X: CRV. pdf: $f(x)$, $\mathbb{P}(X\in B)=\int_B f(x)dx$; CDF: $F(x)=\mathbb{P}(X\le x)=\int_{-\infty}^{x} f(z)dz$. $f(x)=\frac{d}{dx}F(x)$. $E(X)=\int_{-\infty}^{\infty} x f(x)dx$, $E(g(x))=\int g(x)f(x)dx$. Exponential D. $X\sim Exp(\lambda)$: $x\ge 0$, $(\lambda>0)$ pdf: $f(x)=\begin{cases}\lambda e^{-\lambda x}, & x\ge 0\\ 0, & \text{otherw.}\end{cases}$

CDF: $F(x)=\int_0^x \lambda e^{-\lambda z}dz=1-e^{-\lambda x}$. $X\sim Exp(\lambda)\Longrightarrow E(X)=1/\lambda$, $Var(X)=1/\lambda^2$. $X\sim Poisson(\lambda)$, then $T:=$ time until $1°$ $\therefore T\sim Exp(\lambda)$. Memory Less: For $t,s>0$ $X\sim Exp(\lambda)$, $\mathbb{P}(X>t+s \mid X>t)=\mathbb{P}(X>s)$

Normal D: $X\sim N(\mu,\sigma^2)$ if: $x\in\mathbb{R}$, pdf: $f(x)=\frac{1}{\sqrt{2\pi}\sigma}\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, $E(x)=\mu$, $Var[x]=\sigma^2$. Standard N.D: $Z\sim N(0,1^2)$. $\Phi(x)=\mathbb{P}(X\le x)$, $x\sim N(0,1)$. $\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}\cdot e^{-z^2/2}dz$. $\mathbb{P}(X\ge x)=\mathbb{P}(X\le -x)$, $1-\Phi(x)=\Phi(-x)$

If $X\sim N(\mu,\sigma^2)$, then $Y=ax+b\sim N(a\mu+b, a^2\sigma^2)$. If $Y=\frac{X-\mu}{\sigma}$, $Y\sim N(0,1)$. for n big: Binomial$(n,p)\approx N(np, np(1-p))$. Schwarz T.: $\frac{\partial^2}{\partial y\partial x}f(x,y)=\frac{\partial^2}{\partial x\partial y}f(x,y)$. $\iint_B f(x,y)dxdy=\int_a^b\int_{h(y)}^{h(y)} f(x,y)\,dx\,dy$

$x=r\cos\theta$ $dx\,dy=r\,dr\,d\theta$

$y=r\sin\theta$ $r=\sqrt{x^2+y^2}$. DRV: $\langle X,Y\rangle$, $x\in\{x_1,\dots,x_n\}$, $y\in\{y_1,\dots,y_n\}$. Joint pmf: $P_{ij}=\mathbb{P}(X=x_i, Y=y_j)$. $P_{ij}\ge 0$, $\sum_j\sum_i P_{ij}=1$. Joint cdf: $F(x,y)=\mathbb{P}(X\le x, Y\le y)=\sum_{x_i\le x}\sum_{y_j\le y}\mathbb{P}(X=x_i, Y=y_j)$. CRV: $\langle X,Y\rangle$, Joint pdf: $f(x,y)$ s.t.

$f(x,y)\ge 0$

$\mathbb{P}((X,Y)\in B)=\iint_B f(x,y)\,dx\,dy$ $\iint_{\mathbb{R}^2} f(x,y)dxdy=1$. Joint cdf: $F(x,y)=\mathbb{P}(X\le x, Y\le y)=\int_{-\infty}^{y}\int_{-\infty}^{x} f(u,v)\,du\,dv$, $f(x,y)=\frac{\partial^2}{\partial x\partial y}F(x,y)$. $(X,Y)\sim$ Uniform over B. $f(x,y)=\begin{cases}1/\text{Area}(B), & (x,y)\in B\\ 0, & \text{otherwise}\end{cases}$ $\therefore \mathbb{P}((X,Y)\in A)=\frac{\text{area } A}{\text{area } B}$

RV X,Y independent if $\forall A,B\subset\mathbb{R}$, $\mathbb{P}(X\in A, Y\in B)=\mathbb{P}(X\in A)\mathbb{P}(Y\in B)$. Let $F(x,y)=\mathbb{P}(X\le x, Y\le y)$ joint cdf of (X,Y), $F_X(x)$, $F_Y(y)$ cdf's for X,Y respectively $\Rightarrow$ X,Y independent $\Longleftrightarrow F(x,y)=F_X(x)F_Y(y)$

(X,Y) discrete, X,Y independent $\Longleftrightarrow \forall(x_i,y_j)$, $\mathbb{P}(X=x_i, Y=y_j)=\mathbb{P}(X=x_i)\mathbb{P}(Y=y_j)$; (X,Y) continuous: X,Y independent $\Longleftrightarrow \forall(x,y)$, $f(x,y)=f_X(x)f_Y(y)$, or show: $\exists g,h$: $f(x,y)=g(x)h(y)$.

Not Ind if boundaries depend on each other. $X\sim Poisson(\lambda)$, $Y\sim Poisson(\mu)$, X,Y ind. $X\sim Exp(\lambda)$, $Y\sim Exp(\lambda)$, X,Y ind. Continuous $E(h(x,y))=\iint_{\mathbb{R}^2}h(x,y)f(x,y)dxdy$. D: $\sum_i\sum_j h(x_i,y_j)\mathbb{P}(X=x_i, Y=y_j)$

$\mathbb{E}[ah_1(X,Y)+bh_2(X,Y)]=aE(h_1(X,Y))+bE(h_2(X,Y))$. If X,Y ind. and $h(x,y)=h_1(x)h_2(y)$ $\therefore E(h(X,Y))=E(h_1(X))\cdot E(h_2(Y))$. Covariance: $Cov(X,Y)=E((X-E(X))(Y-E(Y)))=E(XY)-E(X)E(Y)$

(i) $Y=X$: $Cov(X,Y)=Var(X)$. (ii) X,Y ind: $Cov(X,Y)=0$. Correlation X,Y. $\rho_{XY}=\frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$, $-1\le\rho\le 1$. $Y=aX+b$, $\rho_{XY}=1$; $Y=-aX+b$, $\rho_{XY}=-1$. X,Y ind. $\underset{\not\Leftarrow}{\Longrightarrow}$ $Cov(X,Y)=\rho_{XY}=0$. Let (X,Y) r. vector $\Longrightarrow$

$E(X+Y)=E(X)+E(Y)$; $Var(X+Y)=Var(X)+Var(Y)-2Cov(X,Y)$. Theorem: $E(X_1+\dots+X_n)=\sum_{i=1}^{n}E(X_i)$, $Var(X_1+\dots+X_n)=\sum_{i=1}^{n}Var(X_i)+2\sum_{i<j}Cov(X_i,X_j)$. If $X_1,\dots,X_n$ ind: $Var(X_1+\dots+X_n)=\sum_{i=1}^{n}Var(X_i)$

$X_1,\dots,X_n$ ind. $\sim$ Bernoulli(p), $\mathbb{P}(X_i=1)=p$, $\mathbb{P}(X_i=0)=1-p$ $\therefore X=\sum_{i=1}^{n}X_i\sim Bin(n,p)$. Indicator: $X_i=\begin{cases}1, & \exists A\\ 0, & !A\end{cases}$. Marginal Distribution: M. pmf: Discrete $\mathbb{P}(X=x_i)=\sum_j\mathbb{P}(X=x_i, Y=y_j)$, $\mathbb{P}(Y=y_j)=\sum_i\mathbb{P}(X=x_i, Y=y_j)$

Continuous: M pdf: $f_X(x)=\int_{-\infty}^{\infty}f(x,y)dy$, $f_Y(y)=\int_{-\infty}^{\infty}f(x,y)dx$. M. cdf: $F_X(x)=\mathbb{P}(X\le x)=F(x,\infty)$. Conditional D: Discrete C. pmf of X given $Y=y_j$. $\mathbb{P}(X=x_i\mid Y=y_j)=\frac{\mathbb{P}(X=x_i, Y=y_j)}{\mathbb{P}(Y=y_j)}=\frac{\text{Joint pmf}}{\text{marginal pmf}}$

continuous: $f(x\mid y)=\frac{f(x,y)}{f_Y(y)}=\frac{\text{J. pdf}}{\text{M. pdf}}$. If X,Y ind: Discrete $\mathbb{P}(X=x_i\mid Y=y_j)=\mathbb{P}(X_i)$, continuous: $f(x\mid y)=f_X(x)$; $\mathbb{P}(X\le x\mid Y=y)=\sum_{x_i\le x}\mathbb{P}(X=x_i\mid Y=y)$, $\int_{-\infty}^{x}f(u\mid y)du$. $E(X\mid Y=y)=\sum_{x_i}x_i\mathbb{P}(X=x_i\mid Y=y)$

$\int_{\mathbb{R}}x f(x,y)dx$; Given $Y>y$: $F(x)=\mathbb{P}(X\le x\mid Y>y)=\frac{\mathbb{P}(X\le x, Y>y)}{\mathbb{P}(Y>y)}$, (ii) $Y=y$: $F(x)=\mathbb{P}(X\le x\mid Y=y)=\frac{\mathbb{P}(X\le x, Y=y)}{\mathbb{P}(Y=y)}$. Conditional Expectation: D: $E(h(Y)\mid X=x)=\sum_j h(y_j)\mathbb{P}(Y=y_j\mid X=x)$, C: $\int_{\mathbb{R}}h(y)f(y\mid x)dy$

$E[\alpha X+\beta Y\mid Z=z]=\alpha E[X\mid Z=z]+\beta E[Y\mid Z=z]$, $E[g(x)Y\mid X=x]=g(x)E[Y\mid X=x]$. If X,Y ind $E[Y\mid X=x]=E[Y]$ $\forall x$. $E[g(X)Y\mid X]=g(X)E[Y\mid X]$. X,Y ind. $E(Y\mid X)=E[Y]$

Tower Property: $E[E[Y\mid X]]=E[Y]$; $E[X]=E[E[X\mid Y]]=\sum_{i=1}^{n}E[X\mid Y=i]\mathbb{P}(Y=i)$. $X=(X_1,\dots,X_d)$ standard Normal $\Leftrightarrow X_1,\dots,X_d$ ind. Def: $X_1,\dots,X_n$ iid from D then $\{X_1,\dots,X_n\}$ random sample of size n

For $\{X_1,\dots,X_n\}$ iid from D: LLN: $E(X_i)=\mu$, then $\lim_{n\to\infty}\frac{X_1+\dots+X_n}{n}=\mu$. (ii) CLT: $E(X_i)=\mu$, $Var[X_i]=\sigma^2$ then $\lim_{n\to\infty}\frac{X_1+\dots+X_n-n\mu}{\sqrt{n}\sigma}=N(0,1)$. $\therefore \lim_{n\to\infty}\mathbb{P}\left(\frac{X_1+\dots+X_n-n\mu}{\sqrt{n}\sigma}\le y\right)=\Phi(y)$. For sample mean $\bar{X}_n=\frac{\sum_{i=1}^{n}X_i}{n}$, $E[\bar{X}_n]=\frac{1}{n}E[X_1+\dots+X_n]$

$\frac{1}{n}(E(X_1)+\dots+E(X_n))=1/n\cdot n\mu=\mu$, $Var(\bar{X}_n)=\frac{\sigma^2}{n}$. M1: $\mathbb{P}(A)=\sum_{i}^{n}\mathbb{P}(A\mid B_i)\mathbb{P}(B_i)$; $\mathbb{P}(B\mid A)=\frac{\mathbb{P}(A\mid B)\mathbb{P}(B)}{\mathbb{P}(A)}$; $\mathbb{P}(B_k\mid A)=\frac{\mathbb{P}(B_k\cap A)}{\mathbb{P}(A)}=\frac{\mathbb{P}(A\mid B_k)\mathbb{P}(B_k)}{\sum\mathbb{P}(A\mid B_i)\mathbb{P}(B_i)}$; $Var[X]=E[(X-\mu)^2]=\sum(x_i-\mu)^2\mathbb{P}(X=x_i)=E[X^2]-(E[X])^2$; $Var(aX+b)=a^2Var[X]$

$X\sim$ Bernoulli(p), $x\in\{0,1\}$, $\mathbb{P}(X=1)=p$, $\mathbb{P}(X=0)=1-p$; $X\sim Bin(n,p)$; $x\in\{0,1,\dots,n\}$, $\mathbb{P}(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$. $E[X]=np$, $Var[X]=np(1-p)$; $X\sim Geo(p)$, $x\in\{1,2,\dots\}$; $\mathbb{P}(X=k)=p(1-p)^{k-1}$, $E[X]=1/p$; $Var[X]=\frac{1-p}{p^2}$, $\mathbb{P}(X>n)=(1-p)^n$

$X\sim Poisson(\lambda)$, $\lambda>0$; $x\in\{0,1,2,\dots\}$, $\mathbb{P}(X=k)=e^{-\lambda}\frac{\lambda^k}{k!}$. $E[X]=\lambda$, $Var(X)=\lambda$. $X\sim$ Uniform(a,b) $f(x)=1/b-a$ $\Longrightarrow E[X]=\frac{b+a}{2}$, $Var[X]=\frac{(a-b)^2}{12}$. LLN: $\bar{X}_n\xrightarrow[n\to\infty]{}\mu$; $\bar{X}_n\sim N(\mu,\sigma^2/n)$ $\bar{X}_n\sim N\left(p,\frac{p(1-p)}{n}\right)$ $n\longrightarrow\infty$. $X=n\bar{X}_n\sim N(np, p(1-p))$

$U_n=\frac{\sum_{i}^{n}y_i-n\mu}{\sigma\sqrt{n}}=\frac{\bar{y}-\mu}{\sigma/\sqrt{n}}$, $\lim_{n\to\infty}\mathbb{P}(U_n\le u)=\int_{-\infty}^{u}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt$ $\forall u$; $\therefore\mathbb{P}(\bar{Y}\le a)=\mathbb{P}\left(\frac{\bar{y}-\mu_{\bar{y}}}{\sigma/\sqrt{n}}\le\frac{a-\mu_{\bar{y}}}{\sigma/\sqrt{n}}\right)=\Phi\left(\frac{a-\mu_{\bar{y}}}{\sigma/\sqrt{n}}\right)$. y_1, y_2 c.r.v. joint density f: $f(y_1,y_2)$ and

marginal d. f. $f_1(y_1)$, $f_2(y_2)\Longrightarrow y_1,y_2$ ind. iff $f(y_1,y_2)=f_1(y_1)f_2(y_2)$ $\therefore$ if y_1,y_2 ind. $f(y_2\mid y_1)=f_2(y_2)$, $f(y_1\mid y_2)=f_1(y_1)$. $V(f_1)=\int_{-\infty}^{\infty}(x-E(x))^2f(x)dx$

$E[Y_1]=\int_{-\infty}^{\infty}x f(x)dx$. Let $y_1,y_2,\dots,y_n$ and $x_1,\dots,x_m$ r.v. s.t. $E[y_i]=\mu_i$, $E[X_j]=\xi_j$. Define $U_1=\sum^{n}a_iy_i$, $U_2=\sum^{n}b_jx_j$. $E[U_1]=\sum^{n}a_i\mu_j$.

$V(U_1)=\sum^{n}a_i^2V(y_i)+2\sum\sum_{1\le i<j\le n}a_ia_jCov(y_i,y_j)$. $Cov(U_1,U_2)=\sum^{n}\sum^{m}a_ib_jCov(Y_i,X_j)$. $X\sim$ Uniform$(0,\theta)$ $\therefore\mathbb{P}(X\ge U)=(\theta-\mu)$, $\mathbb{P}(X\le U)=\theta$. $E[Y^2]=Var[Y]+(E[Y])^2$

χ^2 (chi-square) d.: Sample V.: $s^2=\frac{1}{n-1}\sum^{n}(X_i-\bar{X}_n)^2$; if $X_1,\dots,X_n$ iid $N(\mu,\sigma^2)\Rightarrow\frac{s^2(n-1)}{\sigma^2}\sim\chi^2(n-1)$ freedom; θ-parameter of D. t-distribution: Th) $X_1,\dots,X_n\overset{iid}{\sim}N(\mu,\sigma^2)$ then $T=\frac{(\bar{X}_n-\mu)}{s/\sqrt{n}}\sim t(n-1)$

$T(n-1)\approx N(0,1)$; statistical

$s=\sqrt{s^2}=\sqrt{\frac{1}{n-1}\sum(X_i-\bar{X}_n)^2}\neq\sigma$; $n\to\infty$ (if s close to σ) $\hat{\theta}$ r.v. Inference: sample: $X_1,\dots,X_n\overset{iid}{\sim}D$. Point Estimate $\hat{\theta}$: $T(X_1,\dots,X_n)$ s.t $\hat{\theta}\approx\theta$. $\hat{\theta}\longrightarrow\theta$. Interval e. $[\hat{\theta}_L,\hat{\theta}_U]$, s.t. $\mathbb{P}(\hat{\theta}_L\le\theta\le\hat{\theta}_U)$. Hypothesis T: H_0: $\theta=\theta_0$

H_1: $\theta\neq\theta_0$ P.E: $X_1,\dots,X_n\sim D(\theta)$. Find $\hat{\theta}=T(X_1,\dots,X_n)$ s.t $\hat{\theta}\approx\theta$. $\hat{\theta}$ consistent if $n\to\infty$. Bias: $B[\hat{\theta}]=E[\hat{\theta}]-\theta$. $\hat{\theta}$ unbiased if $B[\hat{\theta}]=0$. $MSE[\hat{\theta}]=E[(\hat{\theta}-\theta)^2]=(B[\hat{\theta}])^2+Var[\hat{\theta}]$ $\therefore$ if $\hat{\theta}$ unb. $MSE[\hat{\theta}]=V(\hat{\theta})$

$Var[\bar{X}_n]=\frac{Var(X_i)}{n}$ c. unb. est. Parameter: pop. m μ; Estimate s. m $\bar{X}_n$; $\bar{X}_n\to\mu$ $\hat{\theta}$ m.o.m consistent (LLN) $\bar{X}_n\overset{CLT}{\sim}N(\mu,\sigma^2/n)$; ρ: σ^2; e: s.v. $s^2\approx\frac{1}{n-1}\sum^{n}(X_i-\mu)^2\xrightarrow{LLN}E[(X_i-\mu)^2]=\sigma^2$; D=Bern(p), p: pop, fraction: p e: $\bar{X}_n$ sample mean

Method of Moments: [continuous] $n\longrightarrow\infty$: $\mu(\hat{\theta})=\bar{X}_n\Rightarrow$ estimator. 1°: $\bar{X}_n\xrightarrow{LLN}\mu$; 2°: $\frac{1}{n}\sum^{n}X_i^2\xrightarrow{LLN}E[X_i^2]=\sigma^2+\mu^2$ $\therefore\sigma^2\approx\left(\frac{1}{n}\sum^{n}X_i^2-\mu^2\right)$ $\therefore\hat{\sigma}=\sqrt{\frac{1}{n}\sum^{n}X_i^2-(\bar{X}_n)^2}$. Method Max L.: [Discrete] $L_\theta(X_1,\dots,X_n)=\prod^{n}f_\theta(X_i)$ f: pmf(X_i)

f: pdf(X_i) $L_\theta(X_1,\dots,X_n)=\prod^{n}f_\theta(X_i)$; Def: $X_1,\dots,X_n\sim D(\theta)$, $\hat{\theta}=\theta$ s.t. maximizes $L_\theta(X_1,\dots,X_n)\Longrightarrow\hat{\theta}$: M.L.E of θ. $X_1,\dots,X_n\overset{iid}{\sim}B(p)$. $P_\theta(x)=\begin{cases}p, & x=1\\ 1-p, & x=0\end{cases}=p^x(1-p)^{1-x}$, $x=0,1$; $L_\theta=\prod_{i=1}^{n}p^{X_i}(1-p)^{1-X_i}=p^{\sum X_i}(1-p)^{n-\sum X_i}$

$\log(L_\theta)=\sum^{n}X_i\log p+(n-\sum^{n}X_i)\log(1-p)$ $\therefore\frac{\partial}{\partial p}\log L_\theta=\sum X_i\frac{1}{p}-(n-\sum X_i)\frac{1}{1-p}=0$; $\sum^{n}X_i=np\Rightarrow\hat{p}=\bar{X}_n$; $X_1,\dots,X_n\overset{iid}{\sim}$ Poisson(λ): $P_\theta(x)=\frac{\lambda^x}{x!}e^{-\lambda}\Longrightarrow\hat{\lambda}=\bar{X}_n$; $U(0,L)$: $f_\theta(L)=\begin{cases}1/L, & 0\le x\le L\\ 0, & \text{otherw.}\dots\end{cases}\Rightarrow\hat{L}=\max\{X_1,\dots,X_n\}$

For θ_1,θ_2 set: $\begin{cases}\frac{\partial L_\theta(X_1,\dots,X_n)}{\partial\theta_1}=0\\ \frac{\partial L_\theta(X_1,\dots,X_n)}{\partial\theta_2}=0\end{cases}$. $X_1,\dots,X_n\overset{iid}{\sim}f(x\mid\theta)$. $\hat{\theta}_{MLE}\overset{def}{=}\text{argmax}\left\{\prod^{n}f(X_i\mid\theta)\right.$. $\vec{X}=(X_1,\dots,X_n)$; $L(\theta\mid\vec{X})=\prod^{n}f(X_i\mid\theta)$; $\ell(\theta\mid\vec{X})=\log(L(\theta\mid\vec{X}))=\sum^{n}\log f(X_i\mid\theta)$; $\hat{\theta}_{MLE}$ solution to $\frac{\partial}{\partial\theta}\ell(\theta\mid\vec{x})=0$

Fisher Inf: $X\sim f(x\mid\theta)\Rightarrow\frac{\partial}{\partial\theta}\log f(x\mid\theta)$ is random; $E\left[\frac{\partial}{\partial\theta}\log f(x\mid\theta)\right]=E\left[\frac{\frac{\partial}{\partial\theta}f(x\mid\theta)}{f(x\mid\theta)}\right]=\int\frac{\partial}{\partial\theta}f(x\mid\theta)dx=\frac{\partial}{\partial\theta}\int f(x\mid\theta)dx=\frac{\partial}{\partial\theta}1=0$ $\therefore I(\theta)=Var\left(\frac{\partial}{\partial\theta}f(x\mid\theta)\right)=E\left[\left(\frac{\partial}{\partial\theta}f(x\mid\theta)\right)^2\right]=-E\left(\frac{\partial^2}{\partial\theta^2}\log f(x\mid\theta)\right)$

Asympt. Normal. of MLE: $X_1,\dots,X_n\overset{iid}{\sim}f(x\mid\theta_0)$ (true p.); $\sqrt{n}(\hat{\theta}_{MLE}-\theta_0)\xrightarrow{n\to\infty}N(0,1/I(\theta))$. Find $\hat{\theta}_L,\hat{\theta}_U$ st. $\mathbb{P}([\hat{\theta}_L,\hat{\theta}_U]$ covers true $\theta)=1-\alpha$, $0<\alpha<1$ confid. coeff. $1-\alpha=90\%\to Z_{\alpha/2}=Z_{0.05}$; $95\%\to Z_{0.025}$; $98\%\to Z_{0.01}$; $99\%\to Z_{0.005}$

Th: given $X_1,\dots,X_n$ iid s.t. $E(X_i)=\mu$, $Var[X_i]=\sigma^2$ size $=n$ $(1-\alpha)$ C. I. EXPECT for μ is $\approx[\bar{X}_n\pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}]$; $Z_{\alpha/2}$ s.t. $\Phi(-Z_{\alpha/2})=\frac{\alpha}{2}$, σ estimated from $s=\sqrt{s^2}=\sqrt{\frac{1}{n-1}\sum^{n}(X_i-\bar{X}_n)^2}$

Mean C.I. $x\%$: Given 100 CIs based on 100 samples, x to cover μ. Spec. Case: D= $B(p)$: $\hat{p}=\bar{X}_n$, $\mu=p$, $\sigma^2=p(1-p)$ $\therefore(1-\alpha)$ C.I for p: $[\hat{p}\pm Z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$ where $\sqrt{p(1-p)}$ ESTIMATED: via $\sqrt{\hat{p}(1-\hat{p})}$

μ_X,σ_X^2 μ_Y,σ_Y^2

$X_1,\dots,X_n\overset{iid}{\sim}D_1(\theta_1)$, $y_1,\dots,y_m\overset{iid}{\sim}D_2(\theta_2)$; $\bar{X}_n-\bar{Y}_m\sim N\left(\mu_X-\mu_Y,\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}\right)$ $\therefore$ CI for $\mu_X-\mu_Y$: $[\bar{X}_n-\bar{Y}_m\pm Z_{\alpha/2}\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}\approx\bar{X}_n-\bar{Y}_m\pm Z\sqrt{\frac{S_X^2}{n}+\frac{S_Y^2}{m}}$. C.I. for p_1-p_2: $\hat{p}_1-\hat{p}_2\pm Z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n}+\frac{\hat{p}_2(1-\hat{p}_2)}{m}}$

$\approx\hat{p}_1-\hat{p}_2\pm Z\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n}+\frac{\hat{p}_2(1-\hat{p}_2)}{m}}$. Hypothesis T: 1° H_0: $p=20\%$; H_1, H_a: $p\neq 20\%$, $H_0\cap H_a=\phi$; 2° Find "test statistic" $\hat{p}$ (point estimate); 3° report p-value $=\mathbb{P}$(Assume H_0 true, observe something (at least as extreme) as the observation)

$\hat{p}\sim N\left(p,\frac{p(1-p)}{n}\right)$; p-value $=P(\hat{p}\ge p_0)=(1-\Phi(c))$ if $\approx 0\Rightarrow$ Not likely if H_0 true $\therefore$ Reject H_0, accept H_a: $p\neq 20\%$. Sign. level: $\alpha\approx 5\%, 1\%$. If p-value $\le\alpha$ reject H_0.

RR: $\{|\hat{p}-20\%|\ge c\}$; $c=Z_{\alpha/2}\times\sigma$. $\hat{p}\sim N\left(p,\frac{p(1-p)}{n}\right)$, if $\hat{p}\in RR$ reject H_0, accept H_a: $p\neq 20\%$. Two sample $\mu_1-\mu_2$, p_1-p_2. I-ERROR: H_0 TRUE BUT REJECTED, II-ERROR... $\mathbb{P}(RR, H_0\text{ TRUE})=\alpha$ $\longrightarrow$ $\mathbb{P}(!RR, H_0\text{ true})$