$f(x)=\sum_{n=0}^{\infty} f^{(n)}(a)\left(\frac{1}{n!}\right)(x-a)^n$

$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$, $\lim_{n\to\infty}(1+\frac{1}{n})^n = e$, $\lim_{n\to\infty}(1-\frac{1}{n})^n = \frac{1}{e}$. **Compounding:** After n years (m-times/annum) $\$1 \longrightarrow (1+\frac{r}{m})^{mn}$. **Continuous C.** (C.C.) $\Rightarrow \lim_{m\to\infty}(1+\frac{r}{m})^{mn} = \lim_{z\to\infty}(1+\frac{1}{z})^{nzr} = e^{nr}$

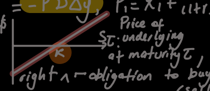
$\$A \xrightarrow{m,n} \$A(1+\frac{r}{m})^{rm}$, $\$Ae^{rn}$. **Convert:** $A(1+\frac{r_m}{m})^{mn} = A(1+\frac{r_{m'}}{m'})^{m'n} = e^{r_c n}$. **Forward Rates:** (i) $e^{r_1 t_1} e^{r_f(t_2-t_1)} = e^{r_2 t_2}$ (ii) $r_f = \frac{r_2 t_2 - r_1 t_1}{t_2 - t_1}$ no-arbitrage condition implied by the current 0-curve

Bond: $\$P$: PV of future cash flows C.C. $P=\sum_{i=1}^{n} c_i e^{-y t_i}$, **PV** $=\frac{FV}{(1+r)^n}$, **Yield:** Discount rate s.t. PV of bonds cash-flows $= \$P$, **Par Yield:** (r) s.t. $\$P$ = Face Value

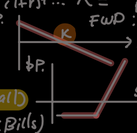
Duration: $\sum_{i=1}^{n} t_i c_i e^{-y t_i} / P$, t_i: time at c_i occurs, y_i: yield (c.c.). For small Δy, $\Delta P = \frac{dP}{dy}\Delta y$, $\Delta P = -\Delta y \sum_{i=1}^{n} t_i c_i e^{-y t_i} = -PD\Delta y$, e.g. $P_1 = X_1 + \frac{X_2}{(1+r)} + \frac{X_3}{(1+r)^2} + \ldots$, $P_2 = X_2 + \frac{X_3}{(1+r)} + \ldots$ $R = \frac{X_1+P_2}{P_1} = 1+r$

$1/year

Risk-free bond $\therefore P_T = \frac{1}{(1+r)^T} \Rightarrow \hat{r}_T = (\frac{1}{P_T})^{-\frac{1}{T}} - 1$ discount rate. **Profit:** **long Fwd Position:** K = Price = at contract t. Delivery Fwd price



SHORT FWD POSITION:



Agree. between 2 parties to buy/sell an asset at a specific price, date. **FWD Contract:** **Future C:** settled daily, margin call, Exchange, standarized. **Options:** Call: right ∧ ~~obligation~~ to buy (sell) an asset; Put: at a certain date/strike price. Profit (**Long-call**)

Principal: payment at maturity T. **(Risk-Free)** (Bills)

Position **(short-call)**

Position **(Long-Put)**

Position **(Short-put)**

. **Fixed Income:** **Coupons:** intermittent payments at $t=1,2,\ldots,T$. **Treasury Rates:** rate of return on Treasury Bonds

Overnight Rates, LIBOR

Repo Rate, $R_c = m\ln(1+\frac{R_m}{m})$, $R_m = m(e^{\frac{R_c}{m}}-1)$, **zero-rate** (spot): R_T earned on A that provides only a payoff at T $\therefore \$P$ (**zero coupon**) $= e^{-T \cdot R_T}$. Ex:

Maturity	Zero rate c.c.
0.5	5.0
1.0	5.8
1.5	6.4
2.0	6.8

PV $=\frac{FV}{e^{rt}}$, **e.g.** 6% coupon semi-annually $T=2 \therefore 3\%$ at 0.5, 1.0, 1.5, 103% at T $\therefore$ **$P** $=3e^{-0.05\cdot0.5}+3e^{-0.058\cdot1.0}+3e^{-0.064\cdot1.5}+103e^{-0.068\cdot2.0} = 98.39$ given Principal = \$100, **Bond Yield:** $3e^{-y\cdot0.5}+3e^{-y\cdot1.0}+3e^{-y\cdot1.5}+103e^{-y\cdot2.0} = 98.39$

Par Yield (c): $\frac{c}{2}e^{-0.05\cdot0.5}+\frac{c}{2}e^{-0.058\cdot1.0}+\frac{c}{2}e^{-0.064\cdot1.5}+(100+\frac{c}{2})e^{-0.068\cdot2.0} = 100$, **c** $=\frac{(100-100d)m}{A}$, **d**: PV(\$1) at T, **A**: PV of annuity of \$1 at each t_i $\therefore$ **d** $= e^{-R_{mn}n}$, **A** $=\sum_{i=1}^{mn} e^{-R_i \frac{i}{m}}$

$\therefore \frac{c}{m}\left(\sum_{i=1}^{mn} e^{-R_i\frac{i}{m}}\right) + 100e^{-R_{mn}n} = 100 \therefore c = \frac{(100-100e^{-R_{mn}n})m}{\sum^{mn} e^{-R_i i/m}} = \frac{(100-100d)m}{A}$, **FWD Rate:** $\frac{R_2T_2 - R_1T_1}{T_2 - T_1}$, **Instantaneous Fwd R:** $R + T\frac{\partial R}{\partial T}$, R: T-year rate

Upward Sloping yield curve: Fwd R. > 0-R > Par Yield, **Downward Sloping Yield Curve:** Par Yield > 0-R > Fwd Rate, **Duration:** $P=\sum_i c_i e^{-R(t_i)t_i}$ $R(t_i)$: 0-R for time t_i

Small parallel Shift: $R(t_i) \longrightarrow R(t_i) + dy$ $\therefore$ $\frac{dP}{P} = \frac{-\sum t_i (c_i e^{-y t_i})}{P} dy$, **D** $=\frac{\sum t_i (c_i e^{-y t_i})}{P}$ D is a weighted average of t_i, weighed by coupon payments

$\therefore$ $\frac{dP}{P} = -D\Delta y$ Duration reflects sensitivity to changes in the yield. **Bond Portfolios:** Duration is weighted average of the bonds in the portfolio by \$

Convexity: $C=\frac{\sum t_i^2 (c_i e^{-y\cdot t_i})}{P} = \frac{1}{P}\cdot\frac{\partial^2 P}{\partial y^2} \Longrightarrow \frac{dP}{P} = -D\Delta y + \frac{1}{2}C(\Delta y)^2$, $\frac{dP}{P} = -\frac{D}{1+y/m}\Delta y$ **Duration-Based Hedge Ratio:** $\frac{P D_P}{V_F D_F}$ { **V_F**: \$ Contract for interest r. futures; **D_F**: Duration of asset underl. at T

P: Value of portfolio being hedged, **D_P**: Duration of portfolio at hedge maturity (T), **Zero Bond:** $P_t = \frac{1}{(1+r)^n} \approx 1 - n\cdot r \longrightarrow D \approx n$ (Asset ∧ no flows). **Perpetuity:** $P=\frac{c}{1+r}+\frac{c}{(1+r)^2}+\ldots = \frac{c}{r} \longrightarrow D = \frac{1+r}{r}$

Futures C: Obligation to buy/sell an under. asset At prespecified price ∧ date

. **Pricing Futures/Forwards:** **S_0**: Spot price today, **F_0**: Futures/Forward price today **T**: time remaining

r: Risk-free interest rate for maturity T. **F_0**: Futures contract price deliverable in T years $\Rightarrow$ **$F_0 = S_0 e^{rT}$**, **r**: T-year risk-free i.r. **Arbitrage Opp.:** S_0: \$40 stock paying (no n-dividend)

e.g. F_0: 3-month \$43, r: 3-month i.r. 5%/annum $\Longrightarrow$ 1) Sell: fwd $|_{\$43}$, 2) Buy: Stock $|_{\$40}$ $\therefore$ 1) Short fwd to sell S in 3-months, 2) Buy(stock + interest) = \$40.5 $\Rightarrow$ \$43 − \$40.5

e.g. F_0: 3-month \$39, r: 5%, S_0: \$40, 1) long fwd at \$39 (agree to buy $S|_{\$39}$ in 3-months) 2) Short-sell the stock at \$40, At T=3-months buy stock at \$39 through fwd contract, deliver the stock to close short-position. If $S \xrightarrow{I}$ **income**: **$F_0 = (S_0 - I)e^{rT}$**, **I**: PV of income during T

When $S \xrightarrow{q}$ **Yield**. **$F_0 = S_0 e^{(r-q)T}$** (Portfolio) (c.c.), **q**: average yield during T. **Value of long Fwd C:** **$(F_0 - K)e^{-rT}$**, **short-Fwd C:** **$(K-F_0)e^{-rT}$**, **K**: Delivery Price

Fwd VS FUTURES: If Maturity \$P. = Asset \$P $\Longrightarrow$ \$FWD = \$Future. If **ρ(i.r., Asset \$P) > +0 $\Rightarrow$ \$Future $\geq$ \$Fwd + ε**, **ρ(i.r., Asset \$P) < −0 $\Rightarrow$ \$Fwd $\geq$ \$Future + ε** $\varepsilon > 0$

Index Arbitrage: **$F_0 > S_0 e^{(r-q)T}$**: Buy stocks underlying index ∧ sell futures; **$F_0 < S_0 e^{(r-q)t}$** Buy Futures, short-sell stocks underlying index. **Currencies:** r_f: i.r. ("yield") foreign risk-free

$F_0 = S_0 e^{(r-r_f)T}$, $F_0 \leq S_0 e^{(r+u)T}$ u: as % S value. storage cost/unit time. Cost of carry (c): Interest cost + storage cost − earned income $\therefore$ For **investment asset** $F_0 = S_0 e^{cT}$, **Consumption A.**

$F_0 \leq S_0 e^{cT}$. Convenience Yield on Consumption asset (y) s.t. $F_0 = S_0 e^{(c-y)T}$. Expected Future Spot Prices: k: expected return required by investor in asset

$F_0 = E(S_T)e^{(r-k)T}$. **No Systematic risk:** $k=r \Longrightarrow F_0 = E(S_T)$, **Positive Systematic Risk:** $k>r$, $F_0 < E(S_T)$, **Negative S.R.:** $k<r$, $F_0 > E(S_T)$

Bond Price: **Cash Price = Quoted Price + Accrued Interest**. **Settlement Price = Quoted Price** $= \frac{\text{Desired Cash Price} - \text{Accrued Interest}}{\text{Conversion Factor}} = \frac{(S_0 - I)e^{rT} - \text{Accrued Interest}}{\text{Conversion Factor}}$

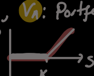
Long-Hedge For Purchase on Asset: **F_1**: \$Future at hedge time, **F_2**: \$Future at Asset Purchase, **S_2**: Asset price at Purchase, **b_2**: Basis at purchase : $S_2 - F_2$

Asset cost: S_2, **Gain on Futures**: $F_2 - F_1$, **Net amount paid:** $S_2 - (F_2 - F_1) = F_1 + b_2$. **Short-Hedge:** **Gain on Futures: $F_1 - F_2$**, **Net Amount:** $S_2 + (F_1 - F_2) =$ **$F_1 + b_2$**

(Fwd) **Optimal Hedge Ratio:** $\rho\frac{\sigma_S}{\sigma_F}$ **σ_S**: σ of $\Delta\$S$, **$\sigma_F$**: σ of $\Delta\$F$ during hedge period, **ρ: $\rho(\Delta S, \Delta F)$**, When settled-daily: **N^*** $=\hat{\rho}\frac{\hat{\sigma}_S S Q_A}{\hat{\sigma}_F F Q_F}$, **INDEX-Future Hedge:**

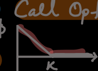
$\beta\frac{V_A}{V_F}$ = Number of contracts that should be shorted, **V_A**: Portfolio Value, **V_F**: Value of 1 F. **Options:** **Call Option** If $S_T \geq K$, **payout** = max $\{S_T - K, 0\}$

Put Option: **payout** = max $\{K - S_T, 0\}$, **Long-Call**

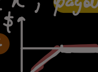


Short-Call

, **Long-Put**



, **Short-Put**



Put-Call Parity:

Buy-call ∧ Sell-Put = Fwd-Payoff $\Longrightarrow$ **$c - P = S_0 - Ke^{-rt}$**. **Option Value** = **Intrinsic value** + **Time value**. **D**: dividends expected in PV, **S_0**: current \$S

	E.call	E.put	A.CALL	A. PUT
S_0	+	−	+	−
K	−	+	−	+
T	?	?	+	+
σ	+	+	+	+
r	+	−	+	−
D	−	+	−	+

(N-D-P-S) **Bounds:** $S_0 - K < S_0 - Ke^{-rT} \leq c = C \leq S_0$, $\max\{Ke^{-rT} - S_0, 0\} \leq p \leq Ke^{-rT}$, $\max\{K - S_0, 0\} \leq P \leq K$, **Put-Call:** $c + Ke^{-rT} = p + S_0$

(D-P-S) **Bounds:** $\max\{D + Ke^{-rt} - S_0, 0\} \leq p$, **Put-Call:** (E) **$c + D + Ke^{-rT} = p + S_0$**, (A) **$S_0 - D - K \leq C - P \leq S_0 - Ke^{-rT}$**, **Trading Option on Under. Stock:**

(Covered Call) (protective put)

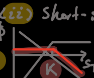
(i) long stock, short (E) c. **(ii)** Short-stock, long (E) c. **(iii)** long-stock, long (E) p **(iv)** Short-stock, Short-(E) p

(i)

(ii)

(iii)

(iv)



. **Bull:** Buy $c^{(p)} - K_1 - T$, Sell $c^{(p)} - K_2 - T$ ($K_1 < K_2$), **Bear:** Buy $c^{(p)} - K_2 - T$, Sell $c^{(p)} - K_1 - T$ ($K_2 > K_1$), **Box:** Bull-call ∧ Bear-put ($K_2 > K_1$)

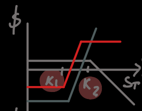
Butterfly: Buy $c^{(p)} - K_1 - T$, Buy $c^{(p)} - K_3 - T$, Sell $2 - c^{(p)} - K_2 - T$, $K_1 < K_2 = \frac{K_1 + K_3}{2} < K_3$. **Straddle:** Buy $c - K - T$ ∧ $p - K - T$. **Bull:** Limits upside ∧ downside risk, Benefit if $S_T\uparrow$

Bear: Limits upside-d. risk, Benefit $S_T\downarrow$, **Butterfly:** Profit if S_T close to K_2, good ΔS_{T_2} small. **Stradde:** Significant profit if $|K - S_T|$ big.

(European, Ignoring Option Prices)

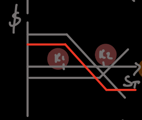
Bull-Spread (Calls)

S_T Range	Long Call (K_1)	Short Call (K_2)	Payoff
$S_T \le K_1$	0	0	0
$K_1 \le S_T \le K_2$	$S_T - K_1$	0	$S_T - K_1$
$K_2 \le S_T$	$S_T - K_1$	$K_2 - S_T$	$K_2 - K_1$



Bear-Spread (Calls)

S_T Range	Long Call (K_2)	Short Call (K_1)	Payoff
$S_T \le K_1$	0	0	0
$K_1 \le S_T \le K_2$	0	$K_1 - S_T$	$K_1 - S_T$
$K_2 \le S_T$	$S_T - K_2$	$K_1 - S_T$	$K_1 - K_2$



Box Spread

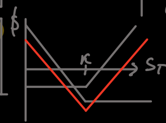
If (E) payoff = $(K_2 - K_1)e^{-r\tau}$

S_T Range	Long Call (K_2)	Short Call (K_1)	Payoff
$S_T \le K_1$	0	$K_2 - K_1$	$K_2 - K_1$
$K_1 \le S_T \le K_2$	$S_T - K_1$	$K_2 - S_T$	$K_2 - K_1$
$K_2 \le S_T$	$K_2 - K_1$	0	$K_2 - K_1$

Straddle

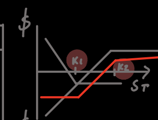
↑$ if $|S_T - K| >> $

S_T Range	Long Call (K_2)	Short Put (K_1)	Payoff
$S_T \le K$	0	$K - S_T$	$K - S_T$
$K \le S_T$	$S_T - K$	0	$S_T - K$



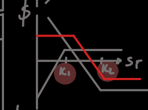
Bull-Spread (Puts)

S_T Range	Long Put (K_1)	Short Put (K_2)	Payoff
$S_T \le K_1$	$K_1 - S_T$	$S_T - K_2$	$K_1 - K_2$
$K_1 \le S_T \le K_2$	0	$S_T - K_2$	$S_T - K_2$
$K_2 \le S_T$	0	0	0

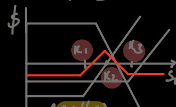


Bear-Spread (Puts)

S_T Range	Long Put (K_1)	Short Put (K_2)	Payoff
$S_T \le K_1$	$K_2 - S_T$	$S_T - K_1$	$K_2 - K_1$
$K_1 \le S_T \le K_2$	$K_2 - S_T$	0	$K_2 - S_T$
$K_2 \le S_T$	0	0	0

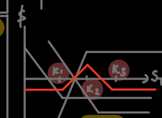


Butterfly Spread



(calls)

S_T Range	Long c-K_1	Long c-K_3	Short 2-c-K_2	Payoff
$S_T < K_1$	0	0	0	0
$K_1 < S_T \le K_2$	$S_T - K_1$	0	0	$S_T - K_1$
$K_2 < S_T < K_3$	$S_T - K_1$	0	$2(K_2 - S_T)$	$K_3 - S_T$
$K_3 \le S_T$	$S_T - K_1$	$S_T - K_3$	$2(K_2 - S_T)$	0



(puts)

S_t : price of the underlying, K : Strike, $T-t$: time to expiry, σ^2 : volatility of the underlying stock, r : risk free rate,