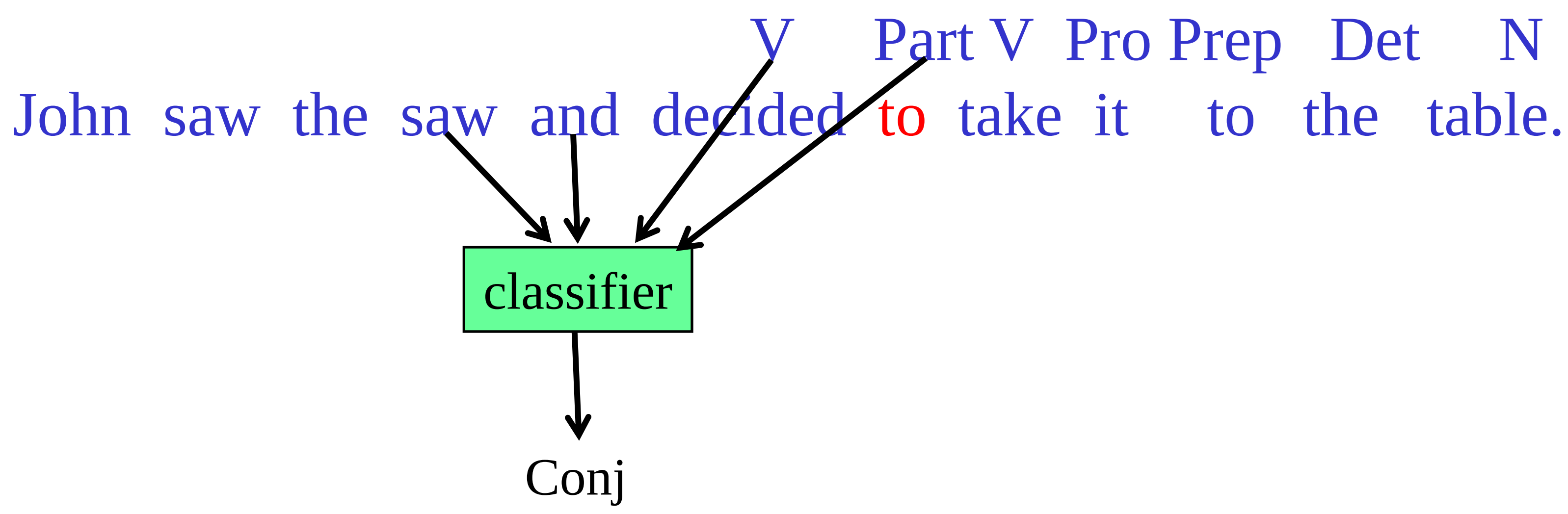
- Disambiguating “to” in this case would be even easier backward.



# Backward Classification



- Disambiguating “to” in this case would be even easier backward.



## Backward Classification

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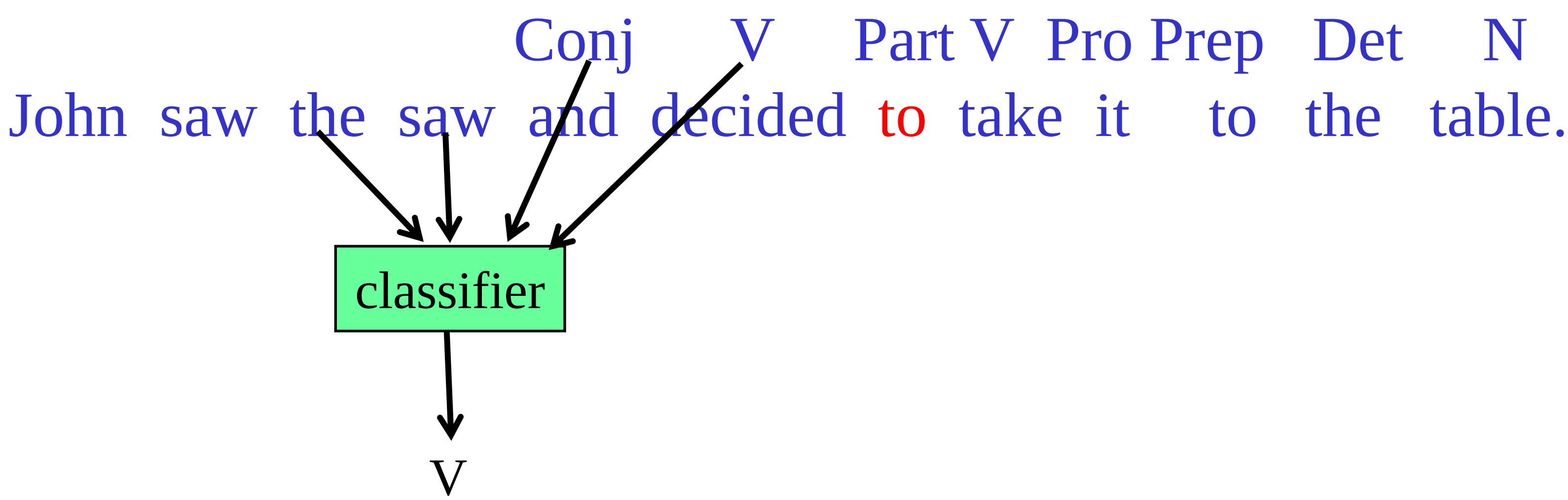
- Disambiguating “to” in this case would be even easier backward.



# Backward Classification

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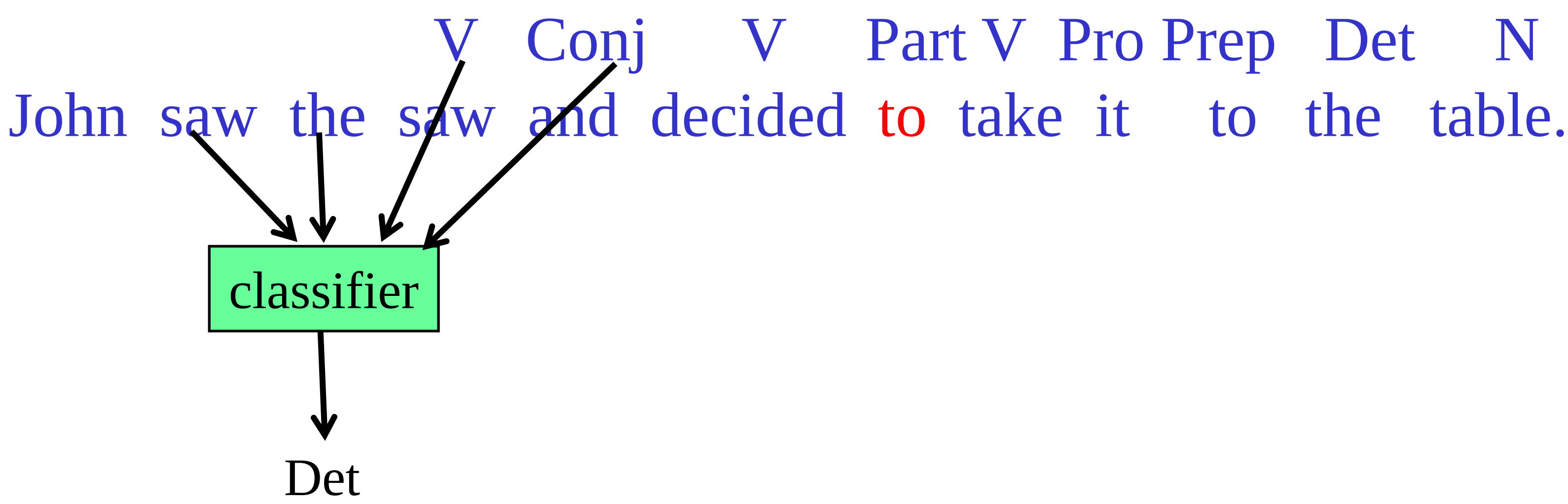
- Disambiguating “to” in this case would be even easier backward.



## Backward Classification

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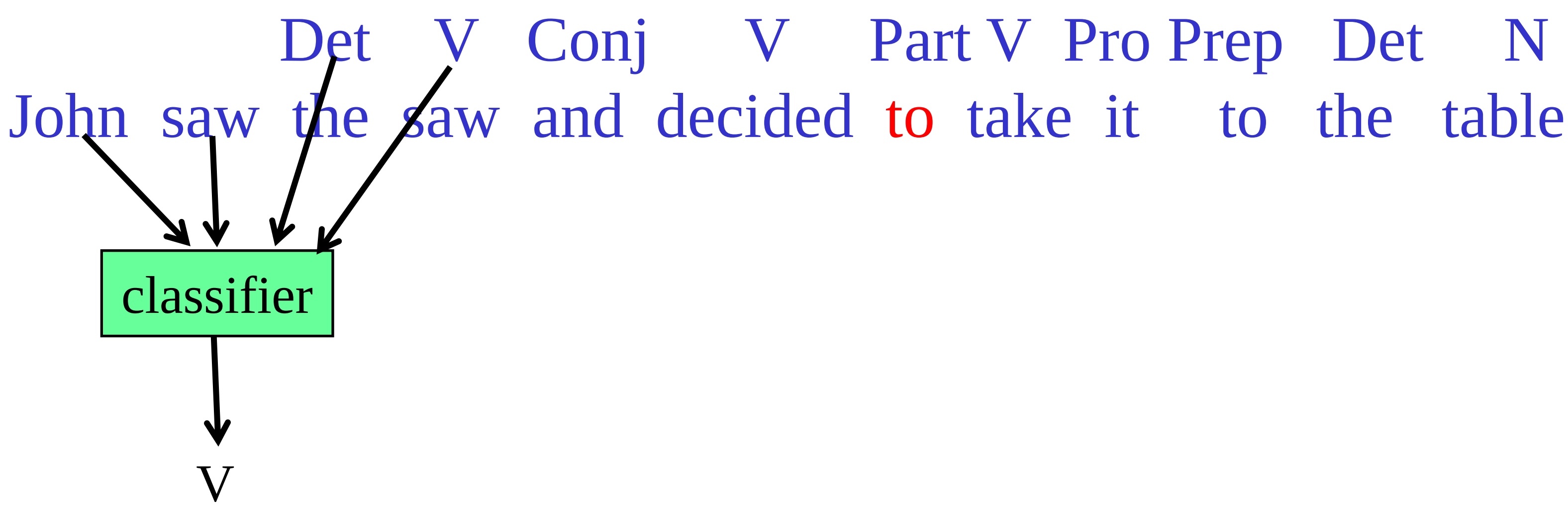
- Disambiguating “to” in this case would be even easier backward.



# Backward Classification

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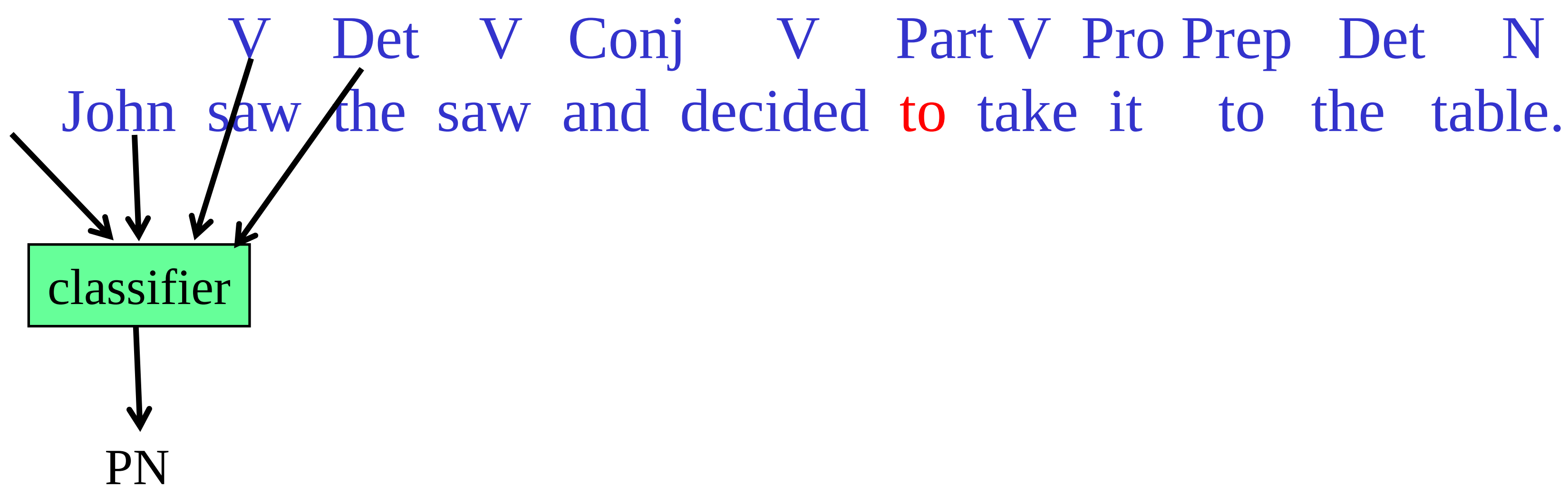
- Disambiguating “to” in this case would be even easier backward.



# Sequence Labeling as Classification

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**Problems**

- Not easy to integrate information from category of tokens on both sides
- Difficult to propagate uncertainty between decisions and “collectively” determine the most likely joint assignment of categories to all of the tokens in a sequence.

# Probabilistic Sequence Models

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- Probabilistic sequence models allow
  - integrating uncertainty over multiple, interdependent classifications
  - and collectively determine the most likely global assignment.

- Generative sequence models: e.g. **Hidden Markov Model (HMM)**

- Later in the lecture: Neural Sequence Models (RNN / LSTM)

# Intuition of HMM decoding: PoS Tagging

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- Choose the tag sequence that is most probable given the observation sequence of n words:

Tag sequence

Best tag sequence $\boxed{\hat{t}_1^n} = \underset{t_1^n}{\operatorname{argmax}} P(\boxed{t_1^n}|\boxed{w_1^n})$ Word sequence

- Bayes' Rule:,

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} \frac{P(w_1^n|t_1^n)P(t_1^n)}{P(w_1^n)}$$

- Drop the denominator

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} P(w_1^n|t_1^n)P(t_1^n)$$

- Assumption 1: word appearing depends only on its own tag

$$P(w_1^n|t_1^n) \approx \prod_{i=1}^{n} P(w_i|t_i)$$

- Assumption 2: the probability of a tag is dependent only on the previous tag

$$P(t_1^n) \approx \prod_{i=1}^{n} P(t_i|t_{i-1})$$

# Hidden Markov Model

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A Hidden Markov Model is a statistical model of hidden, stochastic state transitions with observable, stochastic output. Key features:

- A fixed set of states
  - At each "time", the hidden markov model is in exactly one of these states
- State <u>transition probabilities</u>
  - The starting state can be fixed or probabilistic
- A fixed set of possible outputs
- For each state: a distribution of probabilities for every possible output
  - Also called <u>emission probabilities</u>

Task:

For an observed output sequence, what is the (hidden) state sequence that has the highest probabiliy to produce this output?

# Hidden Markov Model - Example



Every day, Darth Vader is in one of three moods: Good, Neutral or Bad

But, because he wears his mask, we cannot observe it!

# Hidden Markov Model - Example

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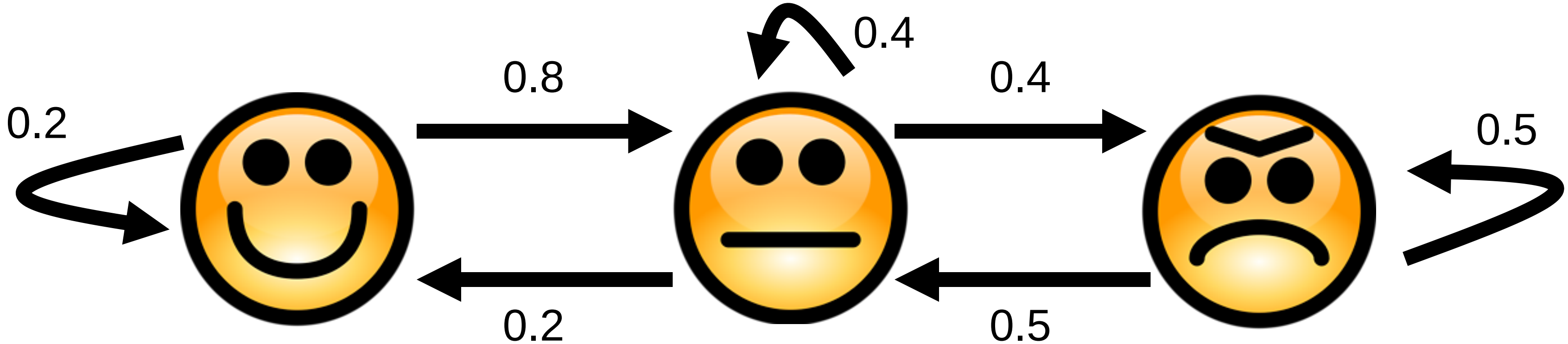
Every day, Darth Vader is in one of three moods: Good, Neutral or Bad

But, because he wears his mask, we cannot observe it!



# Hidden Markov Model - Example


Somehow, you know the odds how his mood changes from day to day:



# Hidden Markov Model - Example

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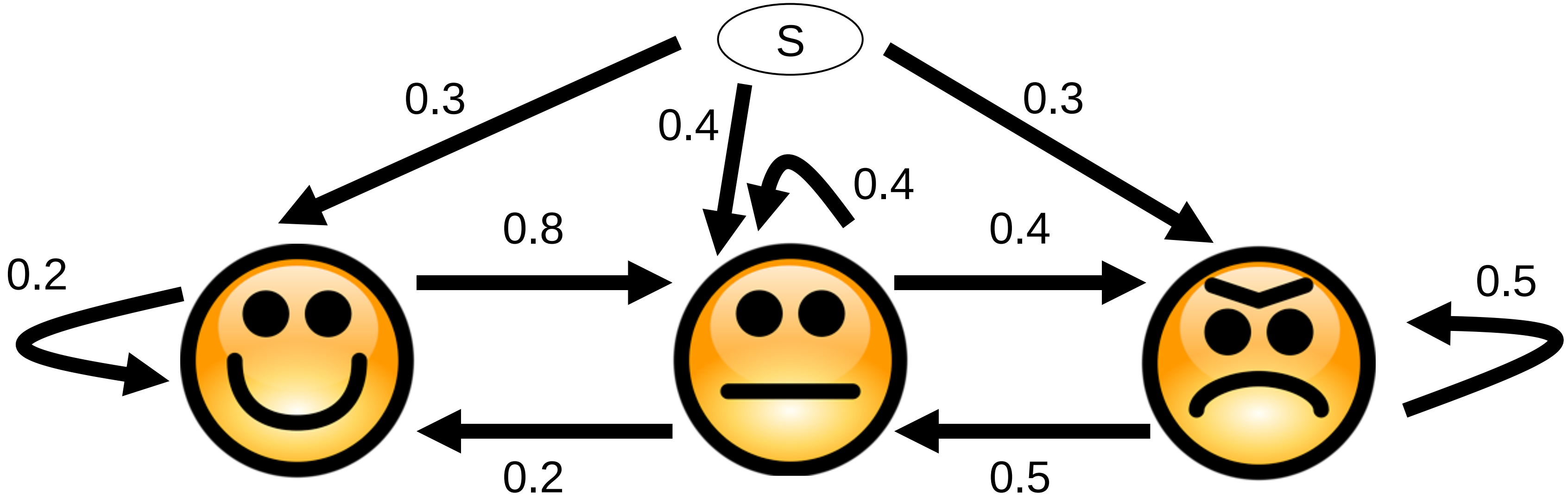
You also know the chances for his mood on day 1:



# Hidden Markov Model - Example

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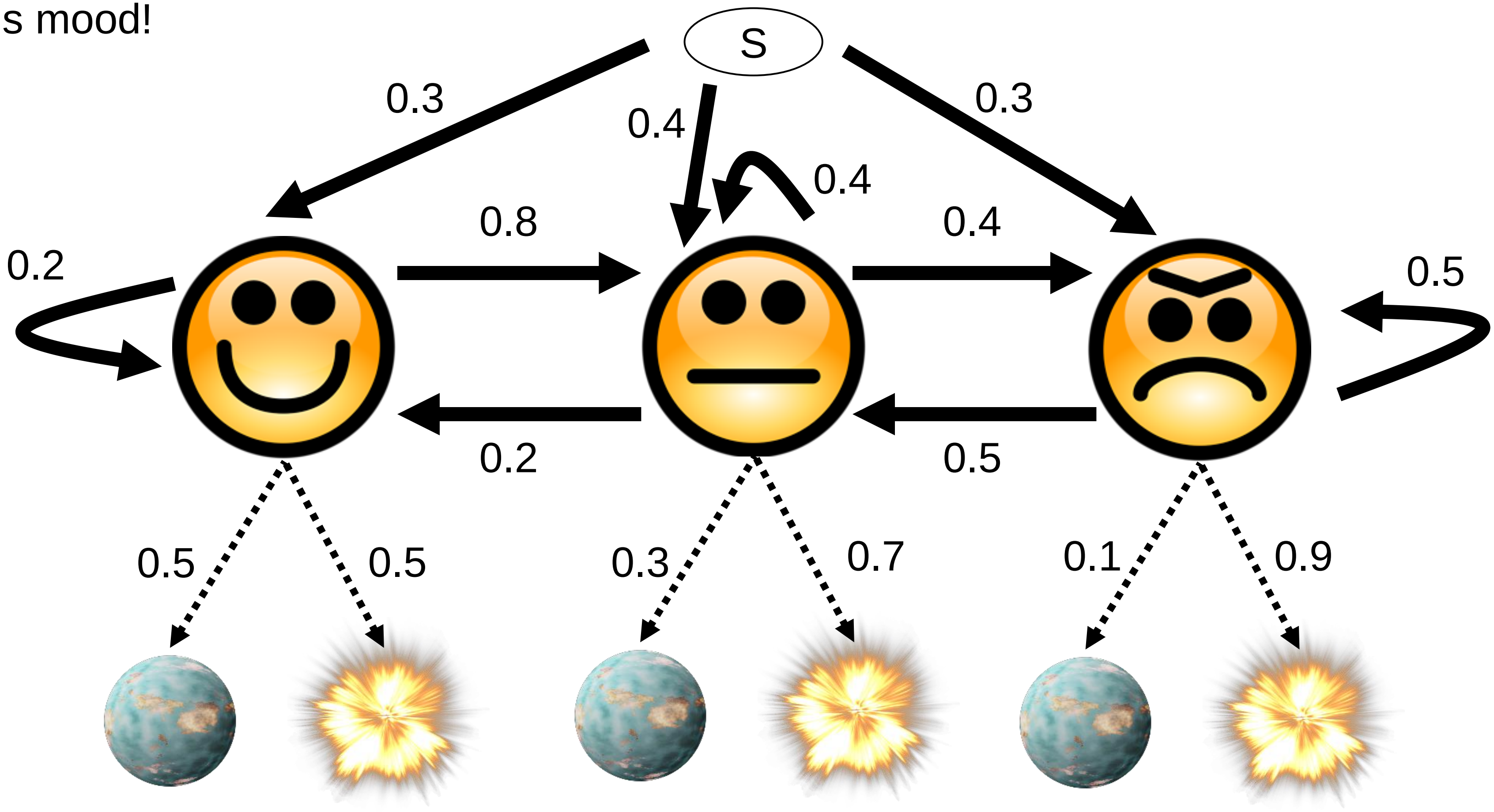
What we CAN observe is if Darth Vader destroys a planet or not, which depends on his mood!



# Hidden Markov Model - Example

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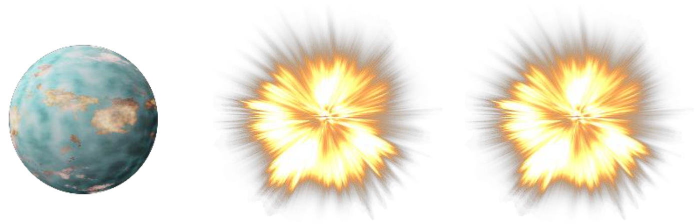
We observe that he does not destroy a planet on the first day, but he destroys a planet each on the second and third day:



Question:

What is the most probable sequence of his mood on these three days?

?

# Hidden Markov Model - Example

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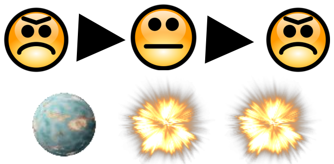
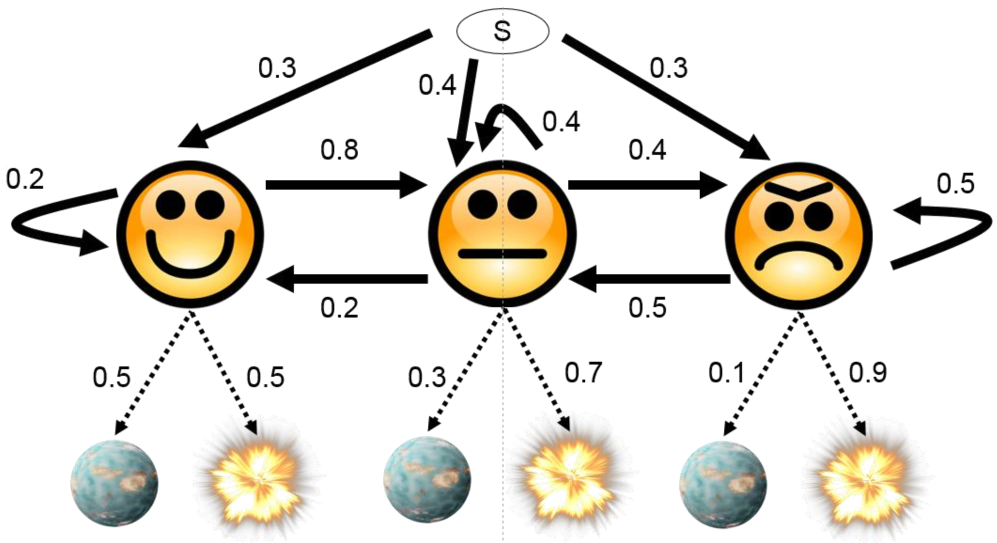
What is the probability for this mood sequence and this observation:





# Hidden Markov Model - Example

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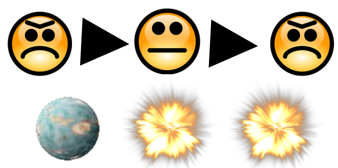
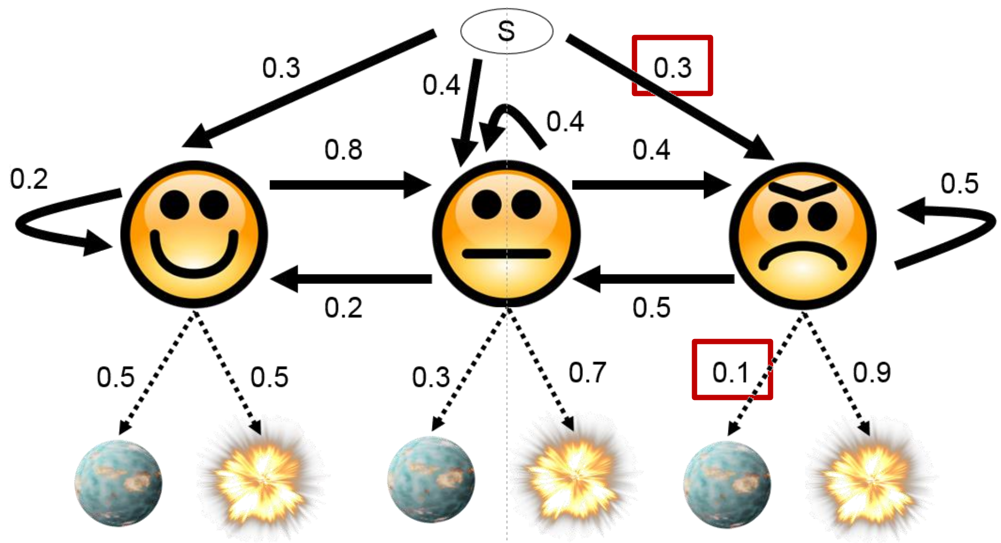
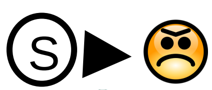
What is the probability for this mood sequence and this observation:






First day:

Transition probability:  0.3

Emission probability:  0.1

Joint probability: 0.3 * 0.1 = 0.03

# Hidden Markov Model - Example

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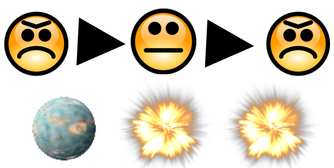
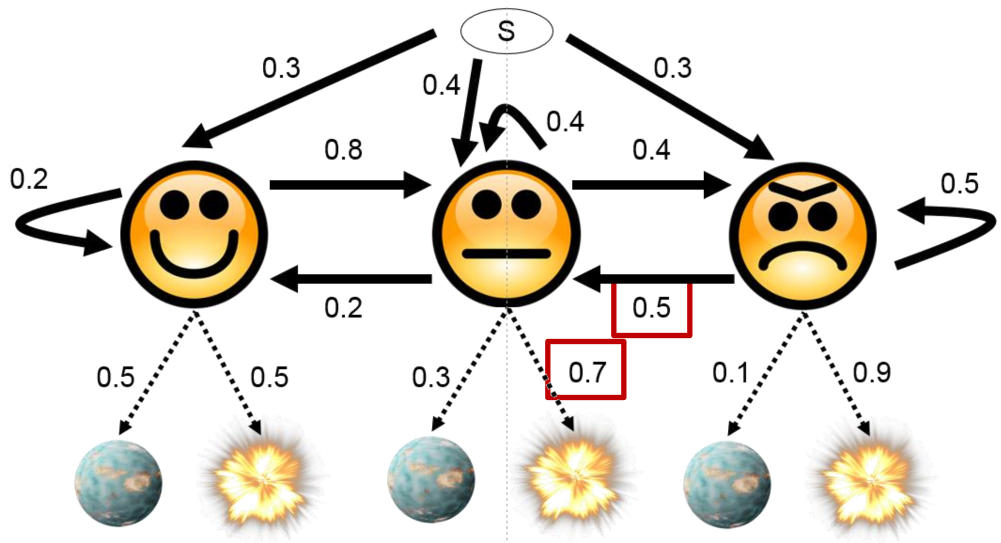
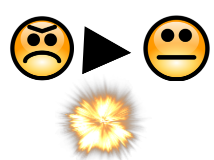
What is the probability for this mood sequence and this observation:



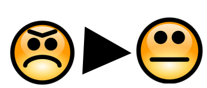



Second day:

Transition probability:  0.5

Emission probability: 0.7

Joint probability: 0.5 * 0.7 = 0.35

Probability for sequence:  0.03 (day one) * 0.35 (day two) = 0.0105

# Hidden Markov Model - Example

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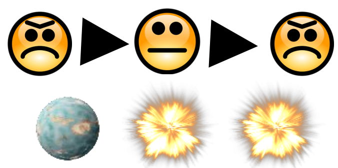
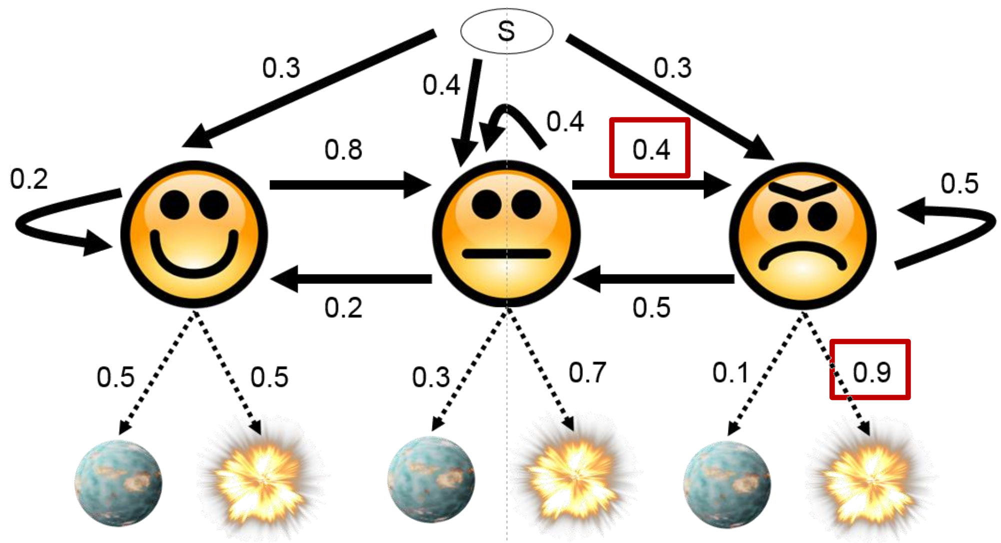
What is the probability for this mood sequence and this observation:



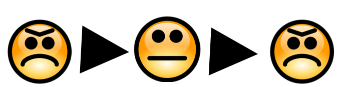



Third day:

Transition probability:  0.4

Emission probability:  0.9

Joint probability: 0.4 * 0.9 = 0.36

Probability for sequence:  0.03 * 0.35 * 0.36 = 0.00378

# Hidden Markov Model – Application to NLP

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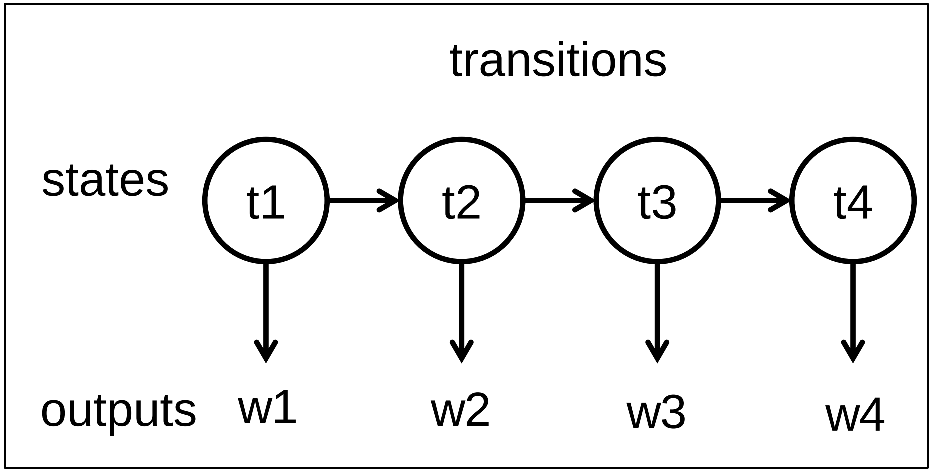
In our POS tagging example, we know the sequence of words, and we want to know the sequence of POS tags!




- (hidden) States: POS tags
- (observable) Outputs: Tokens
- We also need
  - Transition probabilities between states
    - Also: Initial probabilities = Probabilities for the first token of a sentence
  - Emission probabilities for every state

How would our graph from the example look like?

# Hidden Markov Model – Application to NLP

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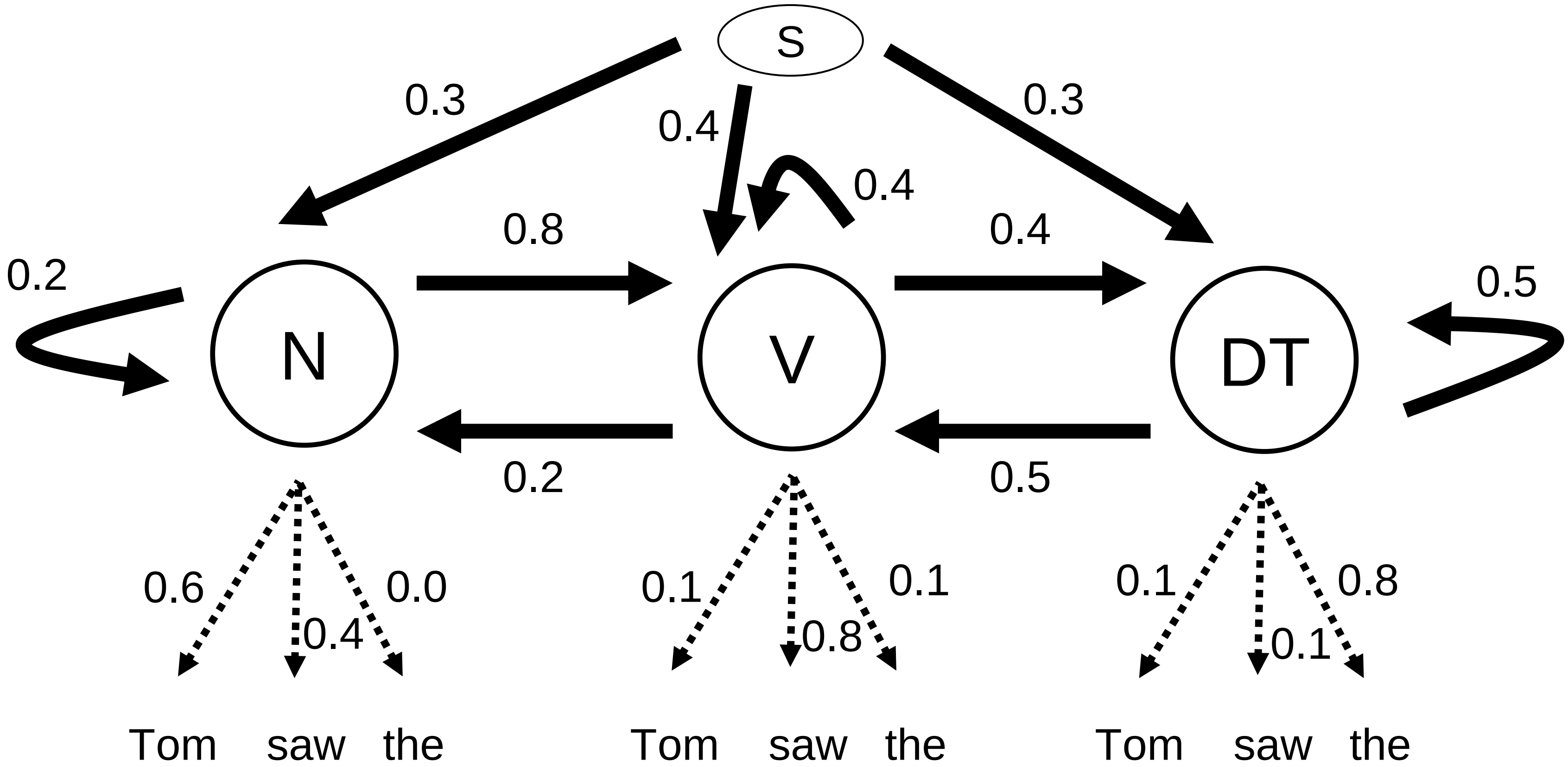


How do we get to the transition and emission probabilities?

# Hidden Markov Model – Application to NLP: Estimating the Probabilities

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- The probabilities are estimated just by counting on a **tagged training corpus**
  - Transition probability: how often the first tag is followed by the second divided by the number of the times the first tag was seen in a labeled corpus

$$P(t_i|t_{i-1}) = \frac{C(t_{i-1}, t_i)}{C(t_{i-1})}$$

  - The emission probabilities: the number of times the word was associated with the tag in the labeled corpus divided by number of the times the first tag was seen in a labeled corpus

$$P(w_i|t_i) = \frac{C(t_i, w_i)}{C(t_i)}$$

# Example

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Input sentence: Janet will back the bill
Correct PoS tags: Janet/NNP will/MD back/VB the/DT bill/NN

Transition probabilities based on WSJ corpus
Rows are labeled with the conditioning event; thus P(VB|MD) is 0.7968.

| | NNP | MD | VB | JJ | NN | RB | DT |
|---|---|---|---|---|---|---|---|
| $<s>$ | 0.2767 | 0.0006 | 0.0031 | 0.0453 | 0.0449 | 0.0510 | 0.2026 |
| **NNP** | 0.3777 | 0.0110 | 0.0009 | 0.0084 | 0.0584 | 0.0090 | 0.0025 |
| **MD** | 0.0008 | 0.0002 | 0.7968 | 0.0005 | 0.0008 | 0.1698 | 0.0041 |
| **VB** | 0.0322 | 0.0005 | 0.0050 | 0.0837 | 0.0615 | 0.0514 | 0.2231 |
| **JJ** | 0.0366 | 0.0004 | 0.0001 | 0.0733 | 0.4509 | 0.0036 | 0.0036 |
| **NN** | 0.0096 | 0.0176 | 0.0014 | 0.0086 | 0.1216 | 0.0177 | 0.0068 |
| **RB** | 0.0068 | 0.0102 | 0.1011 | 0.1012 | 0.0120 | 0.0728 | 0.0479 |
| **DT** | 0.1147 | 0.0021 | 0.0002 | 0.2157 | 0.4744 | 0.0102 | 0.0017 |

Given the observation (output) likelihoods

| | Janet | will | back | the | bill |
|---|---|---|---|---|---|
| **NNP** | 0.000032 | 0 | 0 | 0.000048 | 0 |
| **MD** | 0 | 0.308431 | 0 | 0 | 0 |
| **VB** | 0 | 0.000028 | 0.000672 | 0 | 0.000028 |
| **JJ** | 0 | 0 | 0.000340 | 0.000097 | 0 |
| **NN** | 0 | 0.000200 | 0.000223 | 0.000006 | 0.002337 |
| **RB** | 0 | 0 | 0.010446 | 0 | 0 |
| **DT** | 0 | 0 | 0 | 0.506099 | 0 |

# Hidden Markov Model – Complexity

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Question: What is the most likely state sequence given an output sequence?

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}}\, P(t_1^n|w_1^n) \approx \underset{t_1^n}{\operatorname{argmax}} \prod_{i=1}^{n} \overbrace{P(w_i|t_i)}^{\text{emission}} \overbrace{P(t_i|t_{i-1})}^{\text{transition}}$$

- Naïve solution:
  - brute force search by enumerating all possible sequences of states
  - Complexity $O(s^m)$
    - where m is the length of the input and s is the number of states in the model.
- Better solution: Dynamic Programming!
  - Standard procedure is called the **Viterbi algorithm**
  - Running time is $O(ms^2)$,
    - where m is the length of the input and s is the number of states in the model.

# Viterbi algorithm: Motivation

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- Let $T$: $o_1, o_2, \dots, o_T$ some input sentence
- Let $S$: $s_1, s_2, \dots, s_T$ sequence of tags
- Goal: best sequence of tags given the input sequence
  - $\operatorname{argmax}_{s_1, s_2, \dots, s_T} P(o_1, \dots, o_T, s_T, s_2, \dots, s_T)$
- Example
  - T = The, man, saw, the saw
  - S = {Det, N, V}
  - Possible tag sequences:

-> Det Det Det Det Det -> 0.000003
Det Det Det Det N -> 0.000008
Det Det Det Det V -> 0.000009
…
Det N V Det N -> 0.012

we have $3^5$ (243) sequences: sentence length = 5, #tags = 3

# Viterbi algorithm: Basic idea

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Let us say we have only two possible states, A and B, and some observation o
What is the best possible state sequence of length 5 for this observation o?
It is either:

- the best possible sequence of length 4 that ends with A, followed by A
- the best possible sequence of length 4 that ends with A, followed by B
- the best possible sequence of length 4 that ends with B, followed by A
- the best possible sequence of length 4 that ends with B, followed by B

This is only true because the next state only depends on the state directly before!

So, what is the best possible sequence of length 4 that ends with A? It is either:

- the best possible sequence of length 3 that ends with A, followed by A
- the best possible sequence of length 3 that ends with B, followed by A
- …

# Viterbi algorithm: Example

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**Emission probabilities**

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

**Transition probabilities**

P (DT | DT)= 0.1
P (V | DT) = 0.3
P (N | DT) = 0.6
P (DT | V) = 0.5
P (V | V) = 0.2
P (N | V) = 0.3

P (DT | N) = 0.2
P (V | N) = 0.6
P (N | N) = 0.2
P (DT | q0) = 0.6
P (V | q0) = 0.3
P (N | q0) = 0.1

| | The | man | saw | the | saw | $$ |
|---|---|---|---|---|---|---|
| DT | | | | | | |
| V | | | | | | |
| N | | | | | | |

# Viterbi algorithm: Example

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Emission probabilities

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

Transition probabilities

P (DT | DT)= 0.1
P (V | DT) = 0.3
P (N | DT) = 0.6
P (DT | V) = 0.5
P (V | V) = 0.2
P (N | V) = 0.3

P (DT | N) = 0.2
P (V | N) = 0.6
P (N | N) = 0.2
P (DT | q0) = 0.6
P (V | q0) = 0.3
P (N | q0) = 0.1

| | The | man | saw | the | saw | $$ |
|---|---|---|---|---|---|---|
| DT | 0.6 * 0.5 = 0.3 | | | | | |
| V | | | | | | |
| N | | | | | | |

# Viterbi algorithm: Example

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**Emission probabilities**

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

**Transition probabilities**

| | | | |
|---|---|---|---|
| P (DT \| DT) | = 0.1 | P (DT \| N) | = 0.2 |
| P (V \| DT) | = 0.3 | P (V \| N) | = 0.6 |
| P (N \| DT) | = 0.6 | P (N \| N) | = 0.2 |
| P (DT \| V) | = 0.5 | P (DT \| q0) | = 0.6 |
| P (V \| V) | = 0.2 | P (V \| q0) | = 0.3 |
| P (N \| V) | = 0.3 | P (N \| q0) | = 0.1 |

| | The | man | saw | the | saw | $$ |
|---|---|---|---|---|---|---|
| DT | 0.3 | | | | | |
| V | 0 | | | | | |
| N | 0 | | | | | |

# Viterbi algorithm: Example

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**Emission probabilities**

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

**Transition probabilities**

| | |
|---|---|
| P (DT \| DT) = 0.1 | P (DT \| N) = 0.2 |
| P (V \| DT) = 0.3 | P (V \| N) = 0.6 |
| P (N \| DT) = 0.6 | P (N \| N) = 0.2 |
| P (DT \| V) = 0.5 | P (DT \| q0) = 0.6 |
| P (V \| V) = 0.2 | P (V \| q0) = 0.3 |
| P (N \| V) = 0.3 | P (N \| q0) = 0.1 |

| | The | man |
|---|---|---|
| DT | 0.3 | |
| V | 0 | ? |
| N | 0 | |

= max {
P (man | V) * P (V | DT) * 0.3,
P (man | V) * P (V | V) * 0,
P (man | V) * P (V | N) * 0
}

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# Viterbi algorithm: Example

**Emission probabilities**

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

**Transition probabilities**

| | | | |
|---|---|---|---|
| P (DT \| DT) | = 0.1 | P (DT \| N) | = 0.2 |
| P (V \| DT) | = 0.3 | P (V \| N) | = 0.6 |
| P (N \| DT) | = 0.6 | P (N \| N) | = 0.2 |
| P (DT \| V) | = 0.5 | P (DT \| q0) | = 0.6 |
| P (V \| V) | = 0.2 | P (V \| q0) | = 0.3 |
| P (N \| V) | = 0.3 | P (N \| q0) | = 0.1 |

| | The | man |
|---|---|---|
| DT | 0.3 | |
| V | 0 | ? |
| N | 0 | |

= max {
P (man | V) * P (V | DT) * 0.3,
P (man | V) * P (V | V) * 0,
P (man | V) * P (V | N) * 0
}

# Viterbi algorithm: Example

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**Emission probabilities**

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

**Transition probabilities**

| | |
|---|---|
| P (DT \| DT)= 0.1 | P (DT \| N) = 0.2 |
| P (V \| DT) = 0.3 | P (V \| N) = 0.6 |
| P (N \| DT) = 0.6 | P (N \| N) = 0.2 |
| P (DT \| V) = 0.5 | P (DT \| q0) = 0.6 |
| P (V \| V) = 0.2 | P (V \| q0) = 0.3 |
| P (N \| V) = 0.3 | P (N \| q0) = 0.1 |

| | The | man |
|---|---|---|
| DT | 0.3 | |
| V | 0 | ? |
| N | 0 | |

= max {
P (man | V) * P (V | DT) * 0.3,
P (man | V) * P (V | V) * 0,
P (man | V) * P (V | N) * 0
}

# Viterbi algorithm: Example

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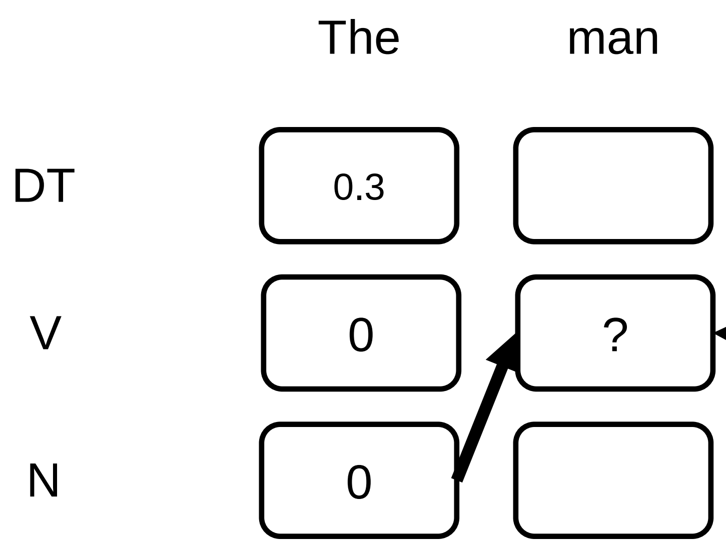
<u>Emission probabilities</u>

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

<u>Transition probabilities</u>

P (DT | DT)= 0.1
P (V | DT) = 0.3
P (N | DT) = 0.6
P (DT | V) = 0.5
P (V | V) = 0.2
P (N | V) = 0.3

P (DT | N) = 0.2
P (V | N) = 0.6
P (N | N) = 0.2
P (DT | q0) = 0.6
P (V | q0) = 0.3
P (N | q0) = 0.1




= max {
P (man | V) * P (V | DT) * 0.3,
P (man | V) * P (V | V) * 0,
P (man | V) * P (V | N) * 0
}

# Viterbi algorithm: Example

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**Emission probabilities**

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

**Transition probabilities**

| | |
|---|---|
| P (DT \| DT)= 0.1 | P (DT \| N) = 0.2 |
| P (V \| DT) = 0.3 | P (V \| N) = 0.6 |
| P (N \| DT) = 0.6 | P (N \| N) = 0.2 |
| P (DT \| V) = 0.5 | P (DT \| q0) = 0.6 |
| P (V \| V) = 0.2 | P (V \| q0) = 0.3 |
| P (N \| V) = 0.3 | P (N \| q0) = 0.1 |

| | The | man |
|---|---|---|
| DT | 0.3 | |
| V | 0 | ? |
| N | 0 | |

= max {
0.1 * 0.3 * 0.3,
0.1 * 0.2 * 0,
0.1 * 0.6 * 0
} = 0.009

# Viterbi algorithm: Example

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**Emission probabilities**

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

**Transition probabilities**

| | |
|---|---|
| P (DT \| DT)= 0.1 | P (DT \| N) = 0.2 |
| P (V \| DT) = 0.3 | P (V \| N) = 0.6 |
| P (N \| DT) = 0.6 | P (N \| N) = 0.2 |
| P (DT \| V) = 0.5 | P (DT \| q0) = 0.6 |
| P (V \| V) = 0.2 | P (V \| q0) = 0.3 |
| P (N \| V) = 0.3 | P (N \| q0) = 0.1 |

| | The | man |
|---|---|---|
| DT | 0.3 | |
| V | 0 | 0.009<br>DT |
| N | 0 | |

= max {
0.1 * 0.3 * 0.3,
0.1 * 0.2 * 0,
0.1 * 0.6 * 0
} = 0.009

Predecessor of the most probable path

# Viterbi algorithm: Example

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Emission probabilities

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

Transition probabilities

P (DT | DT)= 0.1
P (V | DT) = 0.3
P (N | DT) = 0.6
P (DT | V) = 0.5
P (V | V) = 0.2
P (N | V) = 0.3
P (DT | N) = 0.2
P (V | N) = 0.6
P (N | N) = 0.2
P (DT | q0) = 0.6
P (V | q0) = 0.3
P (N | q0) = 0.1

| | The | man |
|---|---|---|
| DT | 0.3 | 0 |
| V | 0 | 0.009 DT |
| N | 0 | 0.054 DT |

# Viterbi algorithm: Example

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Emission probabilities

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

Transition probabilities

| | |
|---|---|
| P (DT \| DT)= 0.1 | P (DT \| N) = 0.2 |
| P (V \| DT) = 0.3 | P (V \| N) = 0.6 |
| P (N \| DT) = 0.6 | P (N \| N) = 0.2 |
| P (DT \| V) = 0.5 | P (DT \| q0) = 0.6 |
| P (V \| V) = 0.2 | P (V \| q0) = 0.3 |
| P (N \| V) = 0.3 | P (N \| q0) = 0.1 |

| | The | man | saw |
|---|---|---|---|
| DT | 0.3 | 0 | |
| V | 0 | 0.009 DT | ? |
| N | 0 | 0.054 DT | |

= max {
P (saw | V) * P (V | DT) * 0,
P (saw | V) * P (V | V) * 0.009,
P (saw | V) * P (V | N) * 0.054
}

# Viterbi algorithm: Example

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Emission probabilities

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

Transition probabilities

| | |
|---|---|
| P (DT \| DT) = 0.1 | P (DT \| N) = 0.2 |
| P (V \| DT) = 0.3 | P (V \| N) = 0.6 |
| P (N \| DT) = 0.6 | P (N \| N) = 0.2 |
| P (DT \| V) = 0.5 | P (DT \| q0) = 0.6 |
| P (V \| V) = 0.2 | P (V \| q0) = 0.3 |
| P (N \| V) = 0.3 | P (N \| q0) = 0.1 |

| | The | man | saw |
|---|---|---|---|
| DT | 0.3 | 0 | |
| V | 0 | 0.009 DT | $6{,}48 * 10^{-3}$ N |
| N | 0 | 0.054 DT | |

= max {
0.2 * 0.3 * 0, ( = 0)
0.2 * 0.2 * 0.009, ( = 0,00036)
0.2 * 0.6 * 0.054 ( = 0,00648)
} = 0,00648 = $6{,}48 * 10^{-3}$

# Viterbi algorithm: Example

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Emission probabilities

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

Transition probabilities

P (DT | DT)= 0.1
P (V | DT) = 0.3
P (N | DT) = 0.6
P (DT | V) = 0.5
P (V | V) = 0.2
P (N | V) = 0.3
P (DT | N) = 0.2
P (V | N) = 0.6
P (N | N) = 0.2
P (DT | q0) = 0.6
P (V | q0) = 0.3
P (N | q0) = 0.1

| | The | man | saw | the | saw | $$ |
|---|---|---|---|---|---|---|
| DT | 0.3 | 0 | 0 | $1{,}62 * 10^{-3}$ V | 0 | |
| V | 0 | 0.009 DT | $6{,}48 * 10^{-3}$ N | 0 | $9{,}72 * 10^{-5}$ DT | $1{,}94 * 10^{-4}$ N |
| N | 0 | 0.054 DT | $2{,}16 * 10^{-3}$ N | 0 | $1{,}94 * 10^{-4}$ DT | |

# Viterbi algorithm: Example

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**Emission probabilities**

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

**Transition probabilities**

| | |
|---|---|
| P (DT \| DT)= 0.1 | P (DT \| N) = 0.2 |
| P (V \| DT) = 0.3 | P (V \| N) = 0.6 |
| P (N \| DT) = 0.6 | P (N \| N) = 0.2 |
| P (DT \| V) = 0.5 | P (DT \| q0) = 0.6 |
| P (V \| V) = 0.2 | P (V \| q0) = 0.3 |
| P (N \| V) = 0.3 | P (N \| q0) = 0.1 |

| | The | man | saw | the | saw N | $$ |
|---|---|---|---|---|---|---|
| DT | 0.3 | 0 | 0 | $1{,}62 * 10^{-3}$ V | 0 | |
| V | 0 | 0.009 DT | $6{,}48 * 10^{-3}$ N | 0 | $9{,}72 * 10^{-5}$ DT | **N** |
| N | 0 | 0.054 DT | $2{,}16 * 10^{-3}$ N | 0 | $1{,}94 * 10^{-4}$ DT | |

# Viterbi algorithm: Example

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**Emission probabilities**

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

**Transition probabilities**

| | |
|---|---|
| P (DT \| DT)= 0.1 | P (DT \| N) = 0.2 |
| P (V \| DT) = 0.3 | P (V \| N) = 0.6 |
| P (N \| DT) = 0.6 | P (N \| N) = 0.2 |
| P (DT \| V) = 0.5 | P (DT \| q0) = 0.6 |
| P (V \| V) = 0.2 | P (V \| q0) = 0.3 |
| P (N \| V) = 0.3 | P (N \| q0) = 0.1 |

| | The | man | saw | the<br>DT | saw<br>N | $$ |
|---|---|---|---|---|---|---|
| DT | 0.3 | 0 | 0 | $1{,}62 * 10^{-3}$<br>V | 0 | |
| V | 0 | 0.009<br>DT | $6{,}48 * 10^{-3}$<br>N | 0 | $9{,}72 * 10^{-5}$<br>DT | **N** |
| N | 0 | 0.054<br>DT | $2{,}16 * 10^{-3}$<br>N | 0 | **DT** | |

# Viterbi algorithm: Example

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Emission probabilities

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

Transition probabilities

P (DT | DT)= 0.1
P (V | DT) = 0.3
P (N | DT) = 0.6
P (DT | V) = 0.5
P (V | V) = 0.2
P (N | V) = 0.3

P (DT | N) = 0.2
P (V | N) = 0.6
P (N | N) = 0.2
P (DT | q0) = 0.6
P (V | q0) = 0.3
P (N | q0) = 0.1

| | The | man | saw V | the DT | saw N | $$ |
|---|---|---|---|---|---|---|
| DT | 0.3 | 0 | 0 | **V** | 0 | |
| V | 0 | 0.009 DT | $6{,}48 * 10^{-3}$ N | 0 | $9{,}72 * 10^{-5}$ DT | **N** |
| N | 0 | 0.054 DT | $2{,}16 * 10^{-3}$ N | 0 | **DT** | |

# Viterbi algorithm: Example

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<u>Emission probabilities</u>

P (the | DT) = 0.5
P (man | V) = 0.1
P (man | N) = 0.3
P (saw | V) = 0.2
P (saw | N) = 0.2

<u>Transition probabilities</u>

| | |
|---|---|
| P (DT \| DT)= 0.1 | P (DT \| N) = 0.2 |
| P (V \| DT) = 0.3 | P (V \| N) = 0.6 |
| P (N \| DT) = 0.6 | P (N \| N) = 0.2 |
| P (DT \| V) = 0.5 | P (DT \| q0) = 0.6 |
| P (V \| V) = 0.2 | P (V \| q0) = 0.3 |
| P (N \| V) = 0.3 | P (N \| q0) = 0.1 |

| | The<br>DT | man<br>N | saw<br>V | the<br>DT | saw<br>N | $$ |
|---|---|---|---|---|---|---|
| DT | 0.3 | 0 | 0 | **V** | 0 | |
| V | 0 | 0.009<br>DT | **N** | 0 | $9{,}72 * 10^{-5}$<br>DT | **N** |
| N | 0 | **DT** | $2{,}16 * 10^{-3}$<br>N | 0 | **DT** | |

# Viterbi algorithm: Example

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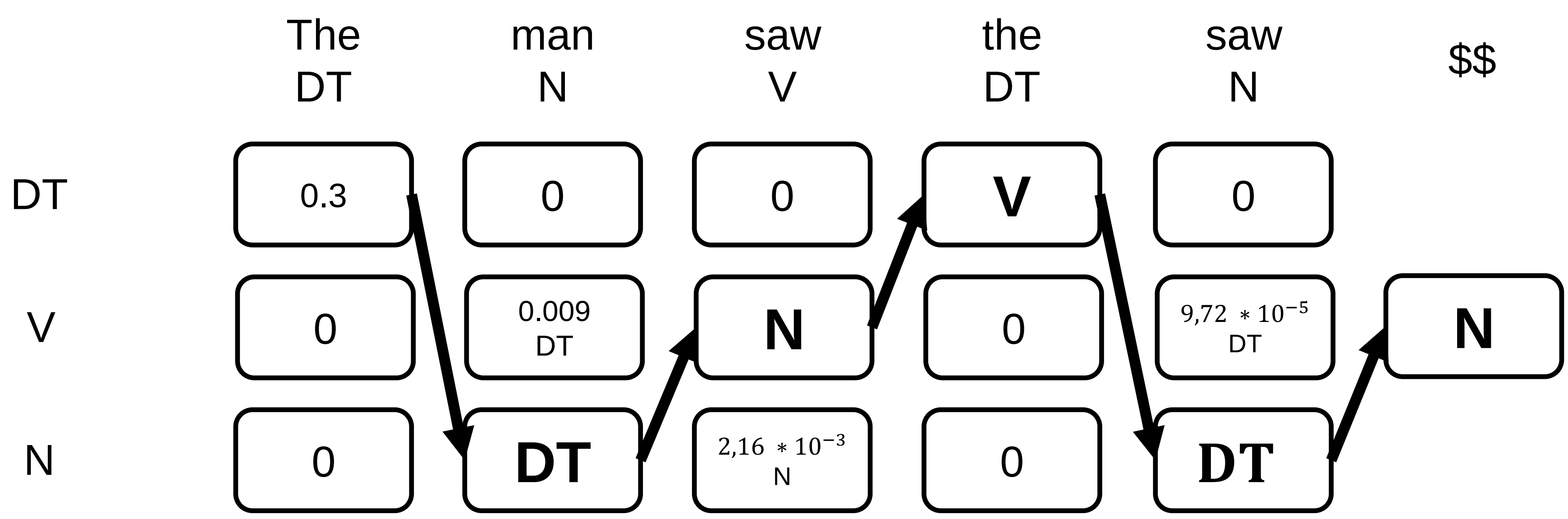
Remember the complexity of the Viterbi algorithm?

Running time is $O(ms^2)$,

- where m is the length of the input and s is the number of states in the model.

Now we see why: For every token (m) we we have to evaluate every POS (s) in combination with every possible predeccessor POS (s), so he have m * s * s operations = $ms^2$



# Summary

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- Sequence Labeling:
  - Input and output are signal sequences
  - No individual classification per signal, but joint classification that minimizes some cost
- Hidden Markov Models
  - Emissions can be observed
  - States are hidden
  - Goal: Find most probable state sequence for a given emission sequence
  - Solve via Viterbi (dynamic programming)

# Next Lecture

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**Information Retrieval Introduction**